THE FOWLER 'JUNIOR' CALCULATOR and CIRCULAR SLIDE RULE.

This combined Calculator and Slide Rule has been put on the market with a view to enabling the Junior Student or Engineer to acquire at a reasonable price an instrument which, in one form or another, has now become almost an essential in industry. It is unique in the sense that it combines in one instrument a calculator of the single dial type, with a Slide Rule in circular form. It is thus possible for the user to become proficient in the operation of either.

It consists as will be seen of a double-faced white disc carrying printed dials which can be revolved with the finger or thumb by means of serrations around its periphery. On one side is printed a replica of our well-known "Universal" Calculator dial, and over this is a transparent disc carrying an indicating line or cursor. This, too, is revolved, independently of the dial, by means of serrations around its edge. A fixed transparent disc on which is engraved a red indicating line or datum completes this side of the instrument.

On the opposite side of the disc is printed a "circular slide rule," which functions in precisely the same way as the ordinary straight rule. A revolving cursor is also fitted here, but no datum, as one is not necessary.

Each side possesses its own special advantages and the choice of which one to use is left to the manipulator. For example, by means of the Calculator side a "Long Scale" may be used when it is desired to obtain an answer to a greater number of significant figures than is possible on the Slide Rule side, and vice versa, the Slide Rule side will be found advantageous when percentage and ratio problems are being dealt with. The two sides will be dealt with individually, firstly by a description of the Scales, and secondly by numerous examples worked out upon them.

How To Use The "Junior" & Circ. Slide Rule

THE CIRCULAR SLIDE RULE

Description of Scales reading inwards from the outer circle.

Nos. 1 and 2 (marked A and B respectively) are for multiplication and division, and correspond exactly to the C and D scales of the ordinary straight slide rule, B being an exact replica of A, and revolving; whilst A remains fixed.

No. 3 is a Scale of Logarithms.

No. 4 is a Scale of Square Roots (extending over two circles).

No. 5 is a Scale of Log. sines of angles from 6° to 90°.

No. 6 is a Scale of Log. tangents of angles from 6° to 45°.

The two outer Scales A and B have a number of special marks, viz. :—$\sqrt{2}$; $\sqrt{3}$; $\log_e 10$; π ; gE (gravity English) ; E.H.P. (electrical horse power) ; $\frac{\pi}{4}$; and gF (gravity French), marked upon them.

All the scales, and sub-divisions are logarithmic, and the distances between the figures 1, 2, 3, 4 . . . 10 gradually diminish, so does the possibility of sub-division. This explains why a long scale permits of greater accuracy of reading than a short one.

Any value may be assigned to the figures of the scales, but the same value must be adhered to in the sub-division. Thus 6 may stand also for ·6, ·06, ·006, 60, 600, etc., but if taken say to represent 60, the sub-divisions would represent 61, 62, etc.; and proportionately for any other value accorded to the prime number. Final readings, as in the case of any scale have to be made by judgment when they come between the lines, and depend largely on the accuracy of the observer. It will be found however that results when they cannot be read on a line of the scale with strict accuracy, can be estimated within a fraction of 1 per cent.

Multiplication.—Example : $a \times b \times c$, etc.

Set arrow on **B** to first factor (a) on **A**. Opposite second factor (b) on **B** product ($a \times b$) on **A**. (It will be noted that the two scales **A** and **B** form a complete multiplication table of which the figure on scale **A**, to which the arrow on **B** is set, is one factor.)

Proceeding with the example, set the cursor to the product ($a \times b$), and turn the dial until the arrow on **B** coincides with this product. Read on **A** opposite the factor (c) on **B**, the product $a \times b \times c$. So on for any number of factors.

When multiplying factors containing decimals the position of the decimal point in the answer is usually best determined by a rough mental calculation. But the following rule is useful :—

Rule.—The number of figures in the product of any two factors equals the number of figures in the two factors, when the product falls between the two arrows to the right of **A**, and one less if it falls to the left of **A**.

Division.—$m \div n$. Set divisor (n) on **B** opposite dividend (m) on **A**. Read result $m \div n$ opposite arrow on **B**.

