INSTRUCTIONS FOR USING THE

CALCULATING SLIDE RULE.*

By "M.I.M.E."

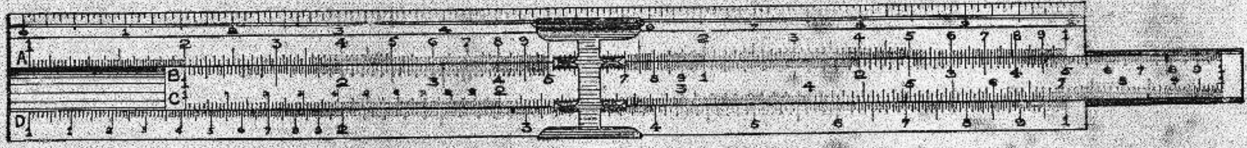

FIG. 1.—RULE WITH METAL CURSOR.

THE "Mannheim" or "Gravet" Slide Rule enables its user—whether engineer, surveyor or actuary; business man or scientist—to rapidly and easily solve the most difficult and intricate problems and formulæ which he is otherwise capable of working out arithmetically by the tedious processes of decimal multiplication and division. The enormous saving of time and mental exertion which it effects can only be fully appreciated by those to whom its use has become habitual, while the degree of accuracy attainable with it is *more than sufficient* for most practical purposes.

Principle of the Rule.—Slide Rules in general are "logarithmic" in principle, and the "Mannheim" is no exception to the rule. The four scales on the front of the Rule—which constitute the *calculating* part of it—are simply tables of common logarithms plotted to scale; the two upper scales, which for convenience we will designate A and B, being the logarithms of 1 to 100 plotted to the scale 12·5 centimetres = 1·00, while the two lower scales, C and D, are the logarithms of numbers 1 to 10 plotted to the scale 25 centimetres = 1·00, which, it is important to observe, is exactly *twice* the scale used for the upper scales A and B.

Multiplication.—As is well known, logarithms are so calculated with regard to natural numbers that *by adding the logarithms of any two numbers together we obtain the logarithm of their product*, or, expressed mathematically:—

$$\log. a + \log. b = \log a \times b.$$

Thus multiplication, with the aid of logarithms, is done *by means of addition*. With the Slide Ru[le] this addition of the logarithms of two numbers is performed by moving two similar sca[les] ... dings on A. to each other until a length corresponding to the logarithm of one of the numbers i... er as it is to on" to a length corresponding to the logarithm of the other, when the combined le... ever. For course correspond to the logarithm of their product. ... of slide ... But ... while ... the

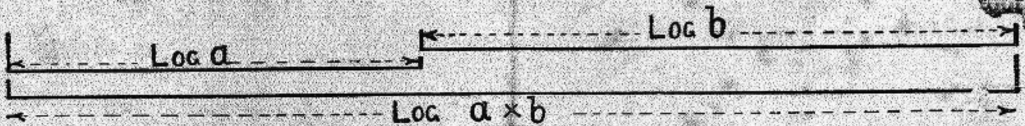

FIG. 2.—MULTIPLICATION (LOG. a + LOG. b = LOG. $a \times b$).

To illustrate this, take a Rule in hand and suppose $a = 2$, and $b = 4$. Then, using the two upper scales A and B, we find, if we move the slide until the left-hand 1 on B lies under 2 on A, that 4 on B will be opposite the product 4 × 2, or 8, on A, as shown in Fig. 1.

Or, again, using the two lower scales C and D, if we set the left-hand 1 on C over 2 on D, the 4 on C will be found to be opposite the required answer, 8, on D. Similarly 2 × 3 will be seen to give 6, 3 × 3 to give 9, 5 × 2 to give 10 (marked only with a 1 on Rule), &c.

It is now only necessary to become thoroughly conversant with *reading the Rule*, and we are then master of it so far as multiplication is concerned, and ease, freedom and rapidity in using it come naturally and quickly as the result of regular practice.

How to read the Scales.—We shall easily learn to do this if we begin by noting first that, *ordinarily, in reading or setting the Rule, no regard whatever is paid to the position of the decimal point in a number.* Thus, whether we multiply 2·25 × 1·4, or 225 × 14, or ·225 × ·14, the procedure is precisely the same, and in each case the result is read from the Rule as 315 (three, one, five, not three hundred and fifteen), the position of the decimal point being subsequently determined either by inspection, or by following the simple "rule for decimal point" enunciated below. Thus, by inspection, we readily see that in the first case, where we multiply 2 and a fraction by 1 and a fraction, the result will be greater than unity but less than 10, so we know the required answer is 3·15; in the second case we see that whether we multiplied 225 by 10 or by 20 the result would have *four figures* before the decimal point, consequently, multiplying by 14 (which is a number between 10 and 20), the answer will be 3150, a cypher being added, of course, for the fourth figure, as the reading on the Rule was 315 *exactly*. This subsequent determination of the decimal point in the result presents no real difficulty in actual work; very frequently we know quite well beforehand where the decimal point will be in the answer (thus we should hardly mistake a 41-horse engine for a 4·1 do., or a 9·5% profit on a transaction for a 95%), and where this is not quite so evident, the matter may be settled with perfect ease by having recourse either to the "rule for decimal point," or to the method of inspection indicated above.

