INSTRUCTION MANUAL FOR

EMBLEM SERIES

TS

SLIDE RULES

EMBLEM TECHNICAL SLIDE RULES
PRACTICAL INSTRUCTIONS

Sect. 1

INTRODUCTION

The World owes the facilities of slide rule calculation to the great British mathematician, Napier, who evolved the system of calculation by logarithms used in all slide rules. Familiarity with logarithms, whilst to be desired, is not an essential pre-requisite to the effective use of the slide rule.

Sect. 2

The Basic Principle

The scales of your slide rule are divided into lengths proportional to the logarithms of the plain numbers figured on the scales. This explains why, with one exception, the scales are not equally divided throughout their length. The exception is the evenly divided scale L, giving the mantissae of logarithms to base 10.

The slide rule effects multiplication of two factors by adding a scale length proportional to one factor to the scale length proportional to the other factor. The sum of the lengths is proportional to the product of the two factors and is read from the figuring of the scale. Division is the reverse process – a scale length proportional to the divisor is subtracted from the scale length representing the dividend, to leave a length proportional to the quotient. The slide rule thus replaces, by a simple mechanical operation, the tedious references to tables of logarithms, writing down values, addition or subtraction, finding the antilog.

Sect. 3

Construction of the Rule

The slide rule consists of three parts: a) the stock or body, b) the slide, moving within grooves in the body and c), the cursor, which is a glass or plastics frame sliding over the whole length of the rule, furnished with one or more reference hairlines to aid accurate setting and reading of scale values.

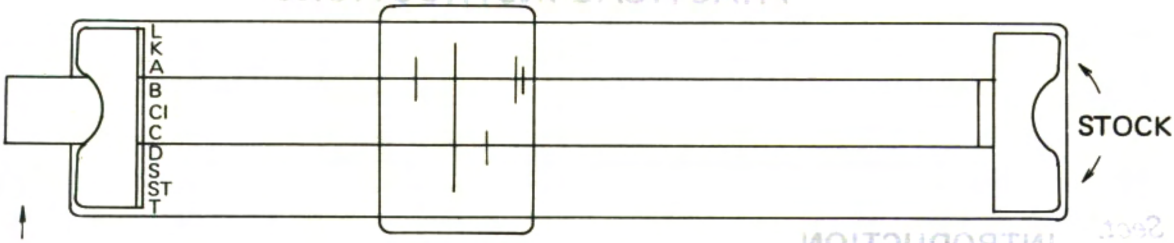

Fig. 1

Sect. *4* **Treatment and Manipulation of the Rule**

The slide rule is a precision instrument, robust and capable, given reasonable care, of a lifetimes service. It must not be exposed for long periods to direct sunlight, nor left on a heated surface such as a radiator. The faces can be cleaned with a soft cloth or paper tissue, with if need be a little soap and water. Detergent fluids should not be used.

If the rule is held as shown in Fig. 2, the slide can be moved easily by pressure and counter pressure at its ends. Holding the rule with pressure on the edges of the body tends to clamp the slide in the groove and to prevent the smooth motion needed for accurate setting. The cursor is moved as needed by lightly holding it between the forefinger and thumb, being careful not to twist it, thus displacing the hairline relative to the scale graduations.

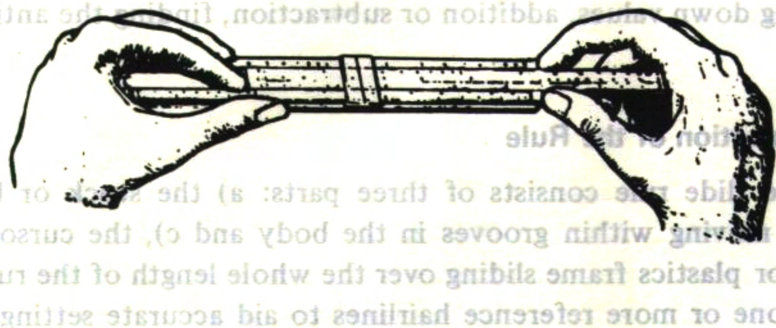

Fig. 2

Sect. 5

The Scales

The slide rule is a precision tool and in design can be simple or, through a series of modifications, complex. The more sophisticated rules have extra scales adapted to special types of calculation and hence, one may choose a rule of the pattern best suited to the class of work commonly met in a particular profession, trade or field of study.

The general principle of scale construction and arrangement is substantially the same throughout, so that mastery of a simple rule enables more complex scale patterns to be understood and used with confidence. The Emblem Series, models 1051 to 1091, shows a typical graduation between simple and fully developed scale patterns, as required for work of differing character. Because the principle of scaling is uniform, it is possible to issue one instruction book for the series. Those working with rules of the less advanced pattern will ignore references to scales not incorporated in the model they are using. The table of scale facilities will help in deciding, as skill and confidence grows, whether a more sophisticated rule would be a good investment, having regard to the nature of the work to be done.

EMBLEM TECHNICAL SLIDE RULES
ARRANGEMENT OF THE SCALES

TRIGONOMETRICAL FACE

Model	Upper Body Panel	Slide	Lower Body Panel
1051	K DF	CF CIF CI C	D A
1052	L K A	B CI C	D S ST T
1053	L K A	B CI C	D S ST T
1081	T_1 T_2 A	B BI CI C	D P S ST
1091	T_1 T_2 ST DF	CF CIF CI C	D P S

LOG-LOG FACE

Model	Upper Body Panel	Slide	Lower Body Panel
1051			
1052			
1053		S LL2 LL3	
1081	LL1 LL2 LL3 DF	CF CIF CI C	D L K
1091	LL01 LL02 LL03 A	B L K C	D LL1 LL2 LL3

Table A

Sect.
6

Scale Arrangement and use of scales C/D

Table A indicates the position of the several scales in the series, using as identification the symbols to be found at the left hand end of the scales as graduated on the rule itself. Each scale or scale pair will be discussed in turn, beginning with scales C/D, which are the most frequently used. These two scales form a pair, similarly divided and figured. At the left hand end of each scale is the figure 1, known as the left hand index (abbr. l.h.i.), whilst at the other end each scale has the figure 10, the right index (r.h.i.).

The scales are logarithmic but the intervals or distances along the scales are marked by natural numbers. The pattern of subdivision must be studied carefully.

Between marks 1 and 2, there are ten main divisions, each figured. Each main division is subdivided into ten parts. In this interval of the scale it is thus possible to locate with the cursor a three figure value, directly, whilst a fourth figure can be estimated by visually subdividing the space between adjacent marks. Fig. 3 shows readings in this range.

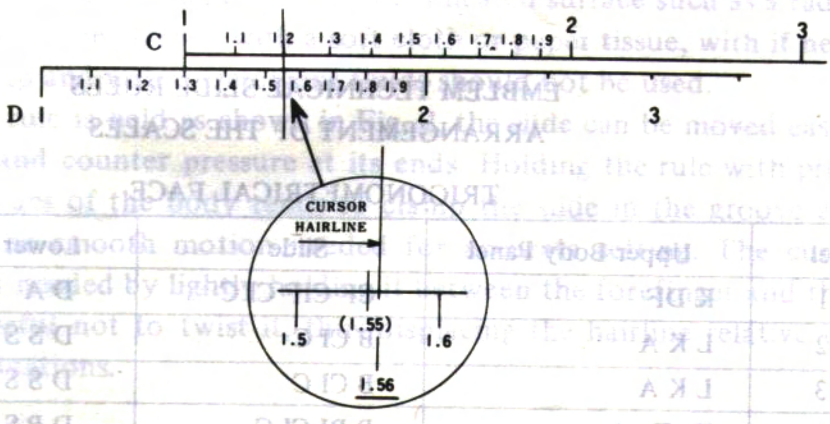

Fig. 3

Because the scale is logarithmically proportioned, distances between marks decrease progressively from left to right. To avoid confusion resulting from a large number of graduations in the reduced space, the pattern of division is changed, between marks 2 and 4. The interval 2 – 3, (and 3 – 4), is divided into ten main divisions, each of which is subdivided into 5 parts. Three figure numbers can still be read – if the value in the units place is even, the third figure is located by a short graduation; if it is odd, it is located by bringing the cursor line mid-way between two adjacent short graduations. Fig. 4 shows examples.

