55/-

CASTELL-Addiator Slide Rule

No. 67/54R SYSTEM DARMSTADT

INSTRUCTIONS

for multiplication
division
addition
subtraction

A.W. FABER - CASTELL, STEIN BEI NÜRNBERG

NOTE.

The Slide Rule **CASTELL** **„System Darmstadt"**

resulted from the work of the Mathematical Institute of the Technical University of Darmstadt under the direction of Professor W a l t h e r

and was brought out by the firm of A. W. FABER **CASTELL** at the instance of Professor Walther.

Description of the Slide Rule.

The Slide Rule **CASTELL System Darmstadt** is a general purpose slide rule. Its logarithmic scales make possible all the calculations which are met with in mathematics and their practice. It carries no special scales such as would be required for commercial, nautical purposes, reinforced concrete, or any other narrow field of activity.

The scales of the System Darmstadt Slide Rule are grouped as follows:

1. The **Main Scales A, B, C, D** (x) and **Cr.**
2. The **Supplementary Scales Cu, P** ($\sqrt{1-x^2}$), **L,** the trigonometrical scales, and the log-log scales.

The Main Scales.

Even the simplest general slide rule has the upper scales, **A** and **B,** and the lower scales, **C** and **D.** Therefore, these are called the **Main Scales** of the rule.

Scales **A** and **B** are exactly alike, and extend from **1 to 100.** Scales **C** and **D** are also alike, and run from **1 to 10.** Scales **A** and **D** are on the body of the rule, and are, therefore, known as the **Rule Scales.** **B** and **C,** being on the slide, are known as the **Slide Scales.**

In addition to these four scales, there is a **reciprocal,** or reversed C, scale on the centre of the slide between B and C. This scale, **Cr,** runs from **10 to 1** (Fig. 1).

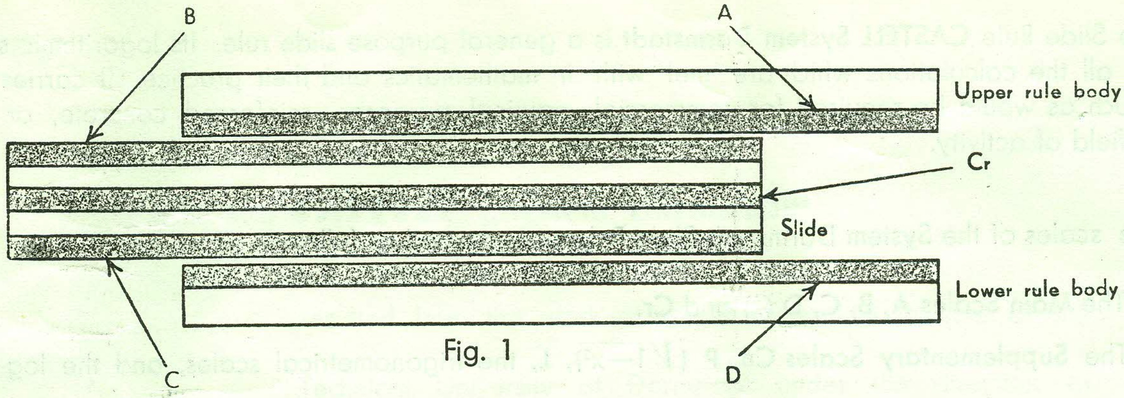


Fig. 1

These five scales are extended a short distance at each end, the extra graduations being a different colour to the main part of the scales.

For all calculations containing only multiplication and division the three scales **C, D,** and **Cr** should be used.

The Supplementary Scales.

Additional scales are provided to facilitate calculations other than multiplication, division, squares and square roots:

The **cube scale Cu** is on the rule face above **A.** It is graduated from **1 to 1000,** and is used with Scale **D** (Fig. 2).

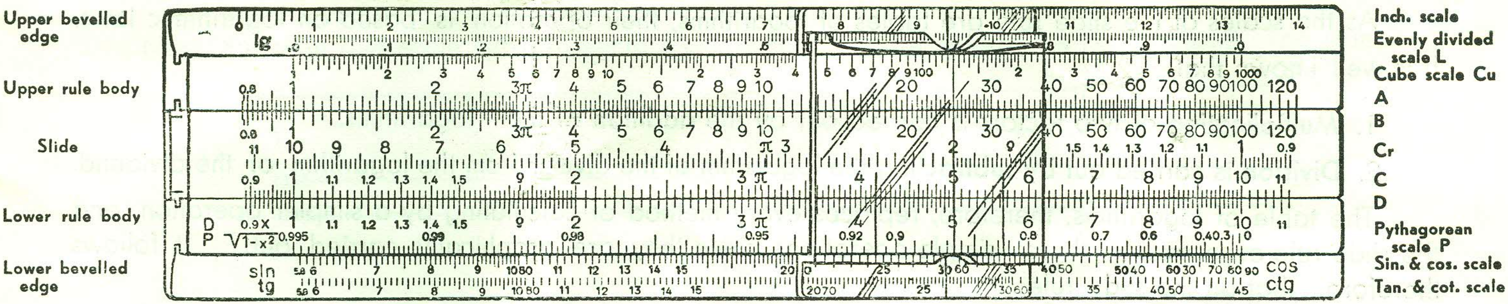


Fig. 2

The **evenly divided scale L** on the bevelled edge of the rule body is used in conjunction with Scale **D** for reading common logarithms.

The **Pythagorean scale P** ($\sqrt{1-x^2}$) is on the rule face below **D,** with which scale it is to be employed. Its uses will be explained later.

The **trigonometrical scales** will be found on the lower bevelled edge of the rule body.

Finally, there is a **log-log-scale,** graduated in three sections, **from 1·01 to 10^5,** on the back of the slide.

The cursor enables these scales to be employed in any combination. The long centre line is generally used, while the two short lines at the sides are provided for a special purpose which will be explained later.

How To Calculate With The Slide Rule.

As the scales of the slide rule are tables of logarithms, their operation is based on logarihtmic laws. It is well known that:

1. **Multiplication** of two factors is carried out by the **addition** of their logarithms.
2. **Division** is carried out by **subtracting** the logarithm of the divisor from the logarithm of the dividend.

The table of logarithms, therefore, replaces every method of calculating by a simpler operation, and the slide rule even avoids these simple operations, since they are graphically carried out. It follows therefore, that on the slide rule:

Multiplication of two numbers is transformed into addition of two lengths.

Division of one number by another is changed to subtraction of one length from another.

Graphical calculation is best explained by using two millimetre scales. In Fig. 3, the addition $35 + 45 = 80$ is worked.

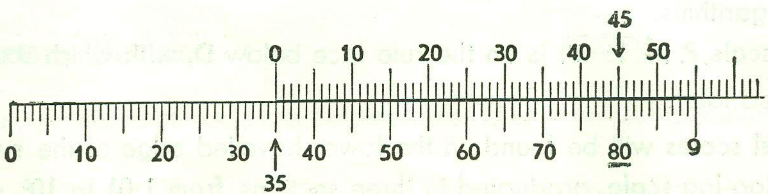


Fig. 3

Fig. 4 shows the subtraction 115 — 53 = 62.

