

Graphic Logarithmic Tables:

A Picture Is Worth A Million ... Numbers¹

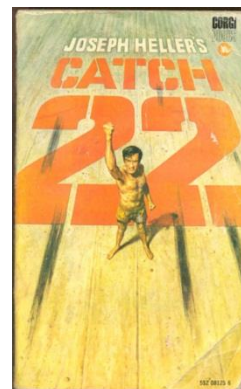


Given what they replaced it seems disparaging, impolite and almost blasphemous to point out that logarithmic tables were sadly error prone and irritatingly awkward to use!

Before John Napier (1550-1617) invented logarithms most forms of computation were enormously time-consuming. In fields like navigation and astronomy it took elite mathematicians of the day² literally years to finish some complex calculations. So before logarithmic tables astronomers, who had to work with very large numbers, typically spent endless days, if not weeks, doing tedious longhand multiplications and divisions. So what on earth could there be about logarithmic tables to complain about?

Catch-22

The work on logarithms undertaken by Napier was a classic “chicken or the egg” Catch-22 style paradox³. First he had to work through all the tedious calculations required to generate all the entries needed for a set of logarithmic tables and then find a way to verify them. But the supreme irony is that once the tables existed, it would have made it much easier to do the calculations needed to create the table entries in the first place! A modern-day Catch-22 analogy would be computer programming. Once a program compiler was developed, programmers no longer had to write programs in machine code. But first someone had to write the compiler in machine code.



Napier, aided by his own “Napier bones”, did set about the enormity of the task with considerable insight and ingenuity. His first challenge was to shake off 16th century mathematical thinking of the time. Before Napier invented logarithmic tables mathematicians of the day relied on fixed sequences. For example, the fixed arithmetic sequence of 0, 1, 2, 3, 4, etc or the fixed geometric sequence of 1, 2, 4, 8, 16, etc. This worked fine when stepping through a series of whole numbers but clearly overlooked all the values in between. So Napier broke with tradition and decided to

¹ Derived from the early 20th century saying: “A Picture Is Worth a Million Words” attributed to the American, Frederick R. Barnard.

² In Napier’s day, such calculation experts were known as “Reckoners”.

³ The impossible paradox famously introduced in Joseph Heller’s 1953 book of the same name.

adopt a kinematic approach for building his tables. This way he would have “no numerical gaps” – i.e. he defined: (i) an arithmetic and continuous movement of a point along a single straight line with a constant speed AND (ii) a proportional and continuous movement of point along a straight line with a proportional speed.

The completeness of his tables was not the only “chicken or the egg” problem Napier had to overcome. He had no precedent for the degree of accuracy the values in his tables needed to have. With this dilemma came another conundrum – for a given degree of accuracy, how much work would that involve to generate the full table? So Napier came up with what today would be called a meta process. Having developed a means of calculating a series of values, he would then evaluate the values obtained to see how they could be used to generate more values i.e. a meta calculation process.

So by adopting a kinematic approach and a meta process Napier achieved an insightful balance between the desired degree of accuracy and the longhand numerical labour needed to generate the table entries.

Published errors

Inevitably a few errors did creep into Napier’s original calculations and indeed into the calculations made by later authors who devised extended versions (e.g. greater number of significant places) or designed new types of logarithmic tables. Some of the earliest errors went unnoticed and were repeated or compounded in many later publications. But even if it had been humanly possible for Napier to generate and in longhand write down all the entries needed for his logarithmic table totally error-free, many mistakes got introduced during the typesetting and printing.

When copying or duplicating a long list of numbers, apart from any other oversights, unintentionally transposing two numbers in a line of numbers is a known human weakness. Then came the mind-numbing task poorly paid typesetters had to face. Typesetting row upon row, upon row of seemingly random numbers in a printing block must have been a really thankless task. It was unintentional but understandable how transcription errors ended up in many published logarithmic tables. For example, errors made in the last decimal place were particularly difficult to spot. Some such mistakes remained unnoticed for decades and were perpetuated when inherited from one publishing house to another and logarithmic tables got reissued or republished.

Logarithmic tables can be a pain

Arguably, given the longhand nature of calculating the table entries and then the mind-numbing typesetting needed to publish them, the odd error was a small price to pay for how logarithmic tables made all kinds of multiplication and division much simpler and momentarily quicker to perform. As it turned out, the immeasurable gains that logarithmic tables made possible far outweighed the more obvious tangible benefits. Top of the list of immeasurable gains were:

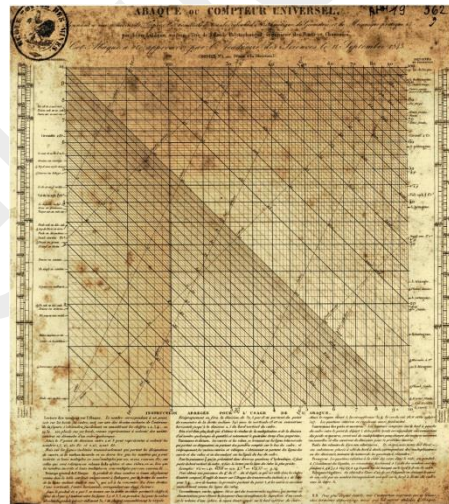
- 1. individuals other than “mathematical geniuses” could now attempt complicated calculations**
- 2. scientists and mathematicians could now (in their lifetimes) unlock and solve many mathematical conundrums which in turn lead to many advancements and discoveries in many different fields**

However, despite being a paradigm shift when compared with the old pre-logarithm ways, the very nature of the concept meant using logarithmic tables had distinct disadvantages. The ritualistic look-up process could be irritating and so long-winded that without fastidious care it could itself easily become error-prone. Also some factors commonly found in calculations could make using early logarithmic tables tricky and a real pain to use. For example, negative numbers or answers needing a high degree of accuracy - say significant to at least 7 or 8 significant places. A workaround for negative numbers was to handle the sign separately. Often the drawbacks got magnified many times over when attempting complex calculations. Finally the sheer number of times an interim solution had to be looked up in a logarithmic table became a test of concentration and patience.

So despite the many obvious advantages, logarithmic tables contained mistakes and using them was error-prone and irritating. But centuries after logarithmic tables were first published an innovative and elegant solution was found for many of the drawbacks and the tedium - **graphic logarithmic tables**.