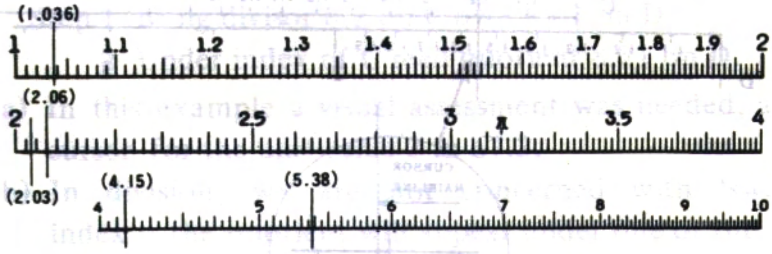

Fig. 4

Finally, in the interval 4 – 10, decreasing space requires a further change in graduation pattern. Ten main spaces, not figured, are each divided by a short graduation into two parts. The value of the short graduations is thus 5. Three figure numbers can be read, but visual estimation of the value of the units figure is necessary if it is other than 5.

The pattern of subdivision is important. After a little practice setting and reading will be found easy and no conscious effort of counting up subdivisions will be needed. One special point of importance to be noted concerns values at the left hand end of the scale – between the mark 1 and 1·1 – do not forget the zero. (Fig. 4).

Scales C/D are the fundamental scales, to which all the other scales on the rule are related.

Sect. 6.1 **Multiplication – Scales C/D.**

Mechanical addition of scale lengths proportional to the logarithms of the factors.

Example

1 Find the product 1·3 x 1·2.

Step 1 Draw the slide to bring l.h.i. (1) on C over 1·3 on scale D.

2 Move cursor over 1·2 on scale C.

3 Under cursor hairline read the product, 1.56 on scale D.

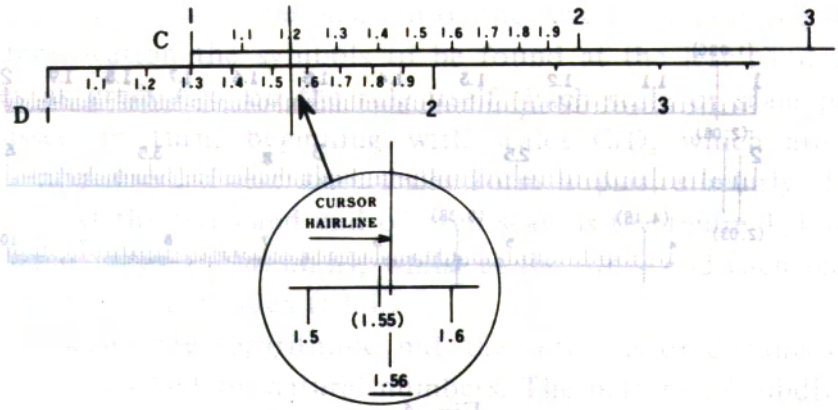

Fig. 5

Example

2 **Multiply 6.45 x 5.24**

Step 1 Draw r.h.i. of scale C over 645 on D

2 Bring cursor to 524 on C

3 Under hairline read product 338 on D.

Two points arise in this example. First, trial will show that here the left hand index of scale C cannot be used. If it were, the slide would project too far out of the body to permit setting the factor 5.24. The slide is therefore 'switched', to change the index. In effect, an identically graduated scale pair is assumed to extend beyond index 1 on D. Switching the index is often necessary when using scale C/D. It can be avoided by the use of other scales, to be discussed later.

The second point to note is that the slide rule scales, except in the case of the LogLog scales to be described later, do not locate the decimal point. This is done by means of an approximation. In examples 1 and 2, the position of the decimal point is obvious. In more complex problesm, the reduction of factors to 'Standard Form' should be used – see section 7.

Sect. 6.2 **Division – Scales C/D**

This is the reverse of multiplication. The scale length representing the divisor is mechanically subtracted from the scale length of the dividend, the scale length thus found represents the quotient.

Example

3 Find 67.3 ÷ 8.2

Step 1 Bring divisor 8.2 on C over 67.3 on D

2 Under index of C read quotient 8.21 on D.

Note: a) In this example a visual assessment was needed, aided by the cursor, for the unit value 3 in 67.3.

Note: b) In division, we are not concerned with 'switching' the index – the quotient will appear under one or the other index, following the initial setting.

Sect. 7

Combined multiplication and division.

Multi-factor problems involving multiplication and division are easily worked on the slide rule.

Example

4 Evaluate $\dfrac{5.34 \times 786.2 \times 18.7}{36 \times 483}$

This will be a convenient point to introduce 'Standard Form'. Since, in the decimal system, we can multiply by 10 simply by moving the decimal point one place to the right, or by 100 by moving it two places to the right; divide by 10 by moving the decimal point one place to the left and so on, we can always write any number, large or small, with a convenient number of digits to the left of the decimal point, adding a term to indicate the powers of ten involved in the change of form. Thus we can write 5793 as 5.793×10^3, or 0.0136 as 1.36×10^{-2}. Applying this to our present example, we re-write the expression as:

$$\frac{5.34 \times 7.862 \times 10^2 \times 1.87 \times 10}{3.6 \times 10 \times 4.83 \times 10^2}$$

Obviously, we can cancel the 10's and leave a more simple expression:

$$\frac{5.34 \times 7.862 \times 1.87}{3.6 \times 4.83}$$

From this we can quickly obtain an approximate answer by rounding off the figures:

$$\frac{5 \times 8 \times 2}{4 \times 5} = 4$$

We now know the order of magnitude of the answer and can place the decimal point without hesitation. So, back to the slide rule!

Step 1 Cursor to 524 on D
2 Bring 36 on C under hairline
3 Cursor to 187 on C
4 Bring 483 on C under hairline
5 Cursor to 7682 on C
(Set 768, visually estimating the units figure, 8)
6 Read 443 on D under the hairling.

This example shows several important procedures. First, by expressing complex expressions in 'Standard Form', we can reduce them to terms involving one digit only before the decimal point. Second, after reduction, the original expression lends itself to the immediate calculation – of the most simple type – of an approx. answer. This approximation will not only permit us to locate the decimal point, it will also help us to avoid the common error, so easily made when working with decimals, of getting an answer ten times too great or perhaps 100 times too small. The third valuable idea shown by the example is the method of working combined multiplication and division problems on the slide rule:

1 Divide and multiply alternately, beginning with division.
2 Take the factors in their order of size, as is usually possible, to reduce slide movement.

Sect. 8 **Scale of Reciprocals, CI**

This scale is an exact counterpart of scale C, but is arranged so that its graduations and figuring progress from right to left. Thus, for any value x found on scale C, we have over it on scale CI the value 1/x. The CI scale is used to reduce slide movement. If we have a multiplier x, we can, if convenient, obtain a product xy by dividing the other factor, y, by 1/x, locating 1/x on scale CI.