Fractions.—Complex fractions are but a series of multiplications and divisions.

Example : $\dfrac{68 \times 9 \times 32 \times 17}{15 \times 12}$

Here one may proceed by multiplying all the figures in the numerator, and dividing this result by those in the denominator, or by taking 68 dividing by 15, then multiplying by 9 and dividing by 12 and afterwards multiplying by 32 and 17 as the judgment of the operator suggests. Taking the first course :—

Set arrow on **B** opposite 68 on **A**. Opposite 9 on **B** read 68 × 9 on **A**. Without noting its value set cursor to this product. Turn arrow on **B** to the cursor and opposite 32 on **B** read product of $68 \times 9 \times 32$ on **A**. Set cursor to this and turn arrow on **B** to cursor.

Then opposite 17 on **B** read product $68 \times 9 \times 32 \times 17$ on **A**. Now without troubling about its value turn cursor to this product and set the first divisor in the denominator (15) on **B** under the cursor. The reading on **A** opposite the arrow on **B** will give the result of this division, but without noting it, turn the cursor to it, and then set the second divisor 12 on **B** to the cursor. Then on **A** opposite the arrow on **B** read the final result of the fraction (1844).

Rule.—The number of figures in the quotient equals the number of figures in the dividend, less the sum of those in the divisor if the last divisor comes between the two arrows to the right of **A**, or one more if it comes to the left of **A**. This is the reverse of multiplication.

In the present case, the last divisor falls between the two arrows to the left of **A**, and as the dividend contains 7 figures, and the divisor 4 figures, we add 1 to the difference between the two, i.e., $7 - 4 = 3$ and $3 + 1 = 4$ figures in the answer (1844).

The above operations take less time to perform than to describe. The example has been taken in detail, for illustration, as once the operations of multiplication and division are grasped, many short cuts in manipulation will be found.

Squares and Square Roots.—In fixing the magnitude of the square of a number remember that the square of any number between 1 and 10 lies between 10 and 100, and the square of any number between 10 and 100 lies between 100 and 10,000. The square of any number less than unity is less than the number.

The square root of any number less than unity is greater than the number.

$$\left(\frac{1}{10}\right)^2 = \frac{1}{100}\,;\ \left(\frac{3}{7}\right)^2 = \frac{9}{49}\,;\ \sqrt{\cdot 64} = \cdot 8\,;\ \sqrt{\frac{1}{100}} = \frac{1}{10}$$

The numbers on Scale **B** are the squares of those on the Square Root Scale, and figures on one are compared with those on the other by means of the cursor. The square root scale extends round two circles, but is only one scale in reality, as will be seen by following round the numbers from 1 to 10.

Rule.—If number has an *odd* number of digits, read square root on smaller circle. If it has an *even* number of digits read root on larger circle of square root scale.

Example: Find Square of 21·5. Set cursor over 21·5 on Square Root Scale. Read 462·25 on Scale 2.

Example: Find Square Root of 2365. Set cursor over 2365 on Scale 2. Read Square Root 48·6 on Square Root Scale.

Cubes.—a^3. Use Scales 1, 2 and 4.

Set arrows on Scales 1 and 2 in line. Set (a) on Scale 4 under cursor. Read a^2 on Scale 1 under cursor. Set arrow of Scale 2 under cursor. Read a^3 on Scale 1 opposite (a) on Scale 2.

Cube Roots. $\sqrt[3]{a}$. Use Scales 1, 2 and 4.

Set cursor to (a) on Scale 1. Turn dial till number on Scale 4, under the cursor, is the same as that on Scale 1 opposite arrow on Scale 2. This number is the cube root of (a).

Circumference of a Circle.—Use Scales 1 and 2.

Set arrow of Scale 2 opposite diameter on Scale 1. Read circumference on Scale 1 opposite π on Scale 2.

Area of a Circle.—Use Scales 1, 2 and 4.

Set arrows of Scales 1 and 2 in line. Set cursor to diameter on Scale 4. Set arrow of Scale 2 under cursor. Read area on Scale 1 opposite $\pi/4$ on Scale 2.