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We next note, in learning to read the Rule, that *the divisions on all four scales, A, B, C and D, read to three figures*; four-figure readings are obtainable by sub-dividing the spaces between the divisions "by the eye," but the division lines themselves all read to *three significant figures only*.

Take now the lower pair of scales, C and D. Commence by regarding the left-hand 1 as 100 (read it as one, nought, nought); the next division will be 101 (read one, nought, one), the next 102, and so on, up to 199 (one, nine, nine), and 200. So far, for additional distinctness, each fifth division is a longer line, and the tenth divisions are numbered, but with smaller figures than those used for the major divisions. Proceeding, we find that from 2 to 4 on the Rule the tenths are only subdivided into five parts each, consequently each fine subdivision corresponds to $\frac{1}{50}$th or $\frac{2}{100}$ths. Hence the first division to the right of 2 is 202, the next 204, and so on up to 400. To obtain 201 we set midway between 200 and 202, 205 is midway between 204 and 206, etc. From 4 (or 400) to the right-hand 1 of the scales the tenths are only subdivided into two parts, so that each subdivision corresponds to $\frac{1}{20}$th or $\frac{5}{100}$ths. Hence the first division past 4 is 405, the next 410, the next 415, and so on, the last division but one on the scale reading 995 (nine, nine, five). With a little practice, and assisted by the illustration given below, the student should now find no difficulty in reading any of the four scales, A, B, C and D, only he must notice that on the two upper scales the divisions between 1 and 2 are *fiftieths* (so that the first division to the right of 1 is 102, the next 104, and so on); from 2 to 5 the divisions are *twentieths* (so that the first division to the right of 200 is 205, the next 210, the next 215, etc.); and from 5 to 10 (or 1) the divisions are *tenths* (so that the first division beyond 500 is 510, the next 520, etc.). The right-hand half of scales A and B is practically a repetition of the left-hand half, and, except in a few special cases (referred to later), it is read in precisely the same manner.

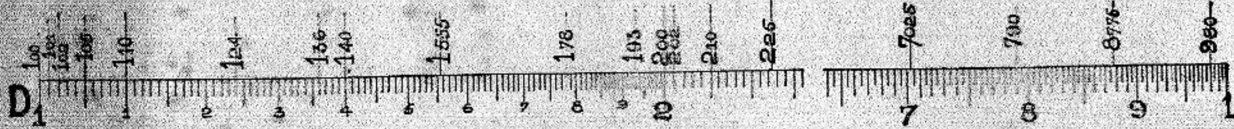

Fig. 3.—Readings on Lower Scales.

In the illustration two or three four-figure readings are indicated. Thus 1555 lies midway between 155 (or 1550) and 156 (or 1560); again, 7025 is midway between 700 and 705, etc.

Multiplication. Further Examples.—We ought now to be able to multiply *any* two numbers together (with a high degree of accuracy), using either the two upper scales or the two lower scales at our discretion. As, however, the two lower scales are most suited for very accurate so... we shall devote our immediate attention more especially to them.

read on scales:—Multiply 1650 × 2·2. Set the left-hand 1 on C over 165 on D. Then opposite mark in the C is read 363 on D. Here the position of the decimal point is evident. Answer = 3630. the *left-ha*, multiply ·785 by 40. Here we find that if we set the left-hand 1 on C over 785 on D angle of w or 400) on C will be off the Rule entirely, rendering it impossible to get a reading for 1 of th... *Whenever this occurs (or would occur), we simply use the right-hand* 1 *of the scale* the s... *of the left-hand* 1. As this is equivalent to using the left-hand 1 and then dividing by 10, it read... *effect whatever on a reading in which the decimal point is ignored.* To multiply ·785 by 40 then, set the right-hand 1 of C over 785 on D, and opposite 4 (or 400) on C we read 314 on D. Hence, answer = 31·4.

Division.—We have seen that multiplication can be performed (with the aid of logarithms) *by adding*; we now notice that division may be done similarly by *subtracting*. To divide any number *a* by another number *b*, we need only *subtract the logarithm of* b *from the logarithm of* a, *and the remainder will be the logarithm of the answer*, or:—

$$\log. a - \log. b = \log. a \div b.$$

In using the Slide Rule for division, we simply *subtract a length corresponding to the logarithm of the divisor from a length corresponding to the logarithm of the dividend (or number to be divided), and the remaining length corresponds to the logarithm of the quotient.* This operation is performed by setting the divisor on the slide to correspond with the dividend on the body of Rule, the result being then read from the body of the Rule. Thus, to divide 8 by 4 using the two upper scales, we set the divisor 4 (or 400) on B under 8 (or 800) on A, and the result 200 (seen to be 2) is read on A, opposite the 1 of slide, as shewn in Fig. 1. The same setting shows that 9 ÷ ·45, ·92 ÷ 460, ·005 ÷ 2·5, etc., give 200 as reading, the actual answers being 20, ·20, and ·002 respectively.