Example

5 Evaluate 718 × 0.202 × 6.75.
First use scales C/D, as already explained.
Step 1 Cursor to 718 on D
2 Bring r.h.i. of C to cursor
3 Cursor to 202 on C
4 Bring l.h.i. of C to cursor

5 Cursor to 675 on C

6 Under hairline read answer 980 on D.

On the second step the slide was moved about 1½ inches; on the fourth, about three inches. In all, slide movement was approximately 4½ inches.

Re-work the example, using scales C, D and CI, solving from the form

718 ÷ 1/.202 × 675.

Step 1 Cursor to 718 on D

2 Bring 202 on CI under hairline

3 Cursor to 675 on C

4 Under hairline read answer 980 on D

One slide movement only, about 1½ inches. The decimal point is located by separating the 10's factors – giving 7.18 × 2.02 × 6.75 × 10^{2-1} which, rounded off, is (7 × 2 × 7) × 10, = 980.

The scale of reciprocals, CI, is particularly useful when we have a constant factor to be multiplied or divided by a series of factors.

Sect. 9 **Scales CF and DF**

These are known as the 'folded' scales. They form a pair, graduated and figured exactly as scales C/D, but moved laterally so that the value $\pi = 3.14159..$ on CF/DF is over index 1 on C/D. This displacement has two advantages. First, for any value x read on scale D, we have (using the cursor), the value $\pi.x$ on CF, with no movement of the slide. Since π is the ratio between the diameter of a circle and its circumference, this relationship between CF and D is clearly timesaving. Second, by displacing the left hand index to the position it has in scales CF/DF, we can always avoid 'switching' the index. If we take the first factor in a product on CF/DF, any other multiplier is at once available on CF/DF or C/D. Let us re-work example 4, with the help of the folded scales.

Step 1 Set 36 on C over 524 on D

(The quotient, which we do not need to read, appears on D under l.h.i. of C and on DF over 1 on CF).

2 Cursor to 7862 on CF

3 Bring 483 on CF under hairline

4 Cursor to 187 on C

5 Under hairline read answer 443 on D.

Slide movement only about 3 inches.

Sect. 10

Scale CIF

This scale is graduated as are CF/DF, but arranged to progress from right to left. It is a scale of reciprocals, related to CF in the same way as is CI to C. Using the CF/DF, CIF and C/D scales, example 5 will be re-worked.

Step 1 Cursor to 718 on DF
2 Bring 202 on CIF under hairline
3 Cursor to 675 on C
4 Under hairline read answer on D 980.

The result is obtained with one setting of the slide and two cursor movements.
The two sets of paired scales, C/D and CF/DF, the CI and the CIF scales together form a 'battery' with which a problem may be tackled. When confidence in using these scales as an assembly of different approaches to a problem has been gained, you will have made a big step forward. You will have begun, perhaps unknowingly, to examine each problem to find the most simple way of solving it – and, never forget, the most simple way is commonly the most accurate.

Sect. 11

Scales A/B

These scales each consist of two equal scale lengths, placed end to end. Although figured differently, each scale length is similarly divided and graduated. Further, each scale length is exactly half the length of scale C or D. Because of this proportionality, for any value x on scale D we have x^2 over it on scale A. (In model 1051 the A scale is below the D scale, so x^2 is immediately under x on D).

With the cursor, without slide movement, we can at once find the squares of numbers:

Thus							
	Scale D	2	3	4	5	1.5	2.5
	Scale A	4	9	16	25	2.25	6.25 and so on.

More thought is needed to read square roots, i.e., given a value x^2, to find x. It is of course the reverse process – locating the radicand (x^2) on A and reading x on D under the cursor. But scale A is in two sections. Which section is to be used in any given example?
Simple, easily remembered rules will help. Count the digits in the radicand. If the number of digits is odd (1, 3, 5 etc.), set the number on the left hand segment of A. When the number of digits is even (2, 4, 6

etc.), set in the right hand segment.

These rules apply to wholly integral numbers and also to numbers partly integral and partly decimal fractions. For values wholly decimal, count up the number of zeros to the right of the decimal point. If the count is odd, use the left hand segment of A. If there is no zero, or an even number of zeros, use the right hand segment.

Example: x^2 = 0.0906 0.49 0.0049
x = 0.301 0.7 0.07

In addition to their value in finding squares and square roots, scales A/B can be used for multiplication and division in the same way as scales C/D. They are also of value in certain problems in which squared terms appear, as will be shown later.

Scale BI

This scale of reciprocals is related to B, the scale of squares, in the same way as is CI to the fundamental scale C. It is thus of value when a calculation, originated on D, is taken up to the A/B scales in a squaring operation.

An example, using simple figures, will make matters clear.

Example

6 Evaluate d^2/y, when d = 8 and y = 2.4

We will solve as $8^2 \times \frac{1}{2.4}$ using scale BI

Cursor to 8 on D
r.h.i. of BI to hairline (short slide movement).
Cursor to 2.4 on BI
Under hairline read 26.7 on A.

Sect. 12 **Scale of Cubes, K**

This is, like scales A/B, related to scales C/D. Whilst scales A/B provide the squares of any number on C/D, scale K provides the cube. To remaind you, scales C/D have the symbol x at the right hand end of the scale, scales A/B have x^2 and scale K, x^3.

Scale K consists of three segments, each one third the length of scales C/D. The division pattern is the same in all three segments, the figuring is progressive between 1 and 1000. This a help in finding the cube root of a number less than 1000, but for larger numbers, or for decimal numbers such as 0.081, you will do well to convert the radicand to standard form.

Example

7 Evaluate $\sqrt[3]{0.1798}$

First, express this as $179.8 \times 1/1000$

The cube root of 1/1000 i.e., of $1/10^3$, is 1/10 and we thus have to solve $\frac{1}{10}\sqrt[3]{179.8}$

The term under the radix can be located on the K scale by the cursor and the cube root read on D as 5.65. Multiplying – mentally – by 1/10 at once gives the required cube root of 0.1798, i.e., 0.565.

Sect. 13 Proportion and Tabulation

When we move the slide so that the index of C stands opposite any value, say x, on scale D, we at once establish an infinite number of pairs of values C : D in the ratio 1 : x. We also set up an identical ratio on the folded scales.

The utility of this is shown by a conversion problem. Suppose we have a series of weights in pounds to convert to their equivalents in kilograms. All we need to do is to set the basic ratio, 1 lb = 0.456 kg in the form 1 (or 10) on C over 0.456 on D. Then, using the cursor only, we can read off as many equivalents (conversions) as we please or require. With the folded scales in the same ratio, it will not be necessary to reset the slide.

Example

8	Given:	lb (on C)	1	3.84	5.62	7.15	8.3	13
	Found:	kg on C	0.456	1.75	2.56	3.26	3.78	
		on DF						5.93

Scale CI similarly provides ratios of the form 1: 1/x.

Sect. 14 Accuracy and Significant Figures.

Because, in the construction of the scales, the intervals between graduations vary and the numerical value of an interval is assessed according to its position in the scale, and because visual estimation is required for values falling between graduations, it is not possible always to read a slide rule scale with absolute accuracy.

A little thought will help you to realise that the accuracy of slide rule working is in general compatible with the accuracy of the data used in the calculations.