Reciprocals.—Values of expressions such as $\frac{1}{a}$, $\frac{1}{a^2}$, $\frac{1}{\sqrt{a}}$, $\frac{1}{\sin a}$, etc., are easily obtained from Scales 1 and 2. Whatever the position of the dial the number on Scale 2 opposite arrow on Scale 1 is the reciprocal of the number on Scale 1 opposite arrow on Scale 2.

Common Logarithms.—Set cursor to number on Scale 2 whose log. is required. Read mantissa of log. on log. Scale No. 3. The characteristic of the log. if positive is one less than the number of figures to left of decimal point; if negative one more than number of cyphers to right of decimal point.

Example: Find log. 2675.

Set cursor to 2675 on Scale 2. Read mantissa of log. 427 on log. scale. There are four whole numbers in 2675. Therefore the characteristic is 3 and the complete log. of 2675 is therefore 3.427.

Example: Find log. of 50.75.

Read mantissa of log. 7055 on log. scale.

As characteristic is 1, log. $50 \cdot 75 = 1 \cdot 7055$.

Hyperbolic Logarithms.—These equal the common logs. $\times$ 2·30258 marked $\log_e{}^{10}$ on Scales 1 and 2.

Nth Powers and Roots.—These are got from the following relationship.

If A is a number and $x = A^n$.

Whether n be a whole number or a fraction

$$\log.\ x = n \log.\ A$$

and the log. values are obtained as explained above.

Sines and Log. Sines.—For angles 6° to 90°.

Example: Sine of 37°. Set cursor to 37° on Sine Scale. Read natural sine on Scale B (·602).

Read log. sine on Scale of Logs. (·779).

Natural or Log. Cosines.—Deduce from cosine of angle = sine of complement.

Example: $\cos 60° = \sin (90-60)° = \sin 30°$.

Natural or Log. Tangents.—6° to 45°.

Set cursor to angle on Scale of Log. Tangents.

Read natural tangent value on B and log. tan values on Log. Scale.

Tangents and Cotangents can be deduced from

$$\text{Tan} = \frac{\text{sine}}{\text{cosine}} \qquad \text{Cotan} = \frac{\text{cosine}}{\text{sine}}$$

Fractions to Decimals.—Set numerator on Scale 1 to denominator on Scale 2. Read decimal value on Scale 1 opposite arrow on Scale 2.

Example: $\frac{15}{16}$. Set 15 on Scale 1 to 16 on Scale 2.

Read ·9375 on Scale 1 opposite arrow on Scale 2.

Decimals to Fractions.—Set arrow on Scale 2 to decimal value on Scale 1.

Read any fraction that coincides on Scales 1 and 2.

Example: ·875. Set arrow on Scale 2 to 875 on Scale 1.

Read on Scales 1 and 2 $\frac{7}{8}$ or any other equivalent fraction such as $\frac{14}{16}, \frac{21}{24}, \frac{35}{40}$, etc.

Proportion.—Set question in fractional form $\frac{a}{b} = \frac{x}{c}$ where x is the unknown and may be the numerator or denominator as convenient. Then by cross multiplication $a \times c = b \times x$ and $x = \frac{a \times c}{b}$

Example: 15 men do a task in 28 days. In how many days should 21 men do it?

Obviously more men will do it in less time and we get the proportion $\frac{21}{15} = \frac{28}{x}$ and $x = \frac{20 \times 15}{21} = 20$ days.

Set arrow on Scale 2 opposite 28 on Scale 1. Turn cursor to 15 on Scale 2. Turn dial till 21 on Scale 2 comes under cursor. Read answer 20 on Scale 1 over arrow on Scale 2.

Percentages.—Example: A man receives £2 9s. interest per annum on £15 10s. What is the rate per cent.?

Reducing to shillings we get 49s. for 340s. If x = rate of interest then $\frac{49}{340} = \frac{x}{100}$

or $x = \frac{100 \times 49}{340} = 14 \cdot 4$ per cent.

This is worked on the slide rule as previous example.