Nomograms showed the way

Like many great ideas, its success comes from its simplicity. A nomogram is a two-dimensional diagram designed to show the approximate “graphic calculation” of a mathematical function. The most basic nomogram having two parallel outer scales representing the values of two quantities involved in a function. Where the lines joining quantities used in the calculation intersect, gives the result of the function. An early advocate of the nomogram was Frenchman *Leon Louis Lalanne* (1812-1892). In 1843, with his “Universal Calculator”, he probably created the first log-log plot.



Sadly some nomograms became so complex that the function(s) they represented were just as difficult to grasp as their inherent longhand mathematical formula. However, it is the same basic idea that a picture is easier to understand than a long list of numbers that led to graphic logarithmic tables being developed.

Graphic logarithmic tables - what are they?

They clearly look (see later compendium) different from the many, many rows and columns of similar numbers in a traditional logarithmic table. Instead they more reflect the ideas Steinhäuser had back in 1807 for a calculation aid based on rods with logarithmic scales. But the differences are much more than just an innovative printing or formatting style. In a graphic logarithmic table:

- **all the entries in the log and antilog sections of a traditional logarithmic table have been integrated into a single, much shorter combined table**
- **the look-up process is simpler, quicker and much more intuitive**

But it is important to set boundaries on what constitutes a graphic logarithmic table as they come in various shapes and sizes and many loosely related “cousins” exist. For example, a Slide Rule (in its many forms) has logarithmic scales that provide answers

without needing to resort to antilogs. However, for practical reasons the logarithmic scales normally found on slide rules only ever represent a subset of all the entries found in any traditional or graphic logarithmic table. By using a subset or leaving out rarely needed parts of the range was one of the few ways slide rule designers could usefully compact the length of a scale and ultimately, the length of the slide rule. Only the largest drum and grid-iron types had enough room for a range of values that got close to the full range of values included in any logarithmic table. Also without a precise definition a plethora of other printed calculating aids and slide charts could arguably have been called types of graphic table.

So unlike slide rules and similar calculating devices, graphic logarithmic tables still rely on a table look-up process and apart from one exception, all known examples exist as some form of printed page or more commonly, a book.

Graphic logarithmic tables – unrelated bedfellows

Using a classification scheme for all printed calculating aids seemed the best way to track and understand the development and use of graphic logarithmic tables. But they defy inclusion into any existing classification scheme for similar aids such as Ready Reckoners, Tabular Calculators, etc. To be sure the issue was raised at the *Mathematisches Forschungsinstitut*⁴ Oberwolfach's 2011 workshop: "*The history of numerical and graphical tables.*" Sadly the distinguished attendees could only confirm that graphic logarithmic tables are virtually unknown and no one had any first-hand knowledge of such tables!

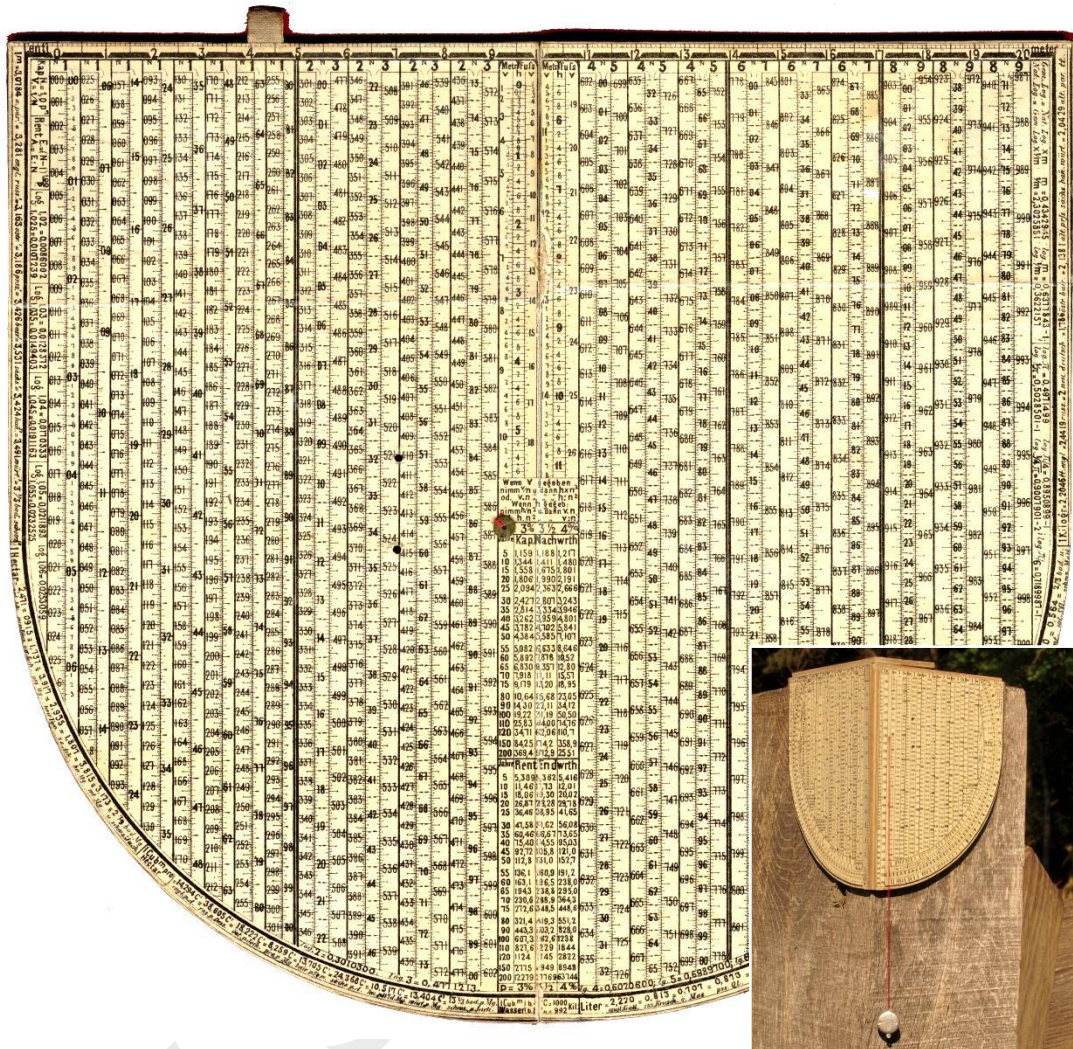
So this compendium of graphic logarithmic tables is ordered by nothing more scientific than the year they were first published. It includes all known examples but others may well exist. Many related calculations aids, often in the form of a grid-iron, have been excluded as they did not fit the strict definition for graphic logarithmic tables. For those that did meet the definition, the salient characteristics are listed with an accompanying example page or pages out of the table.