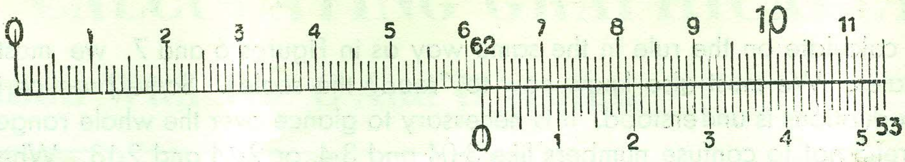


Fig. 4

Now, the slide rule scale is a graphical representation of **logarithms,** as Fig. 5 shows. The number 3

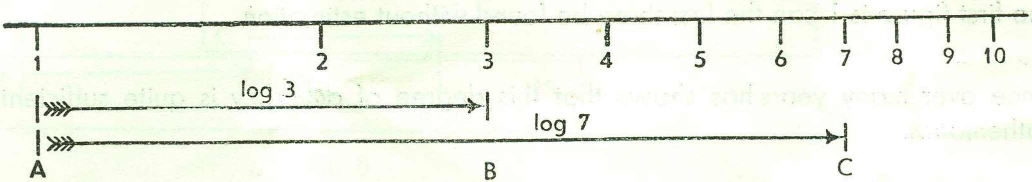


Fig. 5

stands at the extreme end of the length or section log 3; all logarithmic lengths are measured from the beginning of the scale, and this point is marked 1, because log 1 = 0.

When both graphical calculations of Fig. 3 and Fig. 4 have been carried out on the logarithmic scale, the result is not the sum and difference of both numbers, but the **product** and **quotient,** as Figures 6 and 7 show.

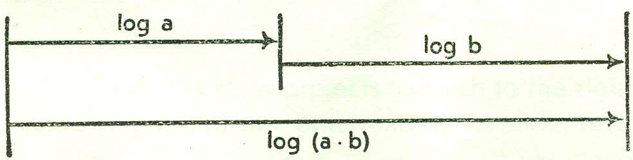


Fig. 6

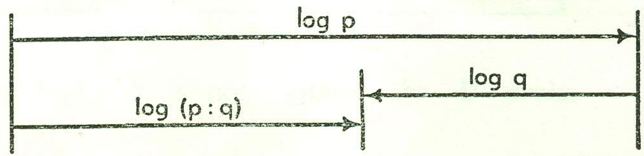


Fig. 7

All other uses of the logarithmic scales are only variations of these two fundamental problems.

If we wish to calculate on the rule in the same way as in Figures 6 and 7, we must "set" the given numbers on the scales. We must also know how to "read" the scales. Some practice is required before the value of the graduations is understood. It is necessary to glance over the whole range of the scales and to be specially careful not to confuse numbers like 3·04 and 3·4, or 2·14 and 2·18. When the graduations are thoroughly understood, it is time to practise the insertion and valuation of the last figure. It is a rule that the first two figures should be set or read with certainty, while the third has to be estimated. Only when the first figure is 1 can the first three be found without estimation.

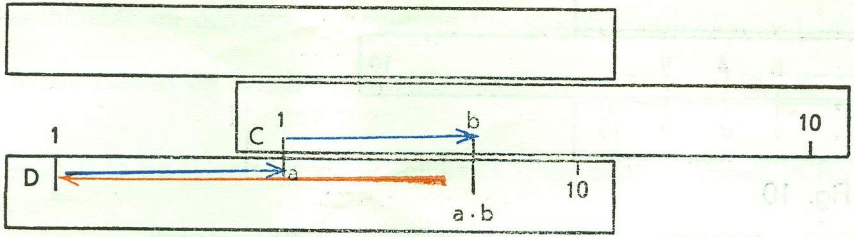
Experience over many years has shown that this degree of accuracy is quite sufficient for all applications in mathematics.

There is no decimal point in slide rule calculations. Therefore, numbers like 13·45; 0·1345; 1345; 1·345 are all read as a row of figures 1—3—4—5. Where the decimal point must be placed in the answer will usually be clear from the problem. But when this is not so, a rough estimate with round numbers will indicate the number of figures in the answer.

CALCULATING GRAPHICALLY.

Multiplication With The C And D Scales.

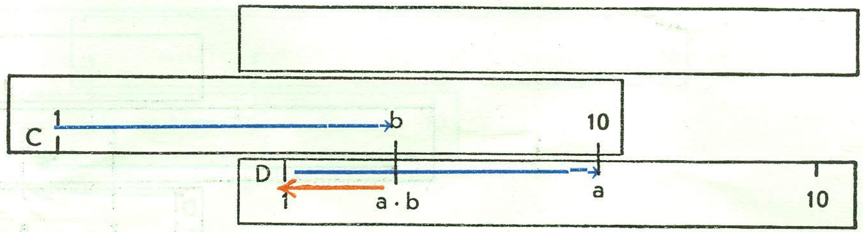
Either



a . b

Fig. 8

Or



a . b

Fig. 9

When the slide projects to much to the right that the factor b is not against the **D** scale, it is necessary to use the other end of the slide, as shown in Fig. 9.

The two scales form a **table;** Scale D is **b-times** the value of Scale C.

Division With The C And D Scales.

Either

$\frac{a}{b}$

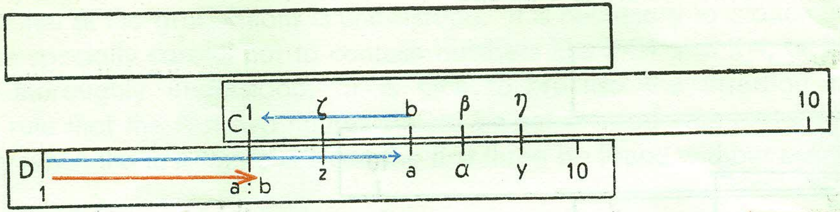


Fig. 10

Or

$\frac{a}{b}$

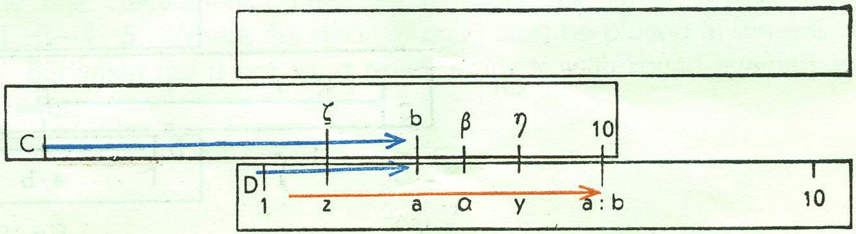


Fig. 11

The answer can only be read at **that** end of the **C** scale which is inside the rule body.