Example 2: A machine A does work in 43 minutes which occupies a machine B 54 minutes. How much per cent. more efficient is A than B. They are inversely as the times occupied and taking B to represent 100

$$\frac{\text{Efficiency A}}{\text{Efficiency B}} = \frac{54}{43} = \frac{x}{100}$$

$\therefore x = 100 \times \frac{54}{43} = 125 \cdot 6$ worked as above Ex.

Example 3: Certain parcels contain respectively 8, 9, 24, and 32 articles. Express as percentages of the whole.

Here $8 + 9 + 24 + 32 = 73$. Set 73 on **B** under 100 on **A**. Opposite 8, 9, 24, and 32 on **B** read 10·8%, 12·3%, 32·8%, and 43·8% respectively.

Note.—The percentages of any number of groups can be read off in this way at one setting.

Gauge Points.—The gauge points $\sqrt{2}$, $\sqrt{3}$, etc., round the circumference of the Scales can be used as factors in any multiplication or division relating to areas or circumferences of circles, etc., and by means of the cursor can be combined with squares, sines, cosines and logs. in almost any desired calculation.

THE CIRCULAR CALCULATOR

This side of the instrument consists of a dial, and a cursor, rotated in each case by their serrated edges. A fixed red datum line is also provided. This side of the instrument is operated in a somewhat different manner to the slide rule, but will present no difficulty to the user after a little practice. Indeed it is quite possible that he will prefer to use it at times, as advantage may be taken of the use of the "Long Scale" No. 4 for multiplication and division. Simplicity of operation, and reading, is also the keynote of the Calculator, and to make this effective it should always be borne in mind that:—

The Red Datum line is only used for the first multiplier and the final answer.

The Dial is turned only for multipliers.

The Cursor is turned only for divisors.

The scales marked on the dial reading inwards are as follows:—

Scale 1. The outer multiplying and dividing scale.
" 2. A scale of reciprocals of numbers on Scale 1.
" 3. A Scale of logarithms of numbers on Scale 1.
" 4. Made up of 3 circles, which give the cube roots of numbers on Scale 1, and which may also be used as a "Long Scale" for multiplication and division.
" 5. A Scale of sines of angles graduated round the *inner*, and then continued round the *outer* circumference of a common circle. The scale ranges from 35 minutes to 90 degrees.
" 6. A scale of tangents of angles from 5 deg. 45 mins. to 45 degs.

Scale No. 1 is $3\frac{1}{16}$ ins. diameter, and thus has a circumference of 9·62 ins. or practically the equivalent of a 10 inch straight rule.

Scale No. 4 has a total length of nearly 20 inches.

Multiplication of Two Factors on Short Scale No. 1.

Example 1: Multiply ·0347 by 2·8.

Set 347 on Scale 1 under red datum line.

This lies between the 30 and 35; the exact point being the 9th division past the 30 to make the 345 and two-fifths of the next division to make the 347.

Set the cursor to 1.

Set dial till 28 comes under cursor.

Read answer (just over 97) on Scale 1 under datum.

By visual inspection it will be seen that the answer must be in the neighbourhood of ·09. Therefore we write our answer as given by the Calculator as ·097. By actual multiplication the correct answer is ·09716 showing how close is the approximation by the instrument.

Multiplication of Two Factors on the Long Scale (No. 4).

Example: Multiply 12·8 by 5·62.

Set 128 under datum.

Set cursor to 1.

Set dial till 562 comes under the cursor (this is the first small division after the 56 on the Long Scale). Read answer just over 71·9 under datum.

Multiplication of Three Factors on the Short or Long Scale.

Example 3: The method is precisely the same whichever scale is used, so it will be described only for the Short Scale.

Find product of ·0347 × 2·8 × 63·5.

Set 347 on Scale 1 (or Scale 4 if using Long Scale) under datum. Set cursor to 1.

Set dial till 2·8 comes under cursor.

All above settings are shown in Example 1.

Set cursor to 1.

Set dial till 635 comes under cursor.

This is the 7th division past the 60.

Read answer 6·17 under datum.

The position of the decimal point is judged by inspection.

By actual multiplication the correct answer is 6·16966 showing a close approximation by the instrument.

Multiplication of Four or more Factors on the Short or Long Scale.