+/-1852: Pressler - Dresden, Germany

| | |
|-----------------------------|--|
| Title: | <i>Ingenieur-Messknecht</i> |
| By: | Maximilian Robert Pressler |
| Type: | Slip cased folding set of tables (with built-in clinometer) printed on escutcheon or shield shaped double-sided stiff cardboard |
| Size: | 20.7cm (longest point i.e. from the Chief to the Base) x 22.4cm (widest point i.e. from the Dexter to the Sinister) x 0.4cm |
| Published by: | Unknown |
| Patents? | None found |
| Style of table(s): | Front and side edges: mixture of tables and conversion factors Back: mainly a four-place graphic table organised in columns |
| Length(s) of graphic table: | ≈ 5.5m |
| Comments: | Probably the <u>first</u> graphic logarithmic table ever published. It was mainly intended for use in the field by forestry workers. But Pressler also claimed it was a universal aid for students |

⁴ The renowned international research centre located in the German Black Forest.

calling it a “*Mathematical Cinderella*” – possibly a cryptic reference to it being able to do all kinds of mathematical chores. Pressler also wrote a book to accompany his Messknecht. Various editions of the Messknecht and the book exist.



1893: Loewe – Bromberg, Germany

| | |
|-----------------------------|---|
| Title: | <i>Rechenscalen für numerisches und graphisches Rechnen</i> |
| By: | Loewe |
| Type: | Hardback book (50 pages) with a brown cover |
| Size: | 23.3cm x 16.3cm |
| Published by: | Verlag des Technischen Versandgeschäfts R. Reiss ⁵ |
| Patents? | None found |
| Style of table(s): | (i) 5-page four-place graphic table organised in columns (ii) Three other tables for trigonometrical functions |
| Length(s) of graphic table: | ≈ 10.4 metres |
| Comments: | Interestingly Loewe also includes “ <i>how to calculate</i> ” instructions for using a pair of dividers. |

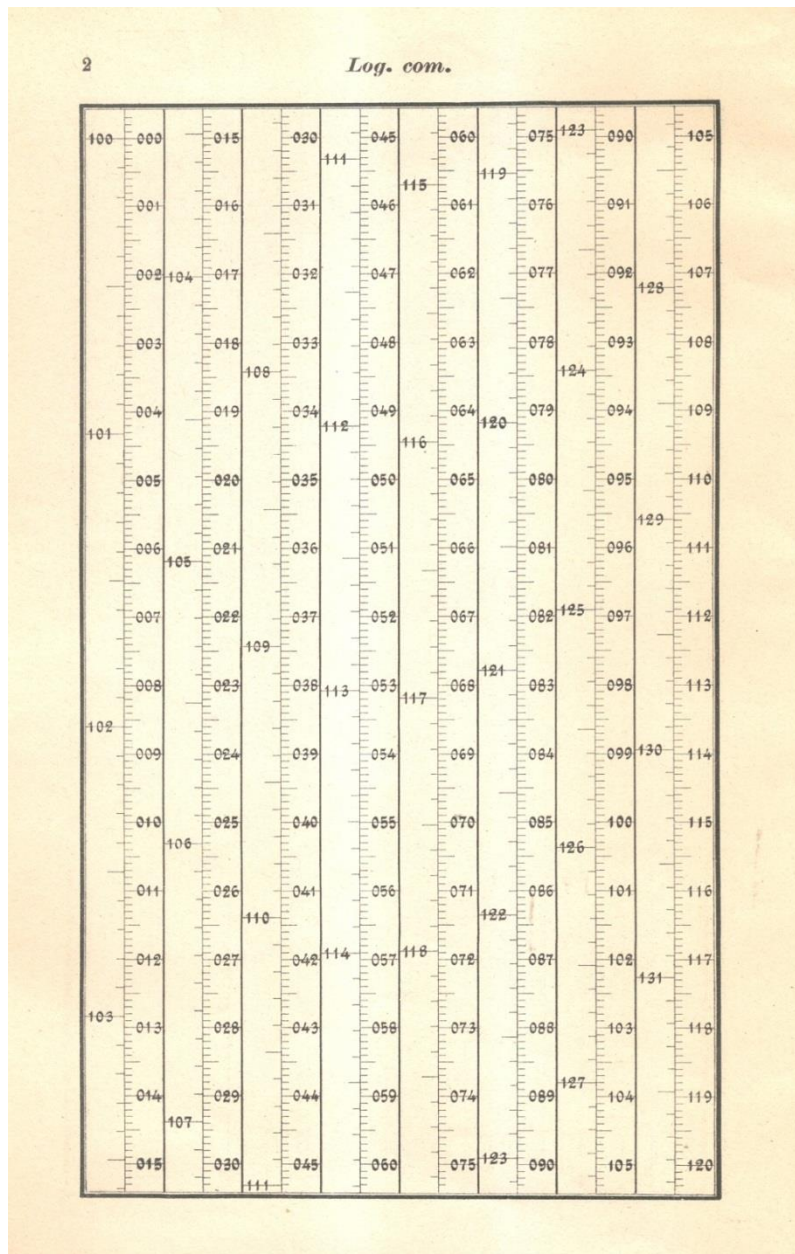
⁵ Nearly two decades later, in 1912, the same Reiss started making slide rules.