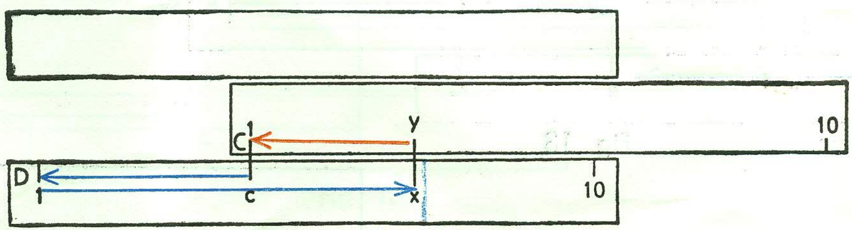
This setting produces a **table** of all pairs of numbers that have the ratio **a ÷ b.**

$$\frac{\alpha}{\beta} = \frac{y}{\eta} = \frac{z}{\xi} = \frac{a}{b}$$

In this manner all conversions requiring the fourth proportion can be solved, such as, for instance:

when metres on C are set to yards on D, read 75 m. = 82 yd.

If $y = \frac{x}{c}$ is to be solved for many values of x, the method shown in Fig. 12 will be the most convenient.



$$y = \frac{x}{c}$$

Fig. 12

Multiplication And Division Using Scale Cr.

The reversed scale **Cr** (which requires care in reading) is a very important scale; it simplifies many calculations and makes others possible.

Between Scales **C and Cr** there is a **reciprocal relationship,** each graduation on **C** being the reciprocal of the one immediately above it on **Cr,** and vice versa. For instance, 30 and 0·0333, 2·5 and 0·4, 125 and 0·008.

If both factors a and b are set in line on **D** and **Cr,** with the help of the cursor, a very convenient method of multiplying is obtained. The answer is always found, either by reading to the left (Fig. 13) or to the right (Fig. 14).

a . b

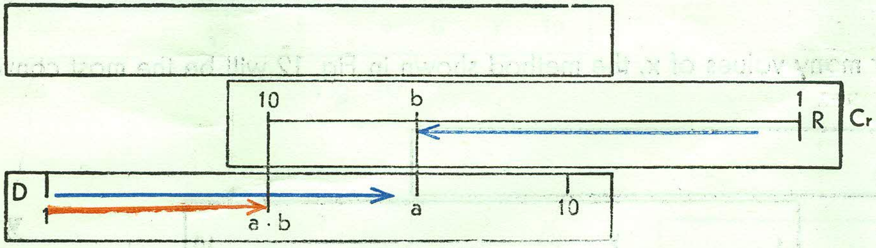


Fig. 13

a . b

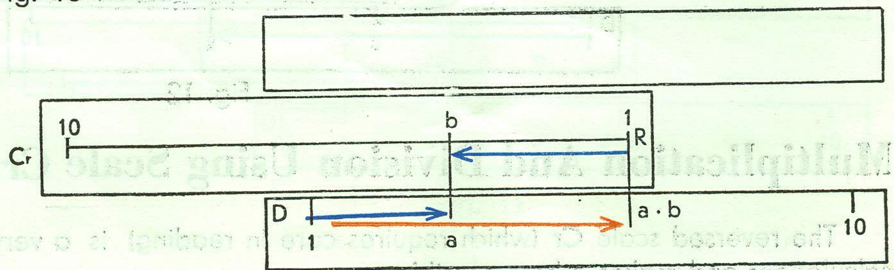
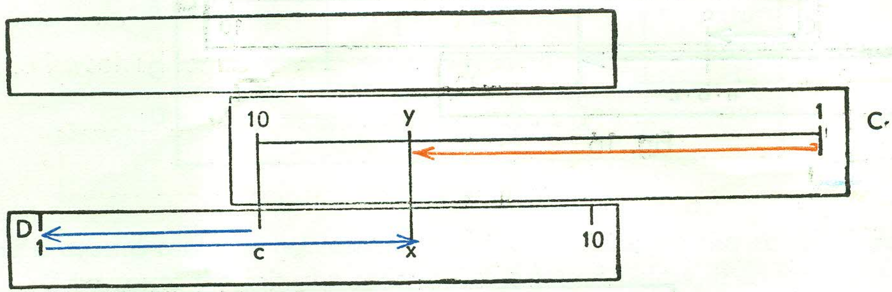


Fig. 14

If $y = \frac{c}{x}$ is to be solved for several values of x, the method shown in Fig. 15 should be employed. With this setting a table is formed which gives all pairs of numbers having c as their product **(inverse proportion).**



$$y = \frac{c}{x}$$

$$x \cdot y = c$$

Fig. 15

The rule is now set to give all possible factors of the number c that satisfy the quadratic equation

$$x^2 + s \times x + c = 0,$$

but the sum must be — s.

With the reverse scale, **Cr,** it is possible in most cases to find the product of three factors with one setting (Fig. 16). Reversing the procedure gives division with two divisors (Fig. 17).

a · b · c

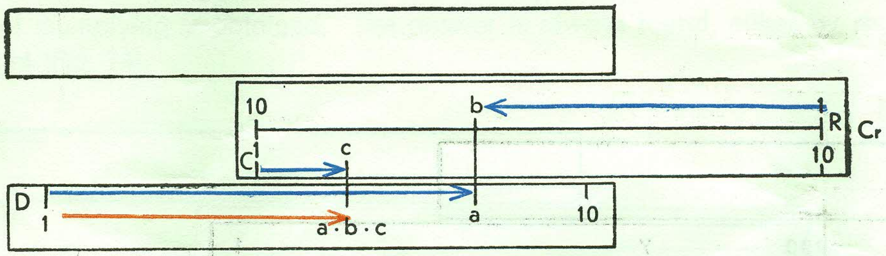


Fig. 16

$$\frac{p}{q \cdot r}$$

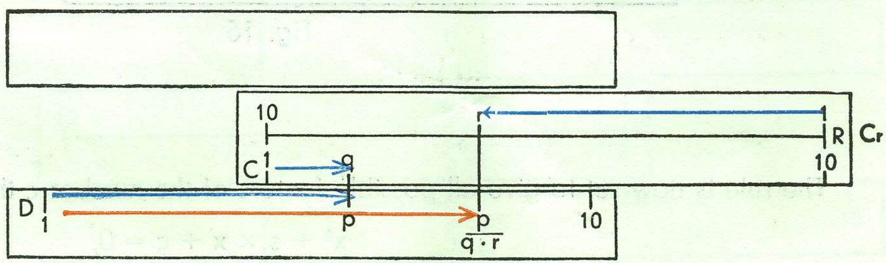


Fig. 17

Squares And Square Roots.

Both the upper scales are graduated to half length. The change over from **D** to **A** (and likewise from **C** to **B**) gives the **square** of the number that is set on **D.** **Square roots** are extracted by reversing this procedure (Fig. 18).