Example 4: Find product of ·0347 × 2·8 × 63·5 × 4·9. Proceed exactly as shown in Example 3 above to find product of ·0347 × 2·8 × 63·5 and then again

Set cursor to 1.

Set dial till 49 comes under cursor.

Read product under datum.

This, if using the Short Scale, comes just short of midway between the 30, and the first division past the 30; and we should estimate the answer as 30·23 (midway being 30·25). The position of the decimal point is mentally estimated as follows:—2·8 is roughly 3 and 3 × ·0347 is roughly ·1, ·1 × 63·5 is 6·35, and 6·35 × 4·9 is roughly 6 × 5, which gives an answer of 30. We thus know that the product will have two whole numbers and will come in the neighbourhood of 30.

If the Long Scale, No. 4, is used instead of the Short Scale, No. 1, the succession of operations is precisely the same, but the setting calls for a little more care, as the factors are spread over a scale extending round three circles, and the answer may also be on any one. The position is decided by the rough mental calculation as explained above and in the problem above the answer (30·23) falls on the middle one of the 3 circles.

Multiplication of an odd number of factors using Scales 1 and 2 in conjunction.

Example 5: Find product of 8·42 × 16·16 × ·422 (3 factors).

Set 842 on Scale 1 under datum.

Set cursor to 1616 on Scale 2.

Set 422 on Scale 1 under cursor.

Read answer 57·4 on Scale 1 under datum.

By actual multiplication the correct answer is 57·42036. The decimal point is fixed mentally in this way: ·422 is roughly ·5; ·5 × 8·42 is roughly 4, and 4 × 16·16 is roughly 56. Therefore there are two whole numbers in the answer.

If we wish to find the product of an *even* number of factors we proceed as in the above example, but make it into an *odd* number by the addition of 1 as a factor, which does not alter the final result.

Thus ·354 × 29·4 × 63·6 × ·862 should be worked as ·354 × 29·4 × 63·6 × ·862 × 1.

Division on Short Scale.—Divide 7,256 by 13·85.

Set 7256 on Scale 1 under datum.

Set cursor to 13·85.

Set 1 to cursor.

Read answer 524 under datum.

It is obvious by inspection that the answer will have three whole numbers, and so we fix the decimal point after the 4. The correct answer is 523·9, and when the example was worked out on the Long Scale this answer was obtained.

Fractions.—Consider first a fraction with two factors in the numerator and one in the denominator and worked out on the Short Scale.

Example 6: Solve $\frac{676 \cdot 9 \times 364}{114 \cdot 2}$

Set dial till 6769 comes under datum.

Set cursor to 1142.

Set dial till 364 comes under cursor.

Read answer 2158 under datum.

Correct answer is 2157·5 and when worked out on the Long Scale the answer came barely 2158.

Consider now fractions with several factors in numerator and denominator.

Example 7: Solve $\frac{19 \cdot 5 \times 66 \cdot 6 \times \cdot 0042}{8 \cdot 9}$

Work this as $\frac{19 \cdot 5 \times 66 \cdot 6 \times \cdot 0042}{8 \cdot 9 \times 1}$

Set 19·5 under datum.

Set cursor to 8·9.

Set 66·6 to cursor.

Set cursor to 1.

Set 42 to cursor.

Read answer ·613 under datum, decimal point being fixed by a rough calculation as previously described. (Correct answer is ·61287.)

Example 8: Solve $\frac{13 \cdot 8 \times 723 \cdot 6}{15 \cdot 8 \times 176 \times 2 \cdot 42}$

This would be worked as $\frac{13 \cdot 8 \times 723 \cdot 6 \times 1 \times 1}{15 \cdot 8 \times 176 \times 2 \cdot 42}$

and, as in Example 7, taking the factors alternately from the numerator and the denominator.

Answer by Calculator 1·487. Correct answer 1·484 (a close approximation).

Rapid Action with the Calculator.—As there are several ways of working a problem with arithmetic so there are several with a Fowler Calculator and movements may be curtailed by using the reciprocal Scale No. 2 in conjunction with the primary Scale No. 1, as shown in the following examples.