100—251
5°—14°

— 3 —

1897: Tichy – Wien, Austria

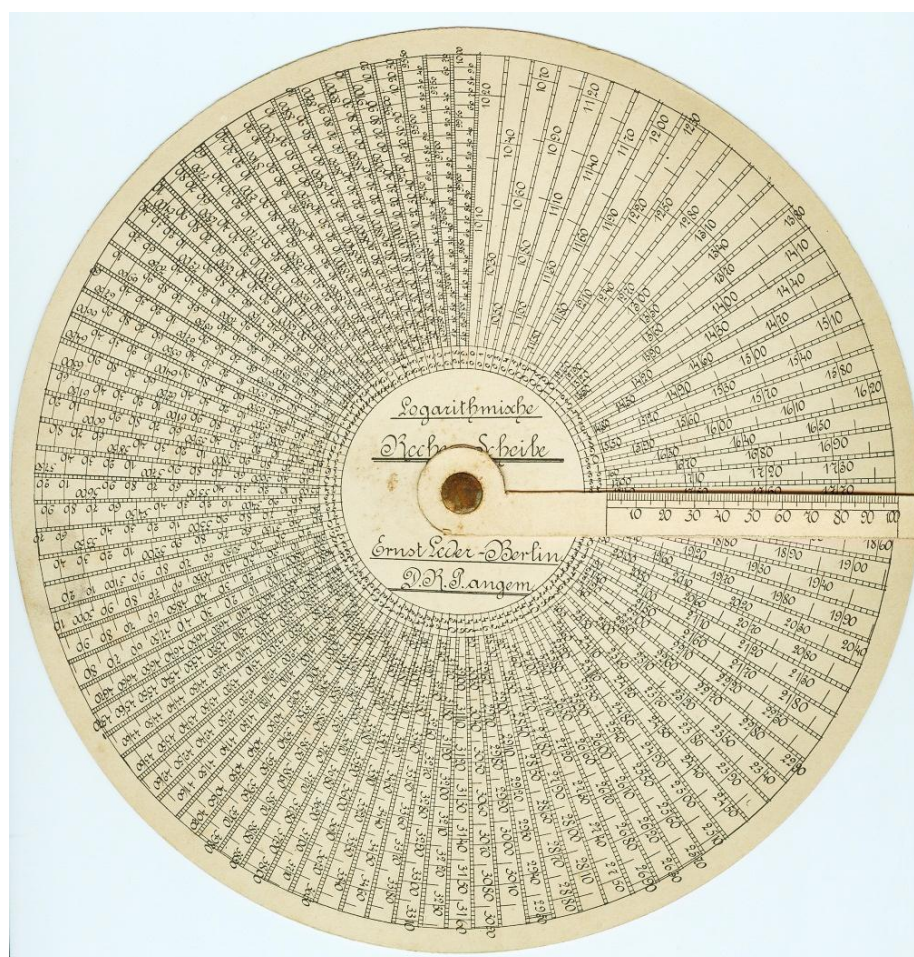
Title: *Graphische Logarithmen-Tafeln*
 By: Anton Tichy
 Type: Hardback book (30 pages) with light fawn cover
 Size: 24.5cm x 16.0cm
 Published by: Verlag des Oesterr. Ingenieur- und Architekten-Vereines, Wien
 Patents? None found
 Style of table(s): (i) Four-place graphic table organised in columns
 (ii) Other tables for trigonometrical functions
 Length(s) of graphic table: $\approx 16\text{m}$
 Comments: -



1908: Leder - Berlin, Germany

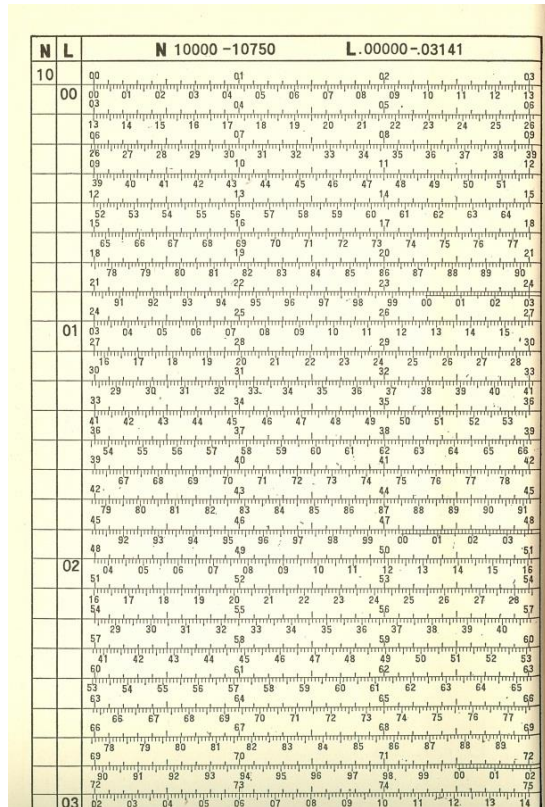
Title: *Die Praxis des Logarithmen-Rechnens*
 By: Ernst Leder
 Type: Circular cardboard chart with a cardboard cursor as part of a hardback book (125 pages) with pale blue linen cover
 Size: Chart: Ø 21cm
 Book: 27.8cm x 21.9cm
 Published by: Verlag der Cito-Rechenmaschinen-Werke G.m.b.H., Berlin
 Patents? DE104927 – 15th August 1899
 DE223529 – 24th June 1910
 Style of table(s): Four-place “graphic table” (antilogs only) organised as radii from the centre of the chart
 Length(s) of graphic table: ≈ 6.1m

Comments: This is the exception to all the other book-style listings. Instead of tables, the entire book consists of advice and worked examples of how to use logarithms in a myriad of calculations. A sleeve, pasted onto the inside back cover, holds a circular graphic antilogarithm chart. Using the chart as an “antilog table” is explained in the book but the author assumes that the looking-up of logarithms (not possible with the chart) is done with a traditional logarithm table.

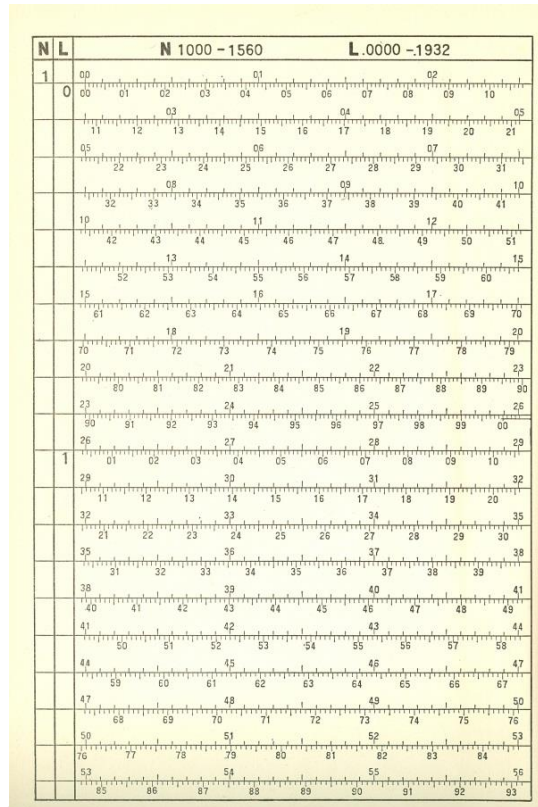


1925: Lacroix and Ragot - New York, USA

Title: *A Graphic Table combining Logarithms and Anti-Logarithms*
 By: Adrien Lacroix and Charles L. Ragot
 Type: Hardback book (52 pages) with green linen cover
 Size: 23.6cm x 15.2cm
 Published by: The Macmillan Company, New York
 Patents? US1610706 – 14th December 1926
 Style of table(s): (i) 40-page five-place without interpolation graphic table organised in rows
 (ii) 6-page four-place graphic table organised in rows
 Length(s) of graphic table: Long version \approx 115m
 Short version \approx 13.8m
 Comments: Book reprinted in 1927, 1936, 1938, 1941, 1942 and 1943.