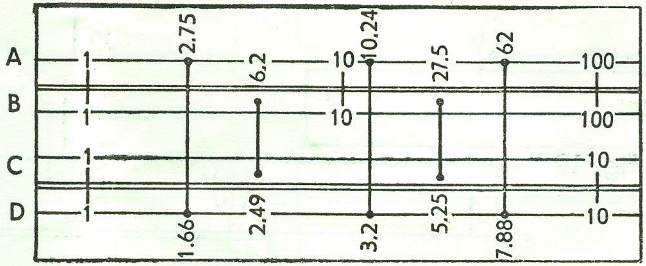


Fig. 18

a^2

$\sqrt{a}$

In extracting a square root it is neccessary to set the number on the correct half of the upper scale. If the figures 6 . . 2 be set to the left, the root of 6·2 is found below, if to the right it is the root of 62. It is also necessary to take notice of the numbered graduations 1 . . . 10 . . . 100. If the number lies outside the scale range 1 to 100, it should be factorised by hundreds to bring the significant figures within these limits.

Example: $\sqrt{1922} = \sqrt{100\times19{\cdot}22} = 10\times\sqrt{19{\cdot}22} = 10\times4{\cdot}38 = 43{\cdot}8$

$\sqrt{0{\cdot}000071} = \sqrt{71\div1000000} = \sqrt{71}\div1000 = 8{\cdot}43\div1000 = 0{\cdot}00843.$

When both the upper and lower scales are used in conjunction many combined calculations are possible, as the following figures will show.

When the calculation contains a **square** it is necessary to **commence** on the **lower** scales so that the answer comes on an upper scale. There are eight alternative:

$(a \cdot b)^2$

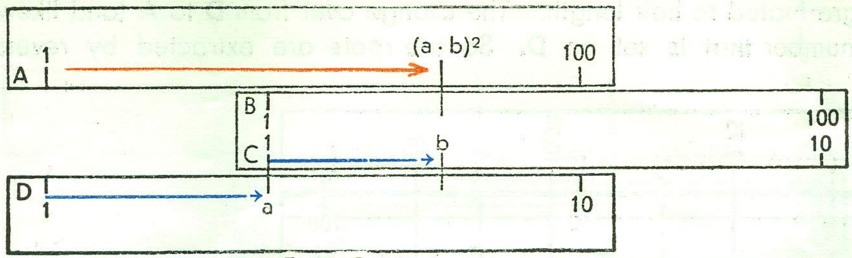


Fig. 19

$\left(\frac{a}{b}\right)^2$

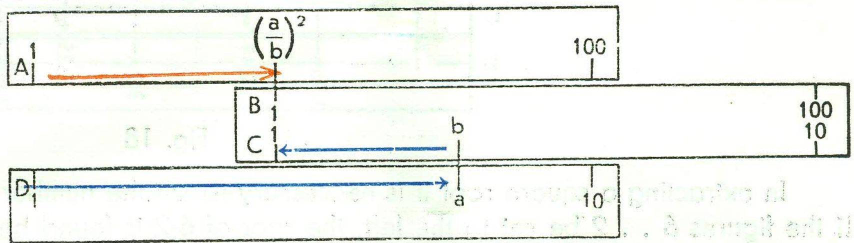


Fig. 20

$a^2 \cdot b$

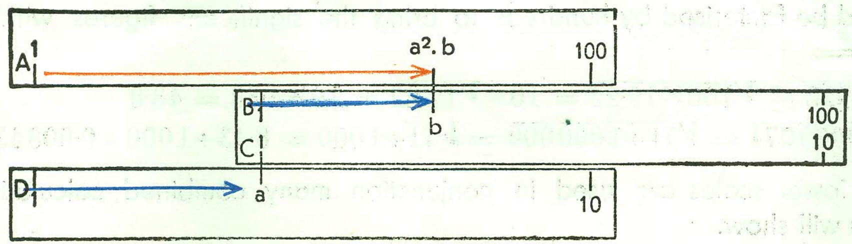


Fig. 21

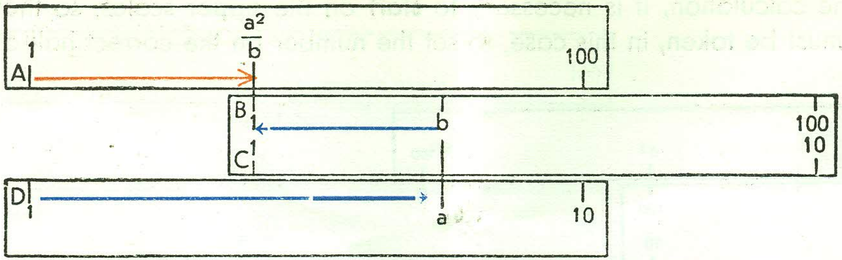


Fig. 22

$$\frac{a^2}{b}$$

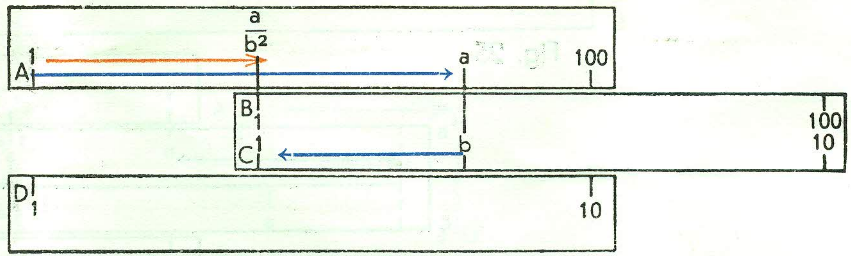


Fig. 23

$$\frac{a}{b^2}$$

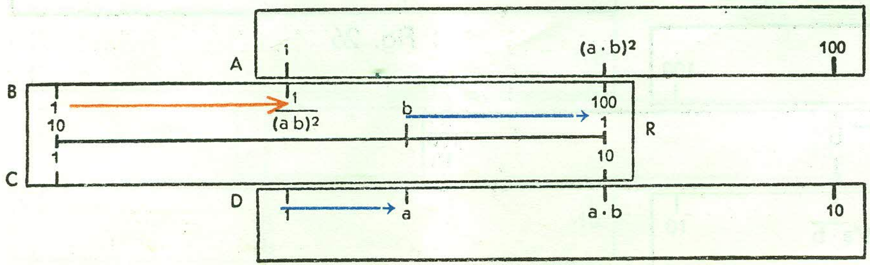


Fig. 24

$$\frac{1}{(a \cdot b)^2}$$

Should there be a **square root** in the calculation, it is necessary to **start** on the **upper** scales, so that the root can be found on a lower. Care must be taken, in this case, to set the number on the correct half of scale **A** or **B.**