Suppose a mass is reported, in an experiment, as 1.05 kg. This does not mean that – except by some lucky chance – the mass is exactly 1.05 kg. It means that, as accurately as the experimental conditions and the equipment used made possible, the mass was found to be 1.05 kg. If the work were fully reported, it would be between 1.055 and 1.045 kg and we would have an experimental error 'built in' to our arithmetic. Again, cloth comes in rolls and the length is stated, and invoiced, in metres. It is very unlikely that any roll will be an exact number of metres in length – neither a few millimetres over or under. An error is once more 'built in'. In practice these errors are very small and relatively unimportant. The errors are, moreover, commonly larger than the error made by a slide rule user in reading his scales. When proper care is taken the error in reading a slide rule value is about 1 in 1000 and in general, slide rule results are accurate to three significant figures.

The main source of error in setting and reading arises in the positioning of the slide. The reference line on the cursor is so fine, by comparison with the graduations, that reading accuracy is very high. Slide setting should always be kept to a minimum and for this reason, it is worth the extra effort to become familiar with the use of the reciprocal and folded scales.

Sect. 15 **The Cursor and its Lines.**
Gauge marks on the scales.

When finding areas and volumes of solids, the factor $\pi = 3.14159..$ which is the ratio between the diameter of a circle and its circumference, occurs as a multiplier. For convenience, your rule has on scales A/B, C/D, BI and CI a 'gauge mark', π, at the value 3.14159. . . Given the diameter of a circle, to find the circumference, we can either a) bring the index of C to the mark π on D, move the cursor to the diameter d on C and read the circumference on D under the cursor hairline, or b) take advantage of the relationship between scales C/D and the folded scales CF/DF and use the cursor to set the diameter d on scale D, reading π d on CF under the hairline.

Example

9 Diameter d = 14.2 cm   What is the circumference?
Set l.h.i. of C to the mark π on D
Cursor to 14.2 on C
Under hairline read circumference 44.6 cm on D.
or,

Bring cursor over 14.2 on D and read circumference 44.6 cm upper hairline on CF.

The long centrally placed reference line of the cursor is already familiar to you. Other lines are to be seen, at unequal distances to the upper left and upper right of the main line. One short line will be found at the lower right hand side of the cursor.

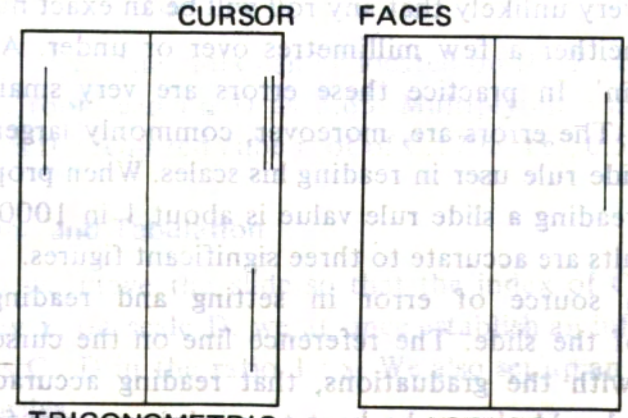

Fig. 7

The upper short line to the left is displaced from the main line by a distance equal to $\pi/4$, i.e., 0.785. It is used in finding the area of a circle, given the diameter, from the relationship $A = \pi/4d^2$, where A is the area and d the diameter.

You know that you can read on A the value of the square of any value set on D. To find a circular area, then, set the diameter d on scale D with the main cursor line (which will then lie over d^2 on scale A) and read the area on A under the auxiliary short hairline (upper left).

To the upper right of the main cursor hairline, over scales A/B, are two short lines, close together. These relate HP, in the British system – the outer line of the pair – or metric horse power (CV or PS), the line nearer the main hairline, to the electrical unit, kW. One HP (British) = 746 watts; one metric horse power = 736 watts.

If we set the main hairline to kW on scale A, we can at once read HP on A under the short (outer) harline or the metric equivalent (inner short line), also on A. Conversely, given British or metric horsepower, the value can be located on A with the appropriate short hairline and the equivalent in kW read on A under the main hairline.

The short line to the lower right is displaced from the main line by the value $\pi/4$ (0.785) and can thus be used to find the diameter of a circle of a given area. Set the area under the main hairline on scale A and read the diameter d on D under the short, lower, auxiliary line.

Models 1053, 1081 and 1091 have scales on both faces, The cursor, too, is double sided on models Nos 1081 & 1091.

On the LogLog face, the cursor has, in addition to the central, main, reference line, a short line to the right of the central line, over the scales CF/DF.

If the main cursor hairline is moved to index 1 on D, then with the slide in 'neutral' position – the index mark 1 of scales A/B, C/D all under the main hairline – the short auxiliary line will be over 36 on DF. The factor 36 is useful in conversions. If a value x is set on D, with the main cursor line, on DF we read under the auxiliary short line, the product 36x.

The factor is used to convert hours to seconds (1 hour = 3600 secs.), or degrees to seconds of arc. Also, note that 1 metre per sec. is 3.6 km per hour.

In financial calculations, for interest or bill discounting, it is common practice to reckon 1 year as 360 days and here again, the line for factor 36 is useful.

The marks ρ'' and ρ' will be found on scale C (Trigonometrical face). These are used when converting to radians an angle expressed in minutes or seconds of arc.

ρ' has the value 3438, or $180/\pi \times 60$ and relates minutes of arc to radians.

ρ'' has the value 206265, or $180/\pi \times 60^2$, and relates seconds of arc to radians.

Example.

10 (a) What is the radian measure of an angle of 23 minutes (23')?
arc $23' = 23/\rho'$ i.e., 0.00669 rad.

(b) What is the radian measure of an angle of 400 seconds (400")?
arc $400'' = 400/\rho''$ i.e., 0.001939 rad.

A common application for radian measure for small angles, using the gauge marks, is the finding of the length of a circular arc, given the radius, r, and the angle subtended by the arc

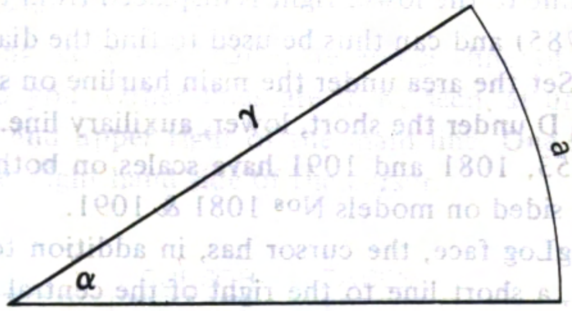

Fig. 8

Here the angle = a/r x ρ' (on ρ'').

$$\text{The length of arc} = \frac{x\,r}{\rho' \text{ (or } \rho'')}$$

Sect. 16 **THE TRIGONOMETRICAL SCALES**

Slide rules for technical work are provided with scales for the setting and reading of sines, tangents and also cosines and cotangents, of angles in degrees and decimal fractions of a degree. Angles are set and read on the trigonometrical scales and the function values set or read on scales C/D or CI.

Scale S.

This is a scale of sines for angles between 5.5° and 90°. Since, in a right angled triangle, the sine of an angle is the cosine of its complement, sinθ = cos (90° – θ), scale S can be read, from right to left, as a scale of cosines. To avoid confusion, sines, reading progressively from left to right, are figured in black. Cosines, read from right to left, are figured in red. For any angle on S, the function value of its sine or cosine can be found on D, immediately under the cursor hairline, when the cursor is set to the angle on S.

Note: Emblem model 1053 has an extra scale of sines, on the slide. This, also, is referred to scale D.

Scales T, T_1 and T_2

These are scales of tangents, the function value again being read on scale D, against the angle in degrees. In models 1052 and 1053, a single tangent scale, graduated between 5.5° and 45° is used. From the

relationship cot $\theta = 1/\tan \theta$, the scale can be read from right to left as a scale of cotangents and is thus figured, in red.