Example 9: Solve $\frac{6734}{9 \cdot 6 \times 142 \cdot 5}$ where there is an *even* number of factors in the denominator.

Set 6734 on Scale 1 under datum.

Set cursor to 96 on Scale 1.

Set 1425 on Scale 2 under cursor.

Read answer 4·92 on Scale 1 under datum (3 movements). Position of decimal point is fixed mentally. Correct answer by multiplication and division is 4·923.

Example 10: Solve $\frac{4276}{3 \cdot 42 \times 18 \cdot 7 \times 32 \cdot 62}$

Here the artifice of inserting the factor 1 is adopted to make the denominator contain an even number of factors thus:—

$$\frac{4276}{3 \cdot 42 \times 18 \cdot 7 \times 32 \cdot 62 \times 1}$$

Set 4276 on Scale 1 under datum.

Set cursor to 342 on Scale 2.

Set 187 on Scale 2 under cursor.

Set cursor to 3262 on Scale 1.

Set 1 under cursor.

Read answer 2·050 on Scale 1 under datum (5 movements).

Exercises with Reciprocal Scale No. 2.—

Example 11: Find decimal equivalent of $\frac{1}{6 \cdot 456}$

Set cursor over 6456 on Scale 1.

Read 1548 on Scale 2 under cursor.

From inspection of the fraction it is obviously between one-sixth and one-seventh and without hesitation we therefore write down its value as 0·1548.

Example 12: Find decimal value of $\frac{1}{3475}$

Set cursor over 3475 on Scale 1.

Read 2878 on Scale 2.

The fraction is manifestly less than $\frac{1}{3000}$ and therefore will require 3 cyphers after the decimal point, so we write it as 0·0002878.

In setting 3475 under the cursor we note it falls between 34 and 35 and that between 34 and 35 there are two graduations each advancing 5 thus, 340, 345, 350. About half-way between 345 and 350 is 347 and a shade past this is 3475.

Reading Scale No. 2 the cursor is just short of the value 288 and so we should estimate it as 2878 with answer as above.

Note that in reading decimal values of fractions less than one-tenth there will be one cypher placed after the decimal point and preceding the number as read from the reciprocal scale. With values less than one-hundredth and greater than one-thousandth, 2 cyphers, and so on.

Example 13: Find decimal value of $\frac{5}{7}$

Set 5 on Scale 1 under datum.

Set cursor to 7 on Scale 1.

Set dial till 1 comes under cursor.

Read ·714 under datum.

Example 14: Find fractional value of ·1428.

Set (anti-clockwise) 1428 on Scale 2 under datum.

Read 7 on Scale 1 under datum.

Fractional value is therefore $\frac{1}{7}$

Example 15: Find fractional value of ·00653.

Set 653 on Scale 2 under datum.

Read 153 on Scale 1 under datum.

Fractional value is therefore $\frac{1}{15300}$

Note.—As many cyphers must follow the 153 as there are cyphers following the decimal point in the given number.

Use of Logarithmic Scale No. 3.—

Example 16: Find log. of 2675.

Set cursor over 2675 on Scale 1.

Read mantissa 427 on Scale 3.

As there are 4 figures in the number all to left of decimal point the characteristic is positive and its value is 3. The complete log. is therefore 3·427.

Example 17: Find log. of 0·023076.

Set cursor over 24076 on Scale 1. This is about one-third of the way between 24 (which represents 240) and the first graduation after it which represents 242.

Read mantissa 3815 on Scale 3.

As the number is less than unity the characteristic is negative, and as there is a cypher to the right of the decimal point its value is 2.

Therefore the log. of 0·023076 is $\bar{2}$·3815.

Hyperbolic Logarithms.—These equal the common logarithms multiplied by $\log_e 10$ marked as a gauge point round the outer circle.

The mantissa of the common log. of the number is first obtained using Scales 1 and 3, and after insertion of the characteristic it is multiplied by the factor $\log_e 10$ in the manner already described.

Examples of Power and Roots.—

Example 18: Find value of $(36·7)^2$.

This can be done by multiplying 36·7 by 36·7 or by the method below.

Set 36·7 on Scale 1 under datum.