$$\frac{1}{a^2}$$

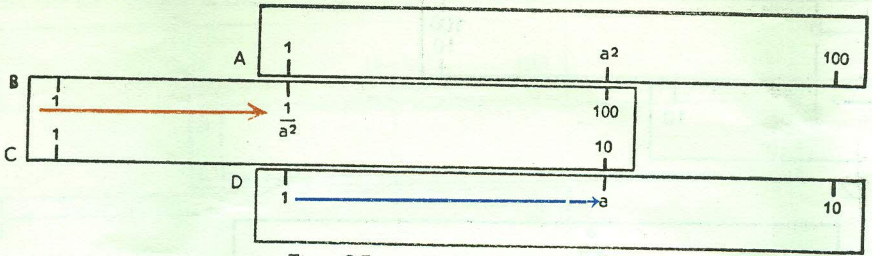


Fig. 25

$$\frac{1}{a^2 \cdot b}$$

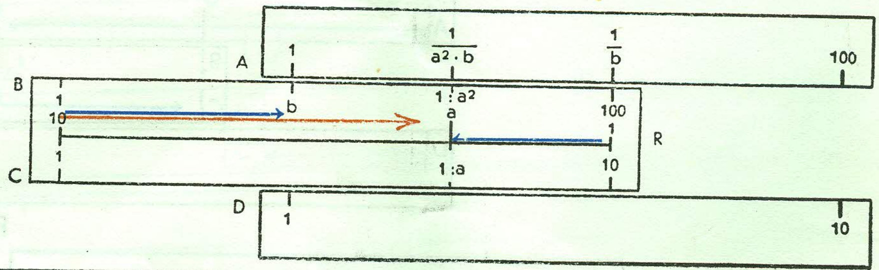


Fig. 26

$$\sqrt{a \cdot b}$$

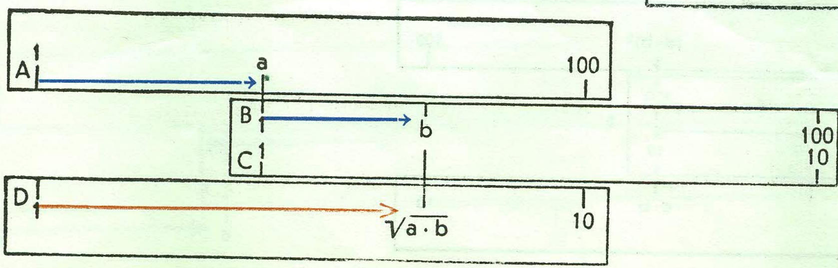


Fig. 27

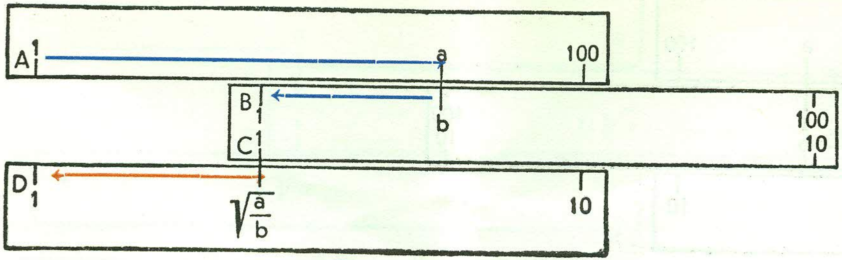


Fig. 28

$$\sqrt{\frac{a}{b}}$$

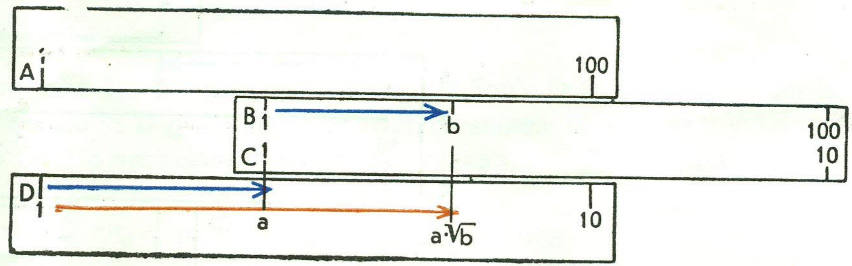


Fig. 29

$$a \cdot \sqrt{b}$$

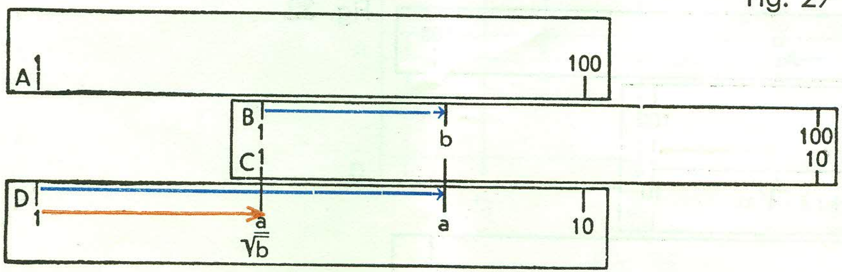


Fig. 30

$$\frac{a}{\sqrt{b}}$$

$$\frac{\sqrt{a}}{b}$$

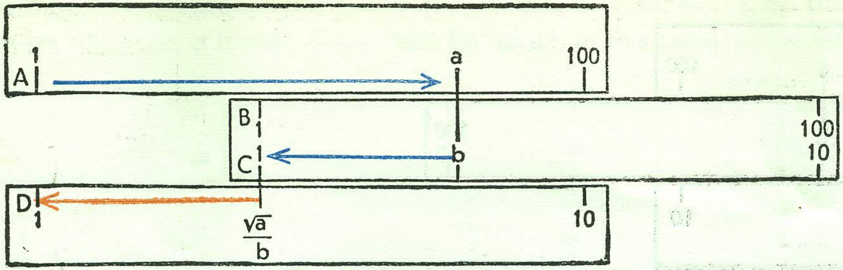


Fig. 31

$$\frac{1}{\sqrt{a \cdot b}}$$

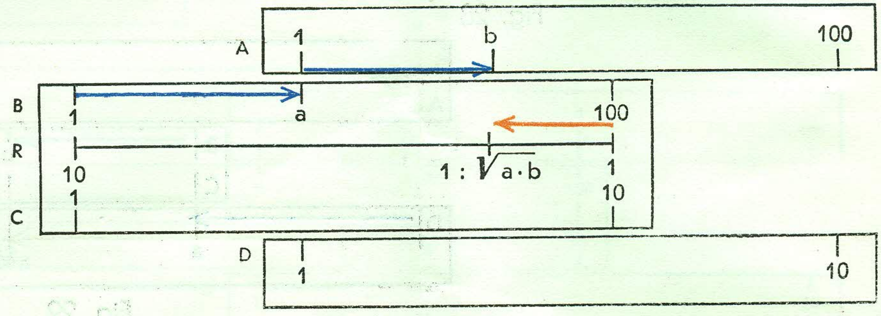


Fig. 32

$$\frac{1}{a \cdot \sqrt{b}}$$

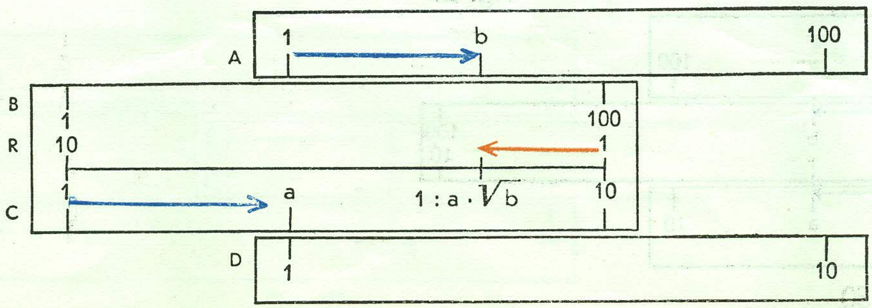


Fig. 33

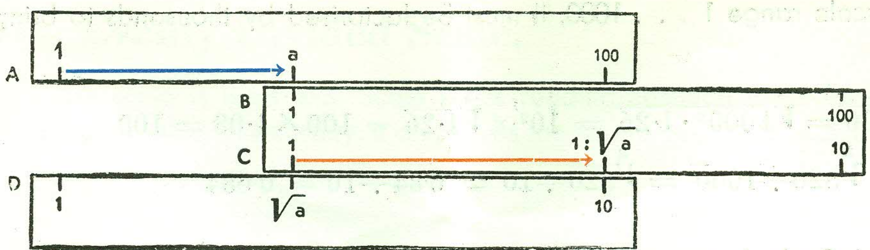


Fig. 34

36 6 216

$$\frac{1}{\sqrt{a}}$$

Cubes And Cube Roots.

Scale **Cu** is graduated in the ratio 1 : 3. In passing over from Scale **D** to **Cu** the number is raised to the **third power,** while passing from **Cu** to **D** gives the **cube root,** as shown in Fig. 35. When setting the number for a cube root on Scale **Cu** it is necessary to watch the values 1 . . . 10 . . . 100 . . . 1000.

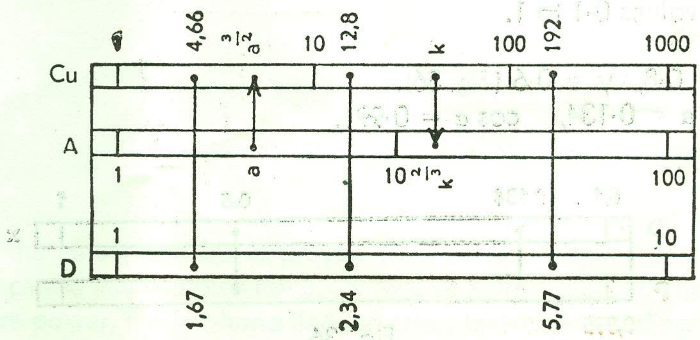


Fig. 35

a^3 $\sqrt[3]{a}$

$a^{\frac{3}{2}}$ $a^{\frac{2}{3}}$

If the number does not lie within the scale range 1 . . . 1000, it must be factorised by thousands to bring it within these limits.

$$\text{Example:}\quad \sqrt[3]{1260000} = \sqrt[3]{1000^2 \cdot 1{\cdot}26} = 10^2 \times \sqrt[3]{1{\cdot}26} = 100 \times 1{\cdot}08 = 108$$