The more advanced rules, models 1081 and 1091, have a two part scale of tangents, T_1 for angles between 5.5° and 45° and T_2 for angles between 45° and 90°. When reading function values for angles on the T scales, remember that tan 45° = 1, so that the function values for angles less than 45° will be < 1, whilst angles greater than 45° will have function values > 1.

Scale ST

This scale is based upon the trigonometrical relationship sine $\theta \simeq \tan\ \theta \simeq \theta$, where θ is in radian measure, the angle is small and the sign $\simeq$ means 'approximately equal to'. The agreement of values for sin θ and tan θ is very good for angles less than 4°. Function values are read on D against the angle on ST. Because scale ST is divided (decimally) in radians, it can be used to find the radian equivalent of an angle in degrees and vice versa. It is also used when solving problems involving small or large angles, with greater accuracy than is possible with scales S and T. Scale ST, progressing from left to right between 0.55° and 6° is figured in black. It is also figured, in red, counter clockwise, between 6° and 89.5°. Function value accuracy for angles less than 5.7° or greater than 84.3° can be enhanced by the use of the approximation $\cos \theta = (1 - \theta^2/2)$, when θ must be taken in radian measure. When θ is large, the approximation $\sin \theta = \theta^2/2$ can be used.

Examined closely, scale ST will be seen to follow the pattern of division of scale S, with, however, lateral displacement of the figured intervals. The displacement represents the factor $\pi/180 = 0.01745$, which is the radian measure of an angle of 1°.

We can by setting the angle in degrees on scale ST, with the cursor, read under the hairline on D the radian measure. Because scale ST is decimally subdivided, this facility is not restricted to the small angles figured in black on ST, but is available for large angles also. Care must be taken in locating the decimal point. As stated, the radian measure of 1° is 0.01745; that of 10° is 0.1745 and that of 50°, 0.8725 and so on.

ANGLES GREATER THAN 90°

The trigonometrical scales give function values in the first quadrant only, i.e., for angles between 0° and 90°. The function for angles in other quadrants can however be obtained by using the relationships, developed in any text on trigonometry and summarised below.

First Quadrant	Second	Third	Fourth
$+\sin\theta°$	$+\sin(180° - \theta°)$	$-\sin(\theta° - 180°)$	$-\sin(360° - \theta°)$
$+\cos\theta°$	$-\sin(\theta° - 90°)$	$-\sin(270° - \theta°)$	$+\sin(270° - \theta°)$
$+\tan\theta°$	$-\tan(180° - \theta°)$	$+\tan(\theta° - 180°)$	$-\tan(360° - \theta°)$

Scale P

This scale, offered in Emblem models 1081, 1091, is often known as the Pythagoras scale. For any value x found on scale D, scale P provides the value $\sqrt{1 - x^2}$. The special facilities offered by this scale will be seen in the examples that follow. Besides its use in trigonometrical work, scale P can be employed to enhance the accuracy of square roots.

Example

11 Evaluate 0.94

Method: Solve as $\sqrt{1 - 0.06}$

Step 1 With cursor set 0.06 in left hand part of scale A. On D, under the hairline will be $\sqrt{0.06} = 0.245$.

Also under the cursor line, on P, will be the value of $\sqrt{1 - 0.245^2}$, i.e., 0.9695, the required result to four figures.

Sect. 17 **Solution of Right Triangles**

In Fig. 9 is shown a triangle, with the right angle, 90°, at γ. As is proved in textbooks on trigonometry, the following relationships hold between the sides and angles:

$\sin\alpha = a/c$ $\quad\sin\beta = b/c$ $\quad a^2 + b^2 = c^2$

$\cos\alpha = b/c$ $\quad\cos\beta = a/c$

$\tan\alpha = a/b$ $\quad\tan\beta = b/a$ $\quad\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

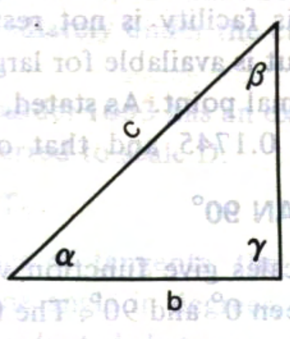

Fig. 9

Fig. 10 shows a right angled triangle for which the hypotenuse and one angle are known. Side y is to be found.

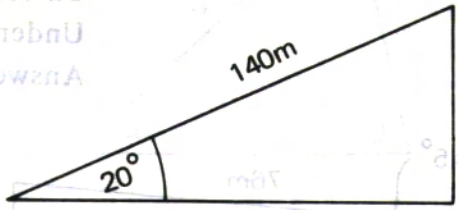

Fig. 10

Here, $y = (140 \times \sin 20°)$m	Method: Cursor to 20° on S
$= (140 \times 0.342)$m	Index 1 of C to cursor
$= 47.89$ m	Cursor to 140 on C
	Under hairline read 47.89 on D.

In Fig. 11 $y = (140 \times \tan 25°)$m	Method: Cursor to 25° on T
$= (140 \times 0.466)$m	Index 1 of C to cursor
$= 65.24$ m	Cursor to 140 on C
	Under hairline read 65.24 on D.

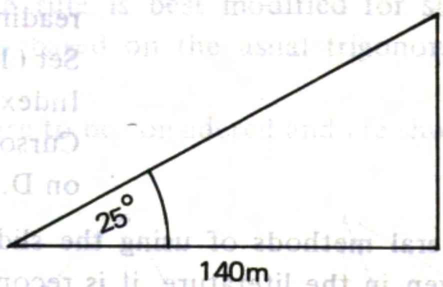

Fig. 11

In Fig. 11a $\cos\theta = 96 \div 120$	Method: Set 120 on C over 96 on D
	Under index of C read the angle (red figures), 37°, on S

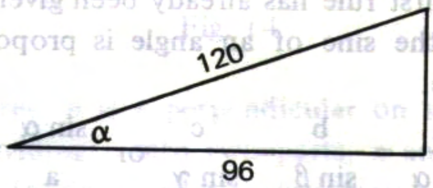

Fig. 11a

In Fig. 12 let the given angle be small, 3.5°. Here we must use scale ST.

$a = (76 \times \sin 3.5°)$m

Method: Cursor to 3.5° on ST
Index of C to cursor
Cursor to 76 on C
Under hairline read a = 464
Answer 4.64 m

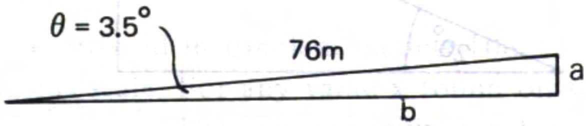

Fig. 12

In this problem, side b could be found by the approximation given in section 16, $\cos\theta = (1 - \theta^2/2)$.

$b = (76 \times \cos 3.5°)$m

$76 \times (1 - \frac{.061^2}{2})$

$76 \times (1 - 0.00187)$

76×0.998

75.8 m

Method: Cursor to 3.5° on ST
read 0.061 rad on D
Under hairline is 0.061^2
i.e., 0.00372, on A
Bring 2 on B under cursor, reading $\theta^2/2$ on A, 0.00187
Set (1 − 0.00187) = 0.998 on D
Index of C to 0.998
Cursor to 76 on C, read 75.8 on D.

N.B. Although several methods of using the slide rule for large angles ($\theta > 84.5°$) are given in the literature, it is recommended that recourse should be had to tables, since 5 or more decimal places are required to achieve accuracy.