Long 40-page version



Short 6-page version

+/-1926: Kübler - Berlin, Germany

Title: *Maximator Logarithmen Tafel*

By: Carl Kübler

Type: Folded card (2 pages)

Size: 27cm x 18.6cm

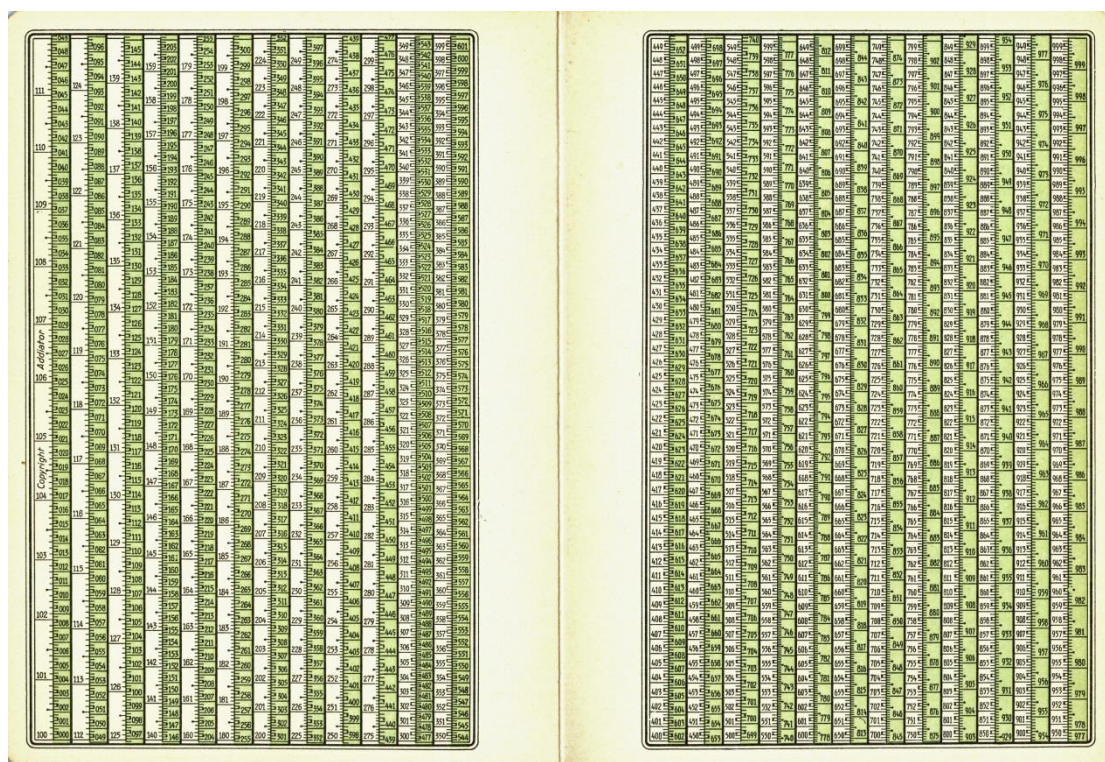
Published by: Addiator GmbH and later A. W. Faber-Castell Vertrieb GmbH

Patents? None found but design Copyrighted by Addiator GmbH

Style of 2-page four place graphic table organised in columns

Length(s) of $\approx 4.1\text{m}$

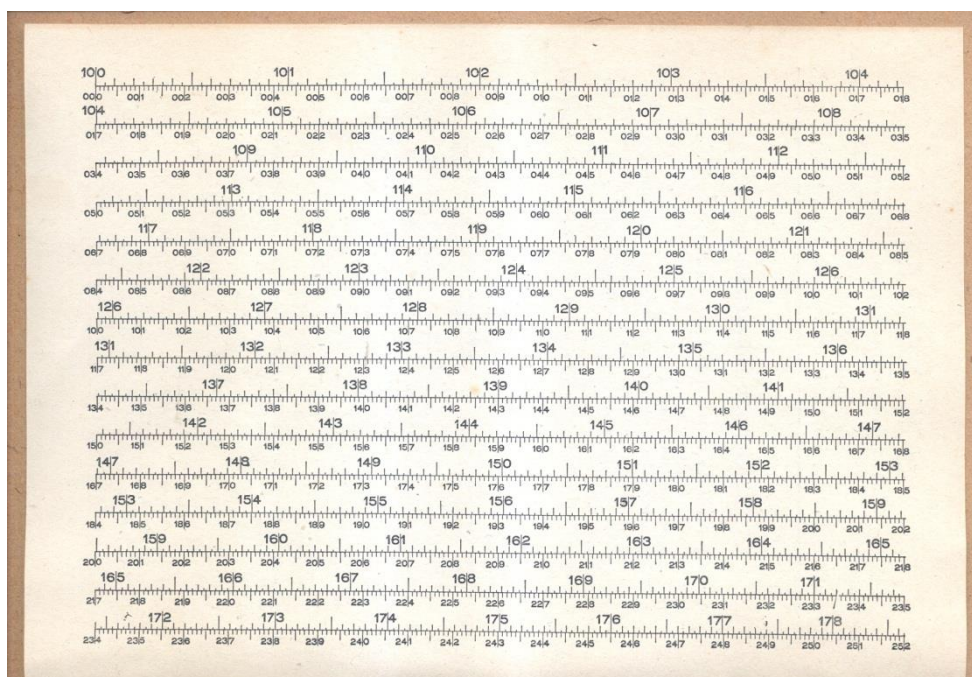
Comments: It was originally sold with the “*Maximator*” - a desk stand mounted mechanical slide adder the company Addiator started selling from around the mid-1920s. However, when Faber-Castell first sourced slide adders from Addiator for their range of hybrid slide rule/Addiators Carl Kübler managed to persuade Faber-Castell that they also had to take his *Maximator Logarithmen Tafel*. So for the early versions (1940-1942) of the Faber-Castell hybrid models 1/22A, 1/54A and 1/87A the *Maximator-Erweiterungs-Tabelle* was inserted behind a glued paper strip at the back of the instruction booklets (booklet no.’s: 1/702, 1/704 and 1/707). It was left out on later models and a generic instruction booklet issued for all Faber-Castell Addiator models.