Set cursor to 36·7 on Scale 2.

Turn dial till 1 comes under cursor.

Read 1347 under datum on Scale 1.

Example 19: Find value of $(16·4)^3$.

Can be done by multiplying 16·4 × 16·4 × 16·4, or by first finding the square as in Example 18 and then multiplying this result on Scale 1 by 16·4.

Finding Nth Powers and Nth Roots of Numbers with logarithms.

Let A be a number and suppose $x = A^n$. Where n may be a whole number or a fraction. Then log. $x = n$ log. A.

Example 20: Find 5th Root of 51·52, i.e., find $(51·53)^{\frac{1}{5}}$

Here n = 1/5th and A = 51·53.

Set cursor over 51·53 on Scale 1.

This is between the 3rd and 4th graduations after 50.

Read mantissa of log. on Scale 3, viz., 713.

The number is more than unity, therefore the log. is positive. There are two figures to left of decimal point therefore value of the characteristic is 1.

Therefore log. of 51·53 = 1·713.

One-fifth of log. 51·53 = 0·3426.

Set cursor over 3426 on Scale 3 and read fifth root of 51·53 on Scale 1, viz., 2·2.

Example 21: Find value of $(2·8)^4$, i.e.,

2·8 × 2·8 × 2·8 × 2·8.

First method. By taking logs.

Set 28 on Scale 1 under cursor.

Read mantissa of log. on Scale 3, viz., ·447.

As only one figure to left of decimal point in 2·8 there will be no characteristic.

Multiply ·447 by 4 mentally to get 4(log. 2·8). This equals 1·788.

·788 is therefore mantissa of log.$(2·8)^4$.

Set 788 on Log. Scale 3 under cursor.

Read 614 on Scale 1 under cursor. The answer will have two whole numbers in front of decimal point.

Therefore $(2·8)^4$ = 61·4.

Second method. Multiply 2·8 by itself four times on Scale 1.

Third method. Multiply 2·8 by itself four times, and then by 1, using Scales 1 and 2 in conjunction as previously described.

Square Roots.—

Example 22: Find the square root of 1849.

Set 1849 on Scale 1 under datum.

Set cursor to 1.

Turn dial until the same number comes simultaneously under the datum on Scale 1 and the cursor on Scale 2. This number, 43, is the square root of 1849.

It will be observed that two values may be obtained when setting in this manner. For instance one can get either 43 coming on Scales 1 and 2, when the unity line on the dial comes opposite the mid-point between the datum and cursor, or we could get 13·6 when the unity line falls midway between the datum and cursor. This second value, 13·6, is the square root of the original number 1849 multiplied by the square root of 10.

Thus 13·6 = $\sqrt{1849} \times \sqrt{10}$.

If we had to find the fourth root of a number, we should first find its square root, as in example above, and then find the square root of this square root.

Cube Roots.—Can be read directly from Scale 1 on one of the three circles which comprise the Long Scale, No. 4.

Example 23: Find cube root of 964.

Set 964 on Scale 1, under datum.

Read 9876 on the outer of the 3 circles of Scale 4.

The cube root is therefore 9·876.

Example 24: Find cube root of 1430.

Set 143 on Scale 1 under datum.

Read 11·275 on the inner of the 3 circles of Scale 4. It is obvious that the cube root lies between 10 and 20 and therefore must be read on the inner circle. Other roots can be obtained by taking logarithms as in Example 20, or if a sixth root was required it could be obtained by taking the square root of the cube root of the number.

Sines, Tangents, etc.—The values of sines, tangents, etc., are read from the scales of angles No. 5 and No. 6 by means of the cursor.

Natural Sin. or Natural Tan. on Scale 1.

Log. Sin. or Log. Tan. on Scale 3.

Cosine, Cotangent, Secant, and Cosecant are deduced from the following relationship:—

$$\text{Cos. A} = \text{Sin }(90—\text{A}) ; \quad \text{Cot. A} = \frac{1}{\text{Tan. A}}$$

$$\text{Sec. A} = \frac{1}{\text{Cos. A}} \quad \text{Cosec. A} = \frac{\text{Sin A}}{1}$$

The Scale of Sines No. 5 extends twice round the circumference of the circle, the inner gives angles between 35 mins. and 5 degs. 45 mins. and its value increases from 0·01 to 0·10, and the outer gives angles between 5 degs. 45 mins. and 90 degs. and values increase from 0·10 to 1·0.