Sect. 18

Solution of any plane triangle

For obtuse or acute triangles, two principal rules govern the method of solution. The first rule has already been given and is the SINE RULE. In any triangle, the sine of an angle is proportional to the side that subtends the angle.

$$\text{Thus: } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \text{ or } \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \text{ (Fig. 13)}$$

(In a right angled triangle, hypotenuse c, $\gamma = 90°$ and the sine of $90° = 1$).

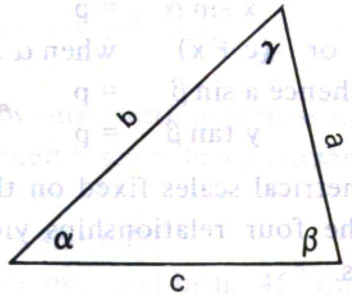

Fig. 13

The sine rule is clearly well adapted to the solution of triangle problems with the slide rule, since it involves a proportionality easily established between scale S and scale D.

The COSINE RULE is usually expressed in the form:

$$a^2 = b^2 + c^2 - 2cb \cos \alpha$$
$$b^2 = c^2 + a^2 - 2ac \cos \beta$$
$$c^2 = b^2 + a^2 - 2ab \cos \gamma$$

To apply the slide rule to expressions of this type is not very efficient and the cosine rule is best modified for slide rule manipulation. The modification is based on the usual trigonometrical proof of the cosine rule.

Two cases are to be considered and are shown in Fig. 14 (a), (b).

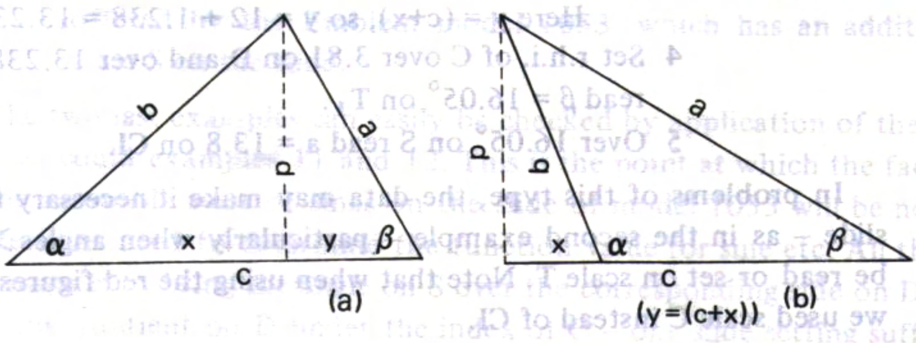

Fig. 14

In both figures, p is a perpendicular on side c, producing two right triangles and dividing c into two parts, x and y. From what has been given above, the following relationships may be deduced:

$\sin\alpha = p/b$	whence $b \sin\alpha$	$= p$	
$\tan\alpha = p/x$	$x \sin\alpha$	$= p$	
$y = (c - x)$	or $(c + x)$		when $\alpha > 90°$
$\sin\beta = p/a$	whence $a \sin\beta$	$= p$	
$\tan\beta = p/y$	$y \tan\beta$	$= p$	

With the trigonometrical scales fixed on the body panel and scales C and CI moveable, the four relationships yielding p are easily used to achieve neat solutions.

Example

12 Given $b = 5$, $c = 8.5$, $\alpha = 38°$, find a and β.

Method: Step 1 Set b = 5 on CI over 38° on S

2 Read x = 3.94 on CI over 38° on T or T_1
$y = (c - x)$, i.e., $(8.5 - 3.94) = 4.56$

3 Cursor to 4.56 on CI, read β on T under hairline. 34.4°

4 Cursor to 34.4° on S, read under hairline a = 5.45 on CI.

Example

13 Given $b = 4$, $c = 12$, $\alpha = 108°$, find a and β.

Step 1 Bring 4 on CI over sin (180° − 108°) on S
Under 1 of C read p = 3.81 on D.

2 Bring 3.81 on C over r.h.i. of D

3 Cursor to (180° − 108°), i.e., 72° on T_1
read x = 1.238 on C

Here $y = (c+x)$, so $y = 12 + 1.238 = 13.238$.

4 Set r.h.i. of C over 3.81 on D and over 13.238 on CI
read $\beta = 16.05°$ on T_1.

5 Over 16.05° on S read a = 13.8 on CI.

In problems of this type, the data may make it necessary to reset the slide – as in the second example – particularly when angles > 45° must be read or set on scale T. Note that when using the red figures of scale T, we used scale C instead of CI.

The scale S on the slide.

The advantages of a moveable scale of sines, preferrably an addition to the fixed scale, is seen when problems necessitating a division by a sine function value, or a similar manipulation, arise.

Example

14 Solve foı x the proportion $\frac{x}{\sin 25^\circ} = \frac{12.36}{\sin 45^\circ}$

Here the obvious solution is cross-multiplying.

Using the fixed S scale, bring cursor to 25° on the sine scale.

l.h.i. to hairline,

Cursor to 12.36 on C,

Turn the rule over and bring 45° on S (moveable), to hairline.

Under r.h.i. of C read 7.4 on D.

Example

15 In a problem in mechanics, resolving a force acting at 22.5° to the horizontal, gave as the vertical component of the force, 18N. What was the force?

The vertical component of the force would be the product of the force and the sine of the angle made by the force with the horizontal axis.

$$\text{So, } x \sin 22.5^\circ = 18 \text{ N.}$$

$$\text{or } x = \frac{18 \text{ N}}{\sin 22.5^\circ}$$

The moveable scale of sines gives an immediate solution:

Bring 22.5° on the moveable S scale to 18 on D.

Under r.h.i. of C read 47 N on D.

Where much work involving the trigonometrical scales is regularly required, consideration should be given to the facilities offered by the Emblem model 1053, which has an additional scale S on the slide.

The two last examples can easily be checked by application of the sine rule, as could examples 11 and 12. This is the point at which the facility of the moveable scale of sines on the slide of model 1053 will be noted. There is no need to determine the function value for sine etc. All that is necessary is to bring the angle on S over the corresponding side on D and read the quotient on D under the index of C – one slide setting suffices. Rules with scale S on the body panel require the side to be set on C over the angle on S. Model 1053 has advantages when, as in survey work, sines appear in the numerator and in the denominator of a combined multiplication and division expression.

Vectors

If, in a right triangle, we are given the two sides containing the right angle and have to find the hypotenuse, R, a situation arising, for example, when finding the resultant of two forces acting at right angles, we can use the Pythagorean relationship and solve as

$$R = \sqrt{a^2 + b^2}$$
$$= a\sqrt{1 + (b/a)^2}$$

Usually, when dealing with vectors, we require not only the value of R but also the angle made by the resultant R with the base line. A neat solution is possible with the slide rule.

Example
16 In a right angled triangle, given a = 4, b = 3, (these being the lengths containing the right angle) Find R and θ.

Step 1 Index of C to 3 on D
2 Cursor to 4 on CI
3 Under hairline read $\theta = 36.8^\circ$ on T
4 Cursor to 36.8° on S
5 Read R = 5 on CI

The j notation

Work in electrical, electronic and telecommunications engineering frequently involves the j notation, i.e., the use of the operator j. ($j = \sqrt{-1}$). Most common is the conversion of a vector expression in rectangular coordinate form to polar coordinate form, e.g., from a + jb to $R\angle\theta$. This work directly applies the process examined in the previous example.