1946: Kienbaum - Gummersbach, Germany

| | |
|-----------------------------|--|
| Title: | <i>Skalog - Der Skalen-Schnellrechner nach Kienbaum, eine graphische Logarithmentafel</i> |
| By: | Gerhard Kienbaum ⁶ |
| Type: | Hardback book (12 pages) |
| Size: | 22.4cm x 15.2cm |
| Published by: | Ingenieurbüro Dipl.-Ing. Kienbaum, Gummersbach |
| Patents? | None found |
| Style of table(s): | (i) 4-page four place graphic table organised in rows (ii) 4-page numeric table for trigonometrical functions |
| Length(s) of graphic table: | ≈ 10.7m |
| Comments: | Probably a private publication by Kienbaum. |

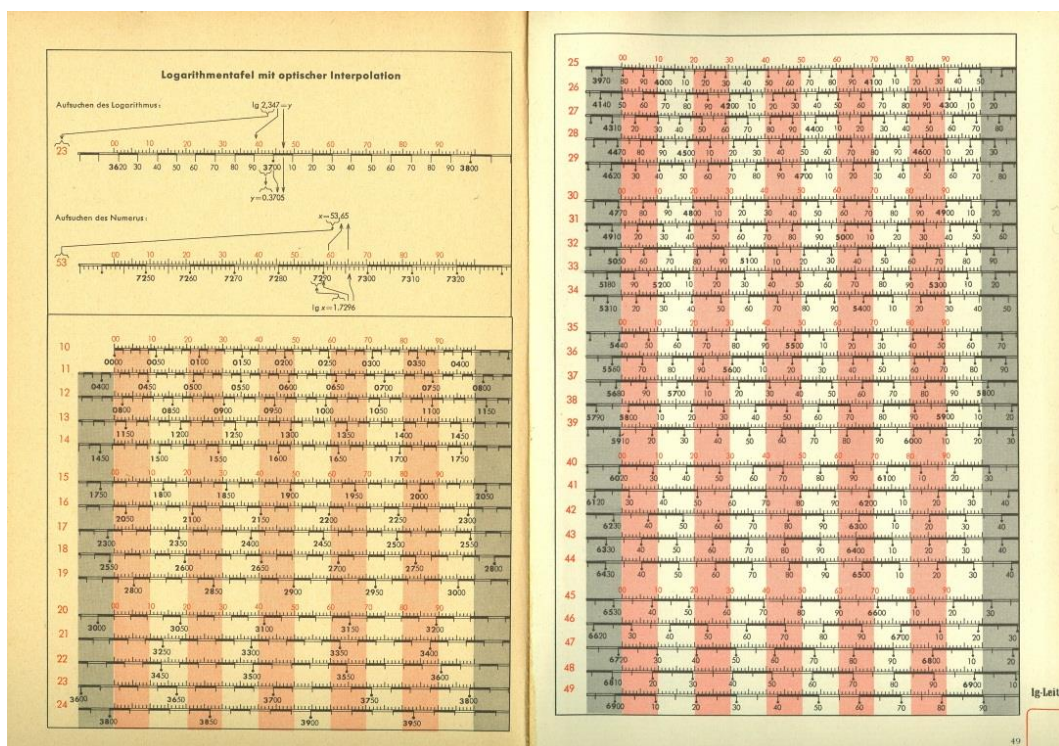
⁶ Started a one-man business that later became one of Germany's leading consulting companies.



1949: Rohrberg - Berlin, Germany

| | |
|-----------------------------|---|
| Title: | <i>Graphische Funktionentafeln</i> <i>Graphical Table of Functions</i> <i>Tables Graphiques des Fonctions</i> |
| By: | Prof. Albert Rohrberg ⁷ |
| Type: | Softback book (30 pages) with pale blue cover |
| Size: | 29.9cm x 20.9cm |
| Published by: | Fachverlag Schiele & Schön, Berlin |
| Patents? | None found but copyrighted in 1949 |
| Style of table(s): | (i) 7½-page four place graphic table organised in rows (ii) Four other tables for trigonometrical functions |
| Length(s) of graphic table: | ≈ 11.7m |
| Comments: | Multi-language: German, English and French. |

⁷ He also designed the special scales and scale layout for the Faber-Castell model 342 Columbus “System Rohrberg” slide rule.



1957: *Obbink* – Den Haag, The Netherlands

Title: *Rekentafel ABACUS Graphische Logaritmentafel*
 By: J. B. Obbink
 Type: Softback book (36 pages) with a mottled grey cover
 Size: 22.5cm x 13.5cm
 Published by: Roos en Roos, Arnhem
 Patents? None found
 Style of table(s): 22-page five-place without interpolation graphic table organised in rows
 Length(s) of graphic table: $\approx 88m$
 Comments: Unlike the rest of the book, the graphic log table pages are printed on a much thicker grade/weight of paper. Probably a private publication by Obbink.

The image shows a slide rule with four logarithmic tables (10, 01, 02, 03) visible. The tables are printed on a yellowish paper and show numbers from 00 to 99 in a grid format. The numbers are arranged in rows and columns, with some numbers having small superscripts or subscripts. The slide rule is placed over the tables, and the numbers are visible through the transparent plastic.

Graphic logarithmic tables – how did they work?

The easiest way to show how such graphic logarithmic tables were used is a worked example. However, the developers of graphic logarithmic tables chose an eclectic variety of ways to achieve the same goal. Despite these design differences, the way they were used, when compared with a traditional logarithmic table, is more or less universal.

The chosen example, 2.5×5 , looks trivial but it is all that is needed to show the generic processes. The only drawback is that the simplicity of the example hides the full tedium of and error-prone nature of the repetitive look-up process when using traditional logarithmic tables for complex calculations. Equally the advantages of a graphic logarithmic table would be amplified many times over for such complex calculations.