$$\sqrt[3]{0{\cdot}32} = \sqrt[3]{320 \div 1000} = \sqrt[3]{320} \div 10 = 6{\cdot}84 \div 10 = 0{\cdot}684$$

If the cube scale be employed with Scale **A**, powers having the exponents $\frac{3}{2}$ and $\frac{2}{3}$ may be found (Fig. 35).

The Pythagorean Scale.

This scale represents the function $y = \sqrt{1-x^2}$. It is employed in combination with Scale **D** $(= x)$, the latter having the range of values 0·1 to 1.

Examples: $x = 0{\cdot}8$, $y = 0{\cdot}6$ (Fig. 36).
$\sin\alpha = 0{\cdot}134$, $\cos\alpha = 0{\cdot}991$.

$\sqrt{1-x^2}$

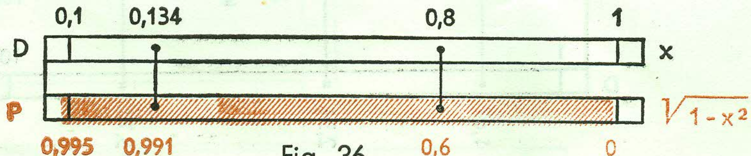


Fig. 36

The Evenly Divided Scale.

This scale is used with Scale **D** for reading **common logarithms,** and may be used in place of a **three-figure table.** Naturally, it only gives the mantissae, the characteristic being found in the usual way.

Example: Log 52 = 1·716 (Fig. 37).
Log x = 3·574 x = 3750

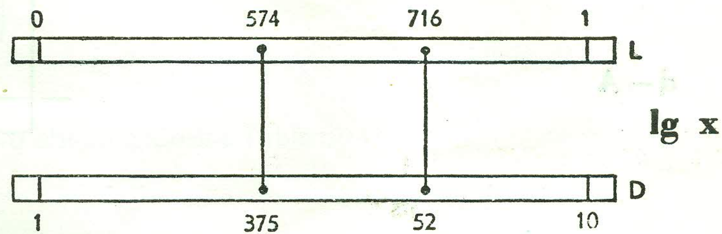


Fig. 37

The Cursor.

The three lines on the cursor can be employed as a scale. When the short right-hand line is set to a **diameter** on **C** or **D,** the centre line will give the **area** on **B** or **A** respectively (Fig.38); and when the right-hand line is on any given **horse-power,** the left-hand line indicates the corresponding **kilowatts.**

H.P.–kW

d – A

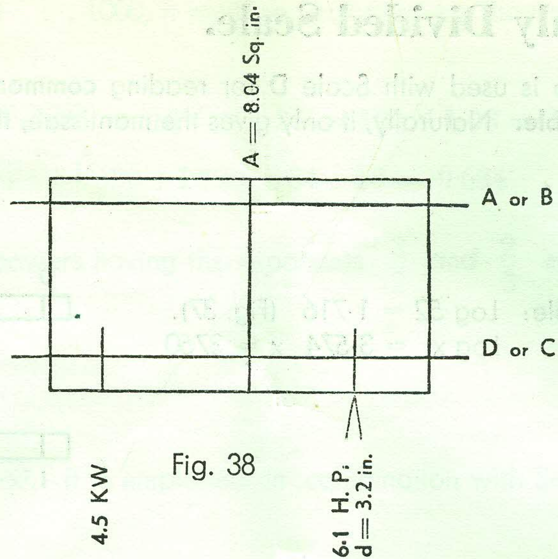


Fig. 38

The Trigonometrical Scales.

Use of the Scales as Tables.