Example 25: Find value of Natural Sin. of 4° 40′.

Set cursor over 4° 40′ on Scale 5.

Read Natural Sin. 0·0813 on Scale 1.

The number on the scale is 813, but as the sines of all angles on the inner circle are between 0 and 0·1 we write down the value as 0·0813.

Example 26: Find value of Natural Sin. of 20° 30′.

Set cursor over 20° 30′ Scale 5.

Read Sin. 0·3500 on Scale 1.

The angle being on outer circle of Scale 5 and exceeding 5° 45′, its value lies between 0·1 and 1·0.

Between 20° and 25° the scale is graduated at intervals of 20′ so that 20° 30′ falls midway in the second interval following 20′.

Example 27: Find value of Cosecant 20° 30′.

Set as above and read value $\frac{1}{\text{Sin } 20° \, 30'}$ which is cosecant 20° 30′ on Scale 2. This value is reading anti-clockwise 2·855.

Example 28: Find value of cosine 48°.

This equals sin (90°—48°) = sin 42°.

Set 42° Scale 5 under cursor.

Read cosine 48° on Scale 1 under cursor = ·669.

On the reciprocal Scale No. 2 is shown the value of the secant of 48° which equals 1·494.

Example 29: Find value of Tan 25° and Cotan 25°.

Set 25° on Scale 6 under cursor.

Read tan 25° = ·466 on Scale 1 and cotan 25° on Scale 2 = 2·145 under cursor.

Note.—If log. values of functions are required they must be read on the Log. Scale No. 3.

In reading the values of log. sines of angles the characteristic of the logs. for all angles between 35 mins. and 5 degs. 45 mins. is 8, and for all angles between 5 degs. 45 mins. and 90 degs. is 9. The mantissa only of the log. is read on Scale 3.

Example 30: Find value of Log. Sin 27° 20′.

Set cursor over 27° 20′ Scale 5.

Read mantissa 662 on Scale 3.

Complete log. sin is therefore 9·662.

Example 31: Find value of Log. Sin 4° 25′.

Set cursor over 4° 25′ on Scale 5.

Read mantissa 8865 on Scale 3.

Complete log. tan is therefore 8·8865.

Mensuration of Circles.

Example 32: Find area of circle 3½ ins. diam.

$$\text{Area} = d^2 \times \frac{\pi}{4}$$

$$= 3·5 \times 3·5 \times ·7854.$$

Set 3·5 on Scale 1 under datum.

Set cursor to 3·5 on Scale 2.

Turn dial till gauge point $\frac{\pi}{4}$ on outer circle comes under cursor.

Read area 9·62 square inches on Scale 1 under datum.

Example 33: Find circumference of circle 9·3 ins. diameter.

Set 93 on Scale 1 under datum.

Set cursor to 1.

Turn dial till π (gauge point on outer circle) comes under cursor.

Read circumference 29·2 under datum Scale 1.

It can of course be worked out on the Long Scale if greater accuracy is required when 29·33 will be obtained.

Example 34: Find diameter of a circle of area 227 square inches.

Diameter = $\sqrt{\text{area}} \times$ C.

C = 1·12838 and is marked as a gauge point on outer circle.

Set 227 on Scale 1 under datum.

Set cursor to 1.

Turn till dial same number comes under datum on Scale 1, as under cursor on Scale 2. This is square root of 227.

Turn cursor to 1.

Turn dial till C comes under cursor.

Read answer 17 under datum on Scale 1.

Discount.—

Example 35: What is the wholesale price of an article subject to 12½ per cent., the retail price of which is 52/6?

Set 1 to datum.

Set cursor to 52·5 (52/6).

Turn dial to 87·5, 12·5 divisions backwards, representing 12½ per cent.).

Read nearly 46 under cursor, which we should estimate as 45/11.