The vector example given could well have had its origin in the following typical problem:
Find the impedance Z and the phase angle of a circuit comprising a resistance of 4 ohms and an inductive reactance of 3 ohms.
Here, $Z^2 = a^2 + b^2$ and $\theta = \sin^{-1} b/2$ or $\tan^{-1} 3/4$

We write $Z^2 = 4 + 3j$

$$Z = 4\sqrt{1 + (3/4)^2}$$

and solve as previously explained, writing our answer as $\underline{5/\,36.8^\circ}$

In all work in trigonometry, it is recommended that a figure be drawn to represent the data. The value of the advice will be seen if we consider the conversion of 4 – 3j to the polar form. A diagram will make clear that $\theta > 90°$, R will not lie in the first quadrant and θ may well have a negative sign. The steps in the slide rule solution will be exactly as for the expression $Z^2 = 4 + 3j$, but the final polar form will be found as $5\underline{/ -36.8°}$

The work in the last sections of this discussion of the use of the trigonometrical scales may appear rather complicated, to the beginner. Many readers will not have occasion to deal with vectors, for example, and should not be discouraged if they find the application of the slide rule to such work difficult. Once again, we advise you to study the nature of your tasks. Determine, by reference to the various sections of this booklet, what slide rule processes are of greatest use to you and practice these until the logical, safe and time-saving techniques become familiar to you.

THE LOGLOG SCALE

Sect. 20 **Scale LL3, LL2, LL1**

These three 'scales' are actually three segments of one continuous LogLog scale, each divided decimally and used, for most purposes, in conjunction with the fundamental scales.

Most readers likely to use the LogLog scale will be familiar with logarithms. An approach to an understanding of the LogLog scale will be a consideration of the solution of an expression such as $y = x^a$, by means of the slide rule. In this expression, a is the exponent or power to which x must be raised to achieve the value y.

We could solve, by using logarithms, as log y = a log x. On the slide rule, however, it would be more convenient to have a scale graduated in intervals proportional to the logarithms of logarithms. Such a scale is the LogLog scale, opened out for convenience into three segments.

With this scale, we solve the given problem in the form:

$$\log\log y = \log a + \log\log x$$

Example: Evaluate $y = x^a$, when a, the exponent, takes the values 2, 3 and 4 and x, the base, is 3.

Method: Set the base, $x = 3$, with the cursor on LL3
Bring index of C to cursor
Move cursor to exponent 2 on C
Under hairline read $3^2 = 9$ on LL3
Move cursor to exponent 3 on C
Under hairline read $3^3 = 27$ on LL3
Move cursor to exponent 4 on C
Under hairline read $3^4 = 81$ on LL3

We have added, mechanically, log 2, log 3 and log 4, as lengths on C, to the log of log 3 on the LogLog scale. This achieves multiplication and we read the result, y, on the LogLog scale, segment LL3. A simple base, $x = 3$ and a simple series of exponents $a = 2$, 3 and 4 were used so that you could at once check the results.

It is important to note that the values figured on the LogLog scale are absolute. The position of the decimal point is fixed and we cannot read the value, for example, 2.8 on LL3 as 28, 2800 or 0.28, as we can on the fundamental scales or on scales derived therefrom.

Note also that the segments of the LogLog scale are so arranged that values corresponding in position on, say, LL3 and LL2 are the 10th power and the 10th root, respectively, of each other. This leads to an immediate display of the values of $y = x^a$ for a progression of decimal variants of a. See Fig. 15.

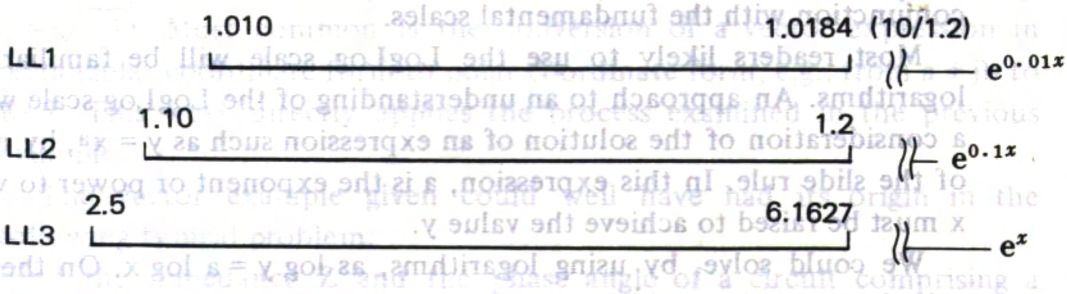

Fig. 15

Sect. 21 **Reading the LogLog scale**

$y = x^a$

(a) When $a > 1$ and the exponent, set on C, does not lie beyond the right hand end of the log.log scale segment in which x is located, the value of y is read on the same loglog segment as is used to set x.

(b) If the exponent on C lies beyond the right hand end of the relevant loglog scale segment, change the index of C from 1 to 10, move the cursor to the exponent on C and read y on the next loglog scale segment.

Example

17 $y = (1.02)^8$

To evaluate, set Cursor to 1.02 on LL1
bring r.h.i. of C to cursor
cursor to 8 on C
under hairline read y = 1.1716 on LL2

(c) When $a < 1$, set the r.h.i. of C to the value of x on the relevant loglog segment and move the cursor to the exponent on scale C. If the exponent is not beyond the left hand end of the loglog segment, y will be read on the same loglog segment as contains x. If, however, the exponent cannot be set on C, change the index of C, bring the cursor to the exponent and under the hairline read y on the adjacent, lower, loglog segment.

Example

18 Evaluate $y = (1.2)^{0.3}$

Set 1.2 on LL2, read y = 1.0562 on LL1, under the cursor line over 3 on C

Roots $y = \sqrt[a]{x}$

With $x > 1$, the slide rule solution follows the form:

$$\text{loglog } y = \text{loglog } x - \log a$$

We therefore bring a on C over x on the appropriate loglog segment and read y under the index of C. Here, again, changing the index of C from 1 to 10 may be necessary and y is then to be read on the next, lower, loglog segment.

Example

19 $y = (1.2)^{\frac{1}{0.3}}$ (or $\sqrt[0.3]{1.2}$)

Here we must read y over the r.h.i. of C. The value of x, i.e., 1.2, is set on LL2 and that of y is read, over 10 on C, from LL1 – 1.0627.

Sect. 22

Scale segments LL03, LL02, LL01

These are the reciprocals of LL3, LL2 and LL1 and are used when solving such problems as $y = x^a$, in which $x < 1$, or $a < 1$ or both x and a are less than unity.

Example

20 Evaluate $y = (0.4)^3$. Here $x < 1$, $a > 1$.

If a value x less than unity is raised to a power greater than unity, the result will be smaller than the value x. Therefore, we will look for our result in such cases on a lower member of the LL03, LL02, LL01 series. Here, we set 0.4 on LL02, with the r.h.i. of C and with the cursor at $a = 3$, on C, read 0.4^3 on LL03, i.e., 0.064.

With $x < 1$ and $a < 1$, the value of $y = x^a$ will be greater than x.

Example

21 $y = (0.25)^{0.3}$

Take 0.25 on LL03, using the l.h.i. of C. Move cursor to 3 on C and under the hairline read 0.66 on LL02.

Sect. 23

The Logarithmic base

The scales previously examined, (the fundamental scales and their derived scales), are divided into lengths proportional to the logarithms of plain numbers. The logarithmic base used is 10 and the logs. are called 'Common Logs'. Any number may be used as the base of a system of logs. For most purposes, base 10 is used but in some branches of applied science, the indeterminate number 2.718. . . (for which the symbol e is written), has advantages and logarithms to base e are 'Natural' or Naperian logarithms. The LogLog scale is based on e and is so positioned with respect to scale D that $e = 2.718\ldots$ on LL3 is immediately under index 1 on D.