Using a traditional table of logarithms

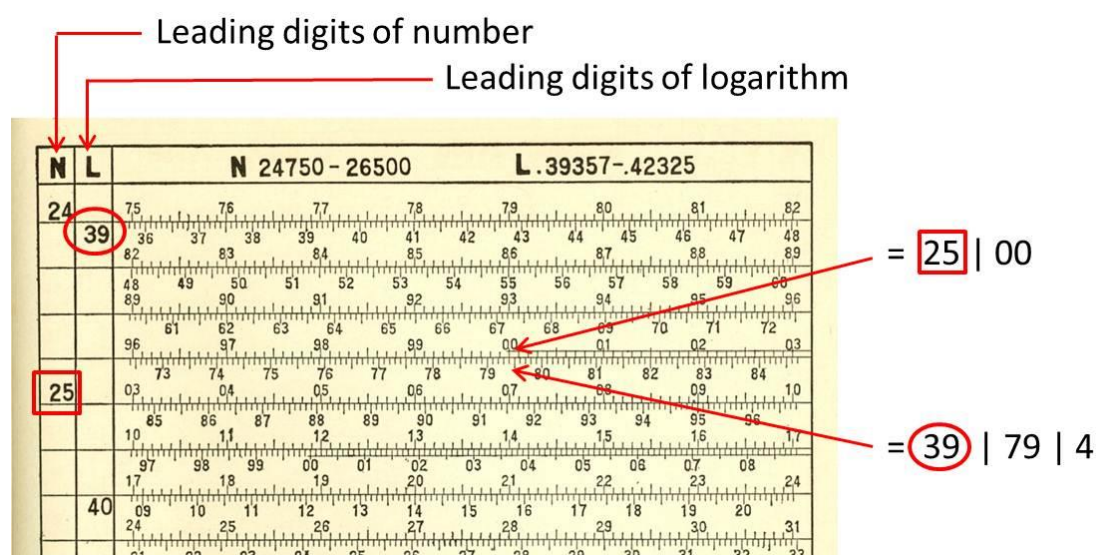
With a table compiled for 5 decimal places⁹ the minimum calculation steps are:

| | |
|--|-------------|
| | = |
| 1. Look-up the logarithm of 2.5 | 0.39794 |
| 2. Look-up the logarithm of 5 | 0.69897 |
| 3. Add the Log of 5 to the Log of 2.5 | 1.09691 |
| 4. Look-up the antilog of the mantissa 09691 | 12500 |
| 5. Use the characteristic "1" before the mantissa to fix the decimal point | 12.5 |

Depending on the notation form/style of the entries (especially the antilog entries) in a traditional table, each look-up step could well have required extra interim interpolation steps to determine the logarithm of each number and the antilog of the resulting mantissa.

Using the graphic logarithmic table by Lacroix and Ragot

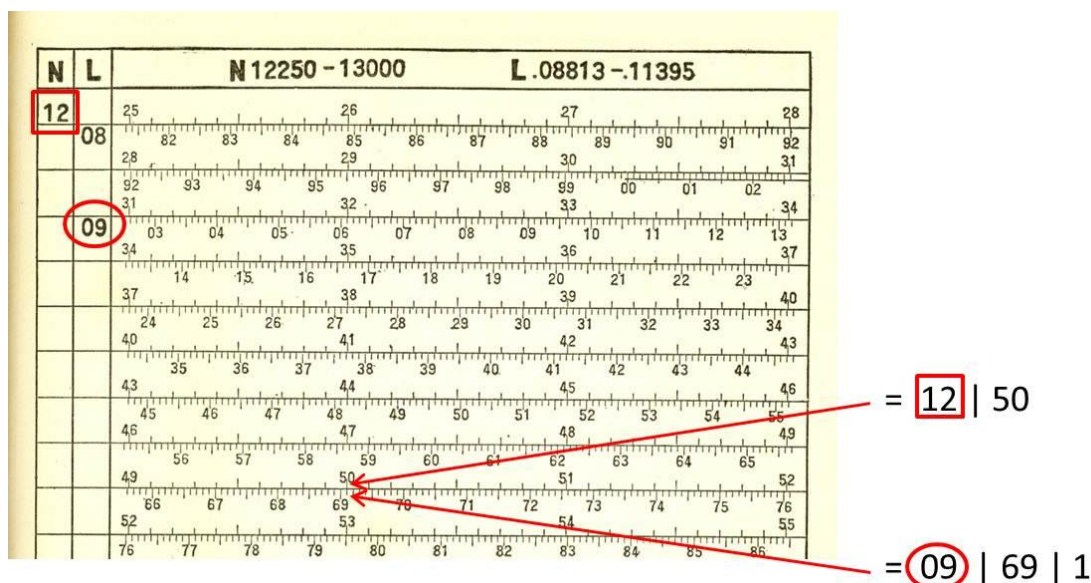
Opting for the longer 40-page *Lacroix and Ragot* table for five decimal places without interpolation, the logarithm of 2.5 can be quickly and easily found.



Ignoring the decimal point and looking up the leading digits "25" in the "N" column is enough to find the right page in the table. In the adjacent column "L" the leading digits of the logarithm, 39, are shown. The next step is to locate the following two 00 digits of the number on the upper scale of graduations for log section 39 of the table. The 3rd and 4th digits, 79, of the logarithm can be found on the lower scale to the left of the tick mark 00. Finally counting the extra divisions/tick marks that come after 79 before lining up with 00 on the upper scale, gives the last digit of the logarithm: 4. So, the complete readout is 39 79 4 or $\log 2.5 = 0.39794$.

The logarithm of 5 can be found equally easily. Adding the two logs (39794 and 69897) gives the same 1.09691 interim answer. But as the log and antilog entries are combined in a graphic logarithmic table, the antilog of the mantissa, 09691, can be quickly and easily "reverse engineered" using the same intuitive process.

⁹ Half of all the traditional logarithm tables ever published were versions for 4 or 5 decimal places.

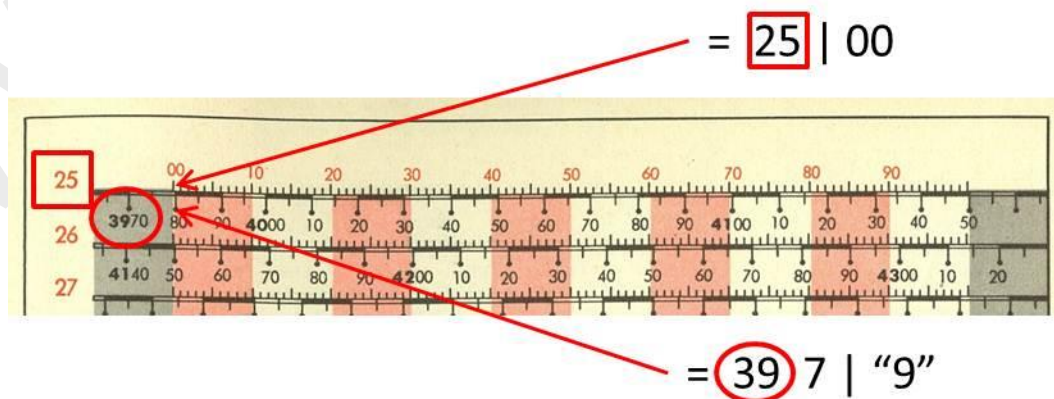


This time looking-up leading “09” digits of the mantissa in column “L” are enough to find the right page in the table and from the column alongside, read off the leading digits of the answer: 12. Having found the next two digits, 69, in the log 09 section, one “tick mark” further for the trailing 1 in the mantissa and the corresponding last two digits of the antilog number can be read off the upper scale – i.e. 50. The characteristic of the mantissa gives the final answer: **12.5**.