Reading the sin-cos table from left to right, with the **Black Numbers,** we obtain a **Sine Table** on Scale **D.**

With large angles the reading becomes uncertain; in this case it is more accurate if the red numbers are used and read on Scale **P.** In Fig. **39,** sin 76⁰ is given as 0·97 on Scale **D,** and as 0·9703 on **P.**

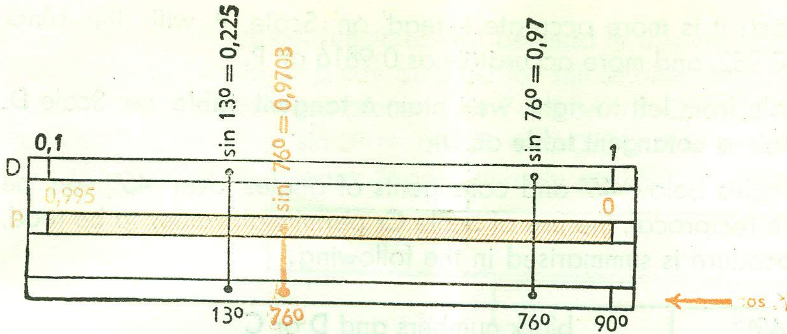
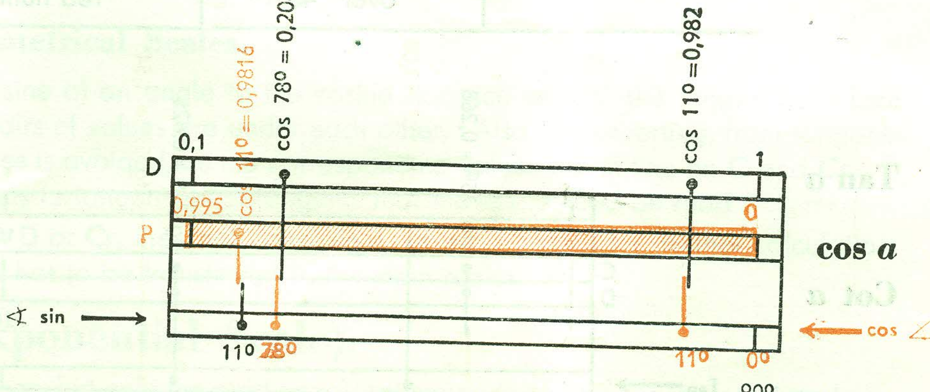


Fig. 39

sin *a*

Reading the **Red Numbers** from right to left, we obtain a **Cosine Table** on **D**



cos *a*

Fig. 40

With small angles the reading is not exact: it is more accurate if read on Scale **P** with the black numbers. In Fig. 40, cos 11° is shown on **D** as 0·982, and more accurately as 0·9816 on **P.**

Reading **black numbers** on the tan-cot scale from left to right, we obtain a **tangent table** on Scale **D.** Reading **red numbers** from right to left, we obtain a **cotangent table** on **D.**

It would appear as if only tangents of angles below 45° and cotangents of angles over 45° can be read. But as tangent and cotangent values are reciprocal, the use of Scale **Cr** permits all values to be read, as shown in the examples of Fig. 41. The procedure is summarised in the following:

Tangents	under 45° over 45°	black numbers and **D** or **C** red numbers and **Cr**
Cotangents	under 45° over 45°	black numbers and **Cr** red numbers and **D** or **C**

Tan *a*

Cot *a*

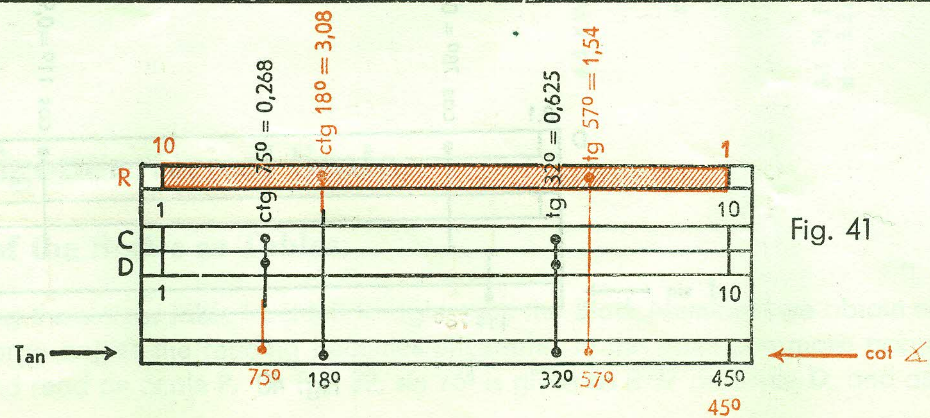


Fig. 41

The trigonometrical functions can be read in this way down to $5^0 \cdot 7$. Then, $\sin 5^0 \cdot 7 \infty \tan 5^0 \cdot 7 \infty 0 \cdot 1$. The following relationship can be used for still smaller angles: $\sin \alpha \infty \tan \alpha \infty \text{arc } \alpha \infty 0 \cdot 01745\, \alpha^0$.

The mark ϱ has been placed at 1-7-4-5 on the **C** and **D** scales; it is employed as shown in Fig. 42. For instance, $\tan 3^0 \infty \sin 3^0 \infty \text{arc } 3^0 \infty 0 \cdot 0524$. The error is less than $0 \cdot 25$ %.

(The symbol ∞ means "approximately equals".)

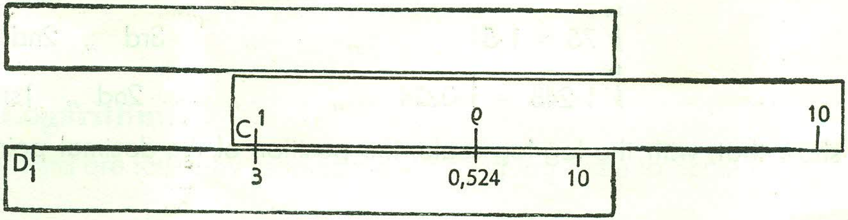


Fig. 42

Calculation with Trigonometrical Scales.

If it is desired to pass from the sine of an angle to the cosine (or vice versa), the angle need not be read. On Scales **D** and **P** these pairs of values are under each other. Also, in converting, from tangents to cotangents the reading of the angles is avoided, as the corresponding values are in line on **C** and **Cr.** It is only when converting sines or cosines to tangents or cotangents that the angle need be read.

As the functions can be found on **D** or **Cr,** they are convenient, in most cases, for further calculation. When the reading is on **P,** however, it has to be transferred to the main scales.

The Log-Log Scale (Exponential Scale).

This scale has manifold applications, but only the most important methods of calculating are given here. The slide is used inverted so that the three sections of the log-log scale move between Scales **A** and **D.**

There is a tenth power relationship between each pair of adjacent sections of the log-log scale, which makes the reading of tenth roots and tenth powers extremely easy.

Example:	$1{\cdot}204^{10} = 6{\cdot}4,$	reading between 2nd and 3rd sections
	$1{\cdot}035^{10} = 1{\cdot}41,$	" " 1st " 2nd "
	$\sqrt[10]{75} = 1{\cdot}54$	" " 3rd " 2nd "
	$\sqrt[10]{1{\cdot}248} = 1{\cdot}0224$	" " 2nd " 1st "

These examples show that, with the log-log scale, the position of the decimal point is definitely fixed.

Powers of e.