Knowing that setting either index of C to a number, x, on D, establishes a ratio between 1 and x, it follows that with 1 on D over e on LL3, we have a ratio between the values on LL3 and D such that, by cursor movement, we can read the value of e^x for any exponent x found on D – reading the value on the LogLog scale. Because LL3, LL2 and LL1 are segments of one continuous scale, the relationship between e and D holds throughout.

Example

22 Evaluate e^2, $e^{0.2}$ and $e^{0.02}$

Bring cursor to 2 on D
Under hairline read 7.4 on LL3 (e^x)
Under hairline read 1.2215 on LL2 ($e^{0.1x}$)
Under hairline read 1.0202 on LL1 ($e^{0.01x}$)

We can also find the power to which e must be raised to obtain a given number. Suppose the number is 4. We must evaluate x in $e^x = 4$. Set cursor to 4 on LL3 and under hairline read 1.1385 on D.

Sect. 24 Values 'off scale'

Occasionally an attempt to evaluate expressions such as $y = x^a$ fails because the given values of a and or x produce a value of y not available on any loglog segment. An example of this is $23^{4.2}$. Although we cannot solve this directly on the slide rule, we can use the index laws and write $y = 23^{2.1} \times 23^{2.1}$, that is, divide the exponent so that y is brought within range.

Thus Cursor to 23 on LL3
Index of C to hairline
Cursor to 2.1 on C
Under hairline read $23^{2.1} = 724$ on LL3
Then $23^{4.2} = 724 \times 724$, i.e., 5.24×10^5

Sect. 25 Hyperbolic Logarithms

Slide rule users faced with problems involving the hyperbolic functions will be well grounded in mathematics and will need little explanation of the value of the loglog scale in their work. Suffice it to say that the basic expressions:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

can be evaluated, for given values of x, at one setting.

Example

23 Find sinh x and cosh x when x = 1.3.

Method: Set cursor to 1.3 on D
Under hairline read $e^{1.3}$ on LL3 = 3.67
Under hairline read $e^{-1.3}$ on LL03 = 0.272

$$\text{Whence } \sinh x \text{ is } \frac{3.67 - 0.272}{2} = 1.699$$

$$\cosh x \text{ is } \frac{3.67 + 0.272}{2} = 1.971$$

Sect. 26

Miscellaneous applications

Confidence and expertise in the use of the loglog scale will be gained by continued practice. The following examples will illustrate situations in which they can be used, drawn from various fields of work.

Example A. A problem in thermodynamics, applying the law: $p_1 v_1 = p_2 v_2{}^n$ requires the evaluation of n when $p_1 = 150$ p.s.i.a. $v_1 = 160$ cu. ft.

$p_2 = 15$ p.s.i.a. $v_2 = 1000$ cu. ft.

Solution: Arrange the data in the form:

$$\frac{150}{15} = \left(\frac{1000}{160}\right)^n$$

i.e., $10 = 6.25^n$

Bring l.h.i. of C over 6.25 on LL3

Cursor to 10 on LL3

Under hairline read n = 1.256 on C

Example B. Find the amount A to which £350 invested at 3½ % Compound Interest would grow in 6 years.

The appropriate formula is: $A = 350\left(1 + \frac{3.5}{100}\right)^6$

Solution: Cursor to 1.035 on LL1

Index of C to hairline

Cursor to 6 on C

Under hairline read 1.229 on LL2

Set 1.229 on D under index of C

Cursor to 350 on C

Under hairline read A = 430 on D

Answer £430

Example C. Find, to four significant figures, the reciprocal of 1.07. Here, at a 'best guess', scales C/CI will only give three figures. We can however use the reciprocal arrangement of the LogLog scale to obtain a better result. Set cursor to 1.07 on LL1 and under hairline read 0.9346 on LL01.

Example D. Evaluate $\sqrt[1.73]{2.58}$. To save slide movement, we can use the CI scale in conjunction with the LogLog scale.

Cursor to 2.58 on LL3

Index (l.h.i.) of C to hairline,

Cursor to 1.73 on CI

Under hairline read 1.76 on LL2.

Example E. An electrical circuit contains a resistance R of 24 ohms and an inductance L of 0.12 henry in series with a battery of negligible impedance and a switch. What is the instantaneous current, i, in amperes at a time t of 0.002 seconds after closure of the switch, if the final current I is 6 amperes?

Using the formula $i = I\,(1 - e^{-Rt/L})$

we have $i = 6\left(1 - e^{\frac{-24 \times 0.002}{0.12}}\right)$

Simplifying the exponent, we have $i = 6\,(1 - e^{-0.4})$

Cursor to 4 on D

Under hairline read $e^{-0.4} = 0.67$ on LL02

Index of C to 0.67 on D

Under 6 on C read 402 on D

Then $i = 6 - 4.02$

$= 1.98$ amperes.

Sect. 27 Quadratic Equations

In schoolwork, quadratic equations are usually solved easily by one or other of the standard procedures. In practical work, the figures arising are often less tractable and solution involves tedious arithmetic. Once again, the slide rule is an effective aid.

To approach solution, we remember the 'formula' method:

Given $ax^2 + bx + c = 0$, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

also, that $x^2 + (b/a)x + (c/a) = 0$ can be written as $(x-r)(x-s) = 0$, where r and s are roots of the equation.

The quantity $b^2 - 4ac$ is the discriminant, D and when

$D < 0$, the roots r and s are imaginary

$D = 0$, the roots are real and equal

$D > 0$, the roots are real and unequal

In practice, we have usually to deal with real roots and if we write the equation in the form $x^2 - (r+s)x + rs = 0$, we can use the slide rule.

We begin by evaluating the discriminant, D, to ascertain the character of the roots. We next study the coefficient. If rs is positive, r and s have like signs. The sum of the roots is, with due regard to sign, the value of the coefficient of x. We need to find on the rule two numbers (r and s) which, added with proper regard to sign, are equal to b and which, multiplied together, have the product c. The method will be clear from an example.

What are the roots of $x^2 - 5.7x - 14.44 = 0$

The discriminant D is positive and the roots r and s are thus real and unequal. The product (rs) is negative, so r and s have different signs. Since the coefficient of x is negative, the larger of the two roots will be negative.

Set index of CI over 14.44 on D.

(This sets up the infinite series of pairs with product 14.44).

Look along CI and D to find a pair of numbers the algebraic sum of which is −5.7. Clearly this pair will be found in the range 1 – 2 on the D scale.

Because we must have regard to sign, and the roots have unlike signs, we must here subtract one number from the other, to obtain −5.7.

Inspection suggests the pair 7.6 on CI and 1.9 on D.

The required roots are then −7.6 and 1.9.

Another example: $x^2 + 7.5x + 8.06 = 0$

Here D is 24.01 and positive. Roots are real and unequal. The coefficient of x is positive and c is positive, so roots have like sign.

Set index of CI to 8.06 on D

Look for pairs on CI and D, with sum 7.5

Try 6.2 on D and 1.3 on CI

The required roots are 6.2 and 1.3.

Changing signs in another example: $x^2 + 1.6x - 11.61 = 0$

D is 49 and positive, so roots are real and unequal.

The product of the roots is negative, the sum positive, thus the roots have unlike signs and the larger is positive.

Set index of CI to 11.61 on D.

Look for pairs with algebraic sum 1.6

Try 4.3 and 2.7

Agreeing signs, roots are +4.3 and −2.7.

The remaining possibility, $x^2 - bx + c = 0$ is a simple application of the foregoing procedure.

With a little practice, the method is a quick and easy approach to the solution of a quadratic, with an accuracy at least equal to that of the originating data. Where refined methods, say, iteration, are to be applied, the slide rule will save time by establishing the starting point.