Superficially the steps look similar to using a traditional table of logarithms. However, the graphic version, with its fewer pages and combined log and antilog entries, is certainly less error-prone and much, much more intuitive to use. Also although both types of tables are compiled for 5 decimal places, only the graphic version has the inherent potential for accuracy to 6 decimal places. The values in the worked example finish exactly on a “tick mark” in the scales. But when needed and much like using a slide rule, accuracy to a sixth decimal place by interpolating between two tick marks would be simple and easy to achieve with this table.

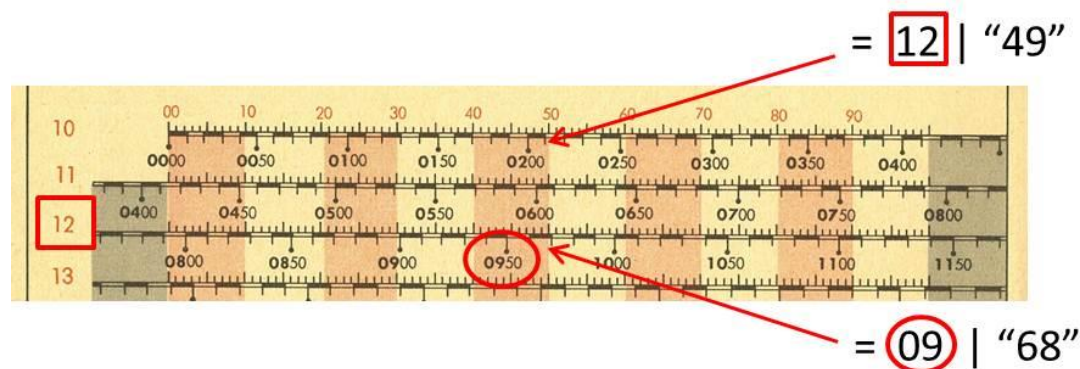
Using the graphic logarithmic table by Koch and Putsbach

By contrast this table needs interpolation to achieve accuracy to four decimal places. But strikingly the table is just 4 pages long – highlighting the compactness possible with graphic logarithmic tables compiled for a limited number of decimal places. Needless to say the logarithm of 2.5 can be just as easily found with this version.



Ignoring the decimal point, the leading digits “25” of the number is on the 2nd page of the table. The next step is to find the following two 00 digits of the number on the upper scale of graduations for line 25. The corresponding logarithmic value for 00 on the upper scale is somewhere between 3970 and 3980 on the lower scale – in fact nearly but not quite 3980. Using interpolation for 4th digit the full readout is 397 “9” or with this table $\log 2.5 = 0.3979$.

Again the logarithm of 5 can be found equally easily. Although this time, the interim answer after adding the two logs (3979 and 6989) is not unsurprisingly slightly less accurate: 1.0968. The antilog of the mantissa, 1.0968, can be just as quickly “reverse engineered” using the same intuitive process.



Again the leading “09” digits of the mantissa are used to find the right page in the table and to read off the leading digits of the answer: 12. The 0968 mantissa lies between two “tick marks”: 0950 and 1000. With interpolation for 0968 the corresponding last two digits of the antilog number can be read off the scale at the top of the page – i.e. 49. The characteristic of the mantissa again gives the final answer but this time the cumulative effect of working to fewer decimal places means the comes out as: **12.49**.

Graphic logarithmic tables – not a panacea

Given my opening “*A Picture Is Worth A Million Numbers*” justification for an alternative to the tedious use of traditional logarithmic tables, graphic logarithmic tables should have superseded them. They did not. Instead graphic logarithmic tables are largely unknown and rare. This could be because schools and educational institutions of the day preferred to stick largely to using conventional (and cheaper) mass-produced books of traditional logarithmic tables. But the more probable explanation is that most examples of graphic logarithmic tables are early 20th century developments and some telling inherent limitations meant they were outdated almost as soon as they were published.

But who were the intended users of graphic logarithmic tables? Clues can be found in the “*Introductions*” of the more well-known graphic logarithmic tables such as *Pressler*, *Lacroix and Ragot* and *Leder*. The advantages commonly quoted are speed and size. Reducing the labour-intensive and error-prone process of using a traditional logarithmic table would have obviously appealed to many professions and trades. Condensing the hundreds of pages of a traditional logarithmic table down to a slim volume would also have been preferable to carrying around a bulky book. A modern-day analogy is how the slim *iPad* is preferred to a bulky laptop computer. Ernst Leder goes on to suggest that graphic logarithmic tables could also be: “*a good tool for*

further education.” However, early 20th century students who could have been attracted by the advantages of a graphic logarithmic table would almost certainly have opted instead for one of the superior aids of the day – such as the slide rule. Although for a brief inexplicable 2-year period renowned German slide rule maker, Faber-Castell, sold 3 hybrid models that included both a mechanical slide adder and a paper graphic logarithmic table!

Ironically having started with a *Catch-22* paradox I conclude with another. Early in their evolution compiling and typesetting traditional logarithmic tables was a challenge. In such times the graphic printing possibilities were extremely crude and virtually non-existent. In contrast, by the 20th century the possibilities for printing complex images and graphically complex figures were bountiful. This meant graphic logarithmic tables were now relatively easy and economical to publish. Their “picture form” would have made it easier to spot typographical errors. So they were undoubtedly inherently less error prone than traditional tables of logarithms. However, by the 20th century demands for accuracy had risen sharply. By their very nature most graphic logarithmic tables, even with the most precise printing or production techniques, only offered 4 or 5 significant places of accuracy. But by now traditional logarithmic tables of 7, 8 or many more significant places had been common place for decades.

Once early 20th century cheaper printing and production techniques became readily available, graphic logarithmic tables could flourish. But sadly by now their level of accuracy had been surpassed and they faced competition from slide rules and other mechanical aids. This meant almost as soon as they became a practical reality, graphic logarithmic tables were outdated and inferior. So unlike the traditional logarithmic table, graphic logarithmic tables (even the often reprinted *Lacroix and Ragot*) never fulfilled their promise and never became popular, well-known or widely used.

Acknowledgements & Bibliography

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