The **exponents** must be set **on** Scale **D.** If they are used in combination with the **lowest** section of the log-log scale, the graduations on **D** must be read as **1 to 10;** with the **middle** section they must be read as **0·1 to 1;** and with the **upper** section as **0·01 to 0·1.**

e^n Example: $e^{1{\cdot}61} = 5.$

Set the cursor line to 1·61 on D and read the answer, 5, on the lowest log-log section.

Example: $e^{0{\cdot}61} = 1{\cdot}84.$

Set the cursor line over 61 on D (0·61) and read 1·84 on the middle section.

Example: $e^{0{\cdot}029} = 1{\cdot}0294.$

Set the cursor over 29 (0·029) on D and read 1·0294 on the upper section.

Roots of e.

$\sqrt[n]{e}$

$$\text{Example:} \quad \sqrt[4]{e} = e^{\frac{1}{4}} = e^{0,25} = 1{\cdot}284.$$

If the exponent of the root be changed to a power exponent, as in the above example, the solution is as in the foregoing. The conversion of the exponents is read from the reciprocal scale. Then the answer 1·284, will be found on the middle section over 1 on **D.**

Hyperbolic Logarithms.

$\log_e a$

Hyperbolic logarithms are found by reading from the log-log scale to Scale **D.**

$$\text{Example:} \quad \text{Log}_e\ 25 = 3{\cdot}22.$$

Set the cursor line on 25 on the lowest log-log section and read $\log_e 25 = 3{\cdot}22$ on **D.**

With this setting we obtain a table of hyperbolic logarithms. There is no movement of the slide.

$$\text{Example:} \quad \text{Log}_e\ 1{\cdot}31 = 0{\cdot}27.$$

Set the cursor line over 1·31 on the middle section and read the numbers 2—7 on **D.** That means 0·27, since it is on the middle section.

$$\text{Example:} \quad \text{Log}_e\ 1{\cdot}0145 = 0{\cdot}0144.$$

The procedure is exactly as before; the numbers 1—4—4, read on **D,** must be taken as 0·0144,as the uppermost section was used in setting.

Powers With Fractional Exponents.

Example: $3{\cdot}75^{2{\cdot}96} = 50.$

Set 3·75 (on the lowest log-log section) over 1 on **D**, move the cursor line over 296 on **D** and read the answer, 50, directly above (Fig. 43).

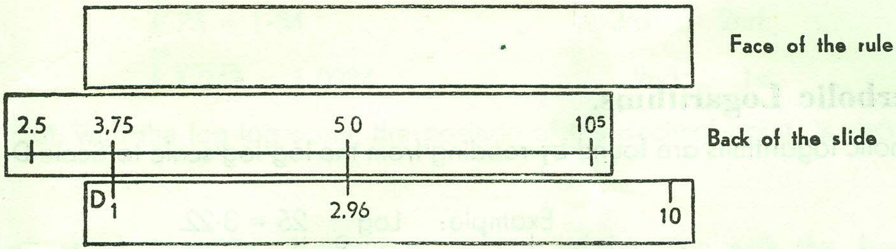


Fig. 43

Example: $1{\cdot}89^{6{\cdot}05} = 47{\cdot}1$

Set 1·89 over 10 on **D**, using the cursor. Move the cursor over 605 on **D** and read the answer, 47·1, above it.

By this method, the change over from the middle section to the lowest is more noticeable.

Example: $1{\cdot}0525^{29{\cdot}4} = 4{\cdot}5.$

Set 1·0525 over 10 on **D**, using the cursor. Move the cursor over 294 on **D** and read the answer, 4·5, above it on the log-log scale.

In the last example, we passed from the highest to the lowest section of the log-log scale. Had the exponent been 2·94, the change over would have been to the middle section (Answer = 1·1623).

There should be no difficulty in selecting the answer on the sections of the log-log scale, as it can be estimated easily.

Roots With Fractional Exponents.

When the root exponent is changed to a power exponent by means of the reciprocal scale, the problem is solved as above. It is possible, however, to obtain the answer without the conversion.

$\sqrt[n]{a}$

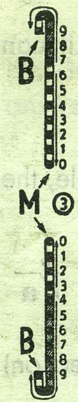
Example: $\sqrt[4\cdot4]{23} = 2{\cdot}04.$

Set 23 on the log-log scale (lower section) over 4·4 on **D** and read the answer, 2·04 (central section) over **D** 10.

Example: $\sqrt[2\cdot08]{1{\cdot}0268} = 1{\cdot}0128.$

By means of the cursor line set 1·0268 on the log-log scale (upper section) over **D** 2·08 and read the answer 1·0128 over **D** 1.

Directions for Use.



To clear register. Pull up the metal slide on the top of the machine, and then push it down again. Before operating, see that the middle Register is set at zero. Should an ↓ sign remain after clearing the machine, insert stylo in the "1" in corresponding column above and pull towards the middle. (Limit "M").

Adding ad subtracting.

Use the respective part of the machine, and proceed as indicated in "Short Rules", inserting stylo of figure required.

Example: 673 + 5269 + 734 = 6676 — 845 = 5831

Insert numbers as you would write them (left to right). Insert stylo in the "6" in third column from right and pull towards middle. Proceed similarly with "7" (2nd column from right) and "3" (righthand column). Machine now shows "673". Now add "5269". The "5" in 4th column from right must be pulled towards middle. Also the "2". The "6" and "9", both being in coloured portion of slide must be pushed away from middle (upwards) and round bend to limit "B". Now insert "734" in the same manner, and sub-total of "6676" will appear in middle register. To subtract "845" use subtraction part. The "8" in 3rd column from right, being coloured, must be pulled away from middle (downwards) and round bend to limit "B". The "4" and "5" are to be pushed upwards to middle. Correct total is now shown in middle register of "5831".

Automatic Stoppage.

If a movement has been made in the wrong direction, i. e., pushed up instead of down or vice-versa, the mistake will be indicated automatically by stoppage of the machine so that the wrong movement will not be continued. In this case, you have merely to leave the stylo in the relative hole and move it to opposite limit, and the correct total will be shown.

Coloured Slide.

For the sake of clarity, the coloured portion of the three right-hand columns is white, and that of the three left-hand colums is black.

Arrow Signal.

Should an "arrow" appear in the middle register, it should be eliminated by inserting the stylo in the "0" of corresponding column — addition or subtraction according to direction of arrow — and moving same, upwards, round bend to limit "B".

Example: 756 + 149 = 905

After insertion of the numbers an ↑ appears in 2nd column from right in middle register, which is eliminated by inserting stylo in "0" in 2nd column from right — addition portion — and pushing it upwards and round to limit "B" when correct answer of "905" will appear. If the operator omits to eliminate the signal, the machine will continue to function correctly, but later a stoppage might occur in the bends, preventing the stylo from reaching limit "B".

Example: 199 + 5 + 8 = 212

After insertion of the "199" and "5" it will be found impossible to move the "8" right round to limit, but by clearing the arrow from "1" in 2nd column, correct answer of "212" will be shown.

1/724e