Again, to divide 725 by 41·5 using scales C and D; the divisor on slide (on scale C, that is) is set over the dividend on D, and the result, about 1747, is read on D. Answer = 17·47.

Rule for Decimal Point.—In multiplication, if the *right-hand* 1 be used, the number of figures before the decimal point in the product of any two numbers is equal to the *sum* of those before the decimal point in the two numbers multiplied together. In division, if the result is read at the *right-hand* end of the slide, the number of figures before the decimal point in the quotient is equal to the *difference* of those before the decimal point in the dividend and divisor. If the *left-hand* 1 be used we must *subtract one* from the *sum* in multiplication, and *add one* to the *difference* in division.

Simple Proportion (or Rule of Three) is very easily solved with the Slide Rule, especially if we use the two upper scales, A and B. Take an example:—Find the value of 58 inches of cloth at each of the following prices per yard, viz., 4/3 and 7/6. Here, we had better work with the money in pence and the lengths in inches; then the stating of our first sum becomes

36 ins. : 58 ins :: 51 pence : x pence.

Set 36 (or 360) on B under 58 (or 580) on A. Then *all* readings on B are to the readings opposite to them on A in the proportion of 36:58. Hence we [illegible] off the figures for both results with this single setting of the slide. Thus, opposite 51 [illegible] B is 822 on A; our first answer is therefore 82·2 pence, or 6/10¼ nearly; again, opposite [illegible] on B is 145 on A; our second answer is therefore 145 pence, or 12/1.

Squares, Cubes, etc.—Readings on scale A are th[illegible] of exactly opposite readings on scale D. Thus 9 on A is exactly opposite 3 on D, 16 on A [illegible]e 4 on D, 225 on A is opposite 15 on D, and so on. *Why* this is so is evident from the e[illegible]tion that *twice the logarithm of* [illegible] *number equals the logarithm of the square of that number*; [illegible]ed with the knowledge that the plotting scale used in determining scales C and D is *exactly twice as large* as that used for scales A and B.

To obtain the square of any number, it is therefore only necessary to set either the left-hand or right-hand 1 of the slide over the given number on D, and take the reading *opposite* on A, which (with the decimal point suitably placed) will be the required square of the number under consideration. Thus, to find the square of 8·5; set [illegible]er 1 of the slide over 8·5 (or 850) on D, and opposite on A we get the reading 7225. Here the position of the decimal point is evident. Answer = 72·25.

Higher powers than the square—cubes, 4th, 5th, and 6th power, and so on—are easily determined by repeated multiplication, but there are one or two short methods which may be noticed here. For example, to determine the cube of 3·5; set 1 on C over 3·5 (or 350) on D, and opposite 3·5 (or 350) on B we obtain as result 428 on A. Answer $3{\cdot}5^3 = 42{\cdot}8$. It will be seen that here we first set the slide so as to obtain the square of 3·5 on scale A, and then, without further setting of the slide, we use scales A and B to multiply $3{\cdot}5^2 \times 3{\cdot}5$ in order to obtain $3{\cdot}5^3$. Again, we have seen how to take the reading for $3{\cdot}5^3$ (428), and the 1 of the slide as now set gives us the reading for $3{\cdot}5^2$ (1225) on A. It follows, therefore, that opposite 1225 on B we have the reading for $3{\cdot}5^4$ ($3{\cdot}5^2 \times 3{\cdot}5^2$) on A, and opposite 428 on B we have the reading for $3{\cdot}5^5$ ($3{\cdot}5^2 \times 3{\cdot}5^3$) on A. These readings are 150 and 525 respectively. Answer, $3{\cdot}5^4 = 150$ and $3{\cdot}5^5 = 525$.

The 6th, 7th, 8th, 9th and 10th powers of any number a may be determined with only twice setting the slide, by multiplying a^3 by a^3, a^3 by a^4, a^4 by a^4, a^4 by a^5, and a^5 by a^5 respectively. For example, to raise 2·0 to the 10th power:—Set left-hand 1 of slide over 2 (or 200) on D; then opposite 2 on B we have the reading for 2^3 (8, or 800) on A, and opposite 800 on B we have the reading 32 (or 320) for 2^5 ($2^2 \times 2^3$). Now again setting the slide, so as to multiply 32 by itself ($a^5 \times a^5$), we get the reading 1024. Answer, $2^{10} = 1024$.

Square Root, Cube Root, etc.—Readings on scale A being the *squares of opposite* readings on scale D, it follows that readings on D are the *square roots* of opposite readings on A. Hence, with the Slide Rule, it is almost as easy to extract the square root of a number as it is to determine its square or 2nd power. One little additional difficulty presents itself, however. For instance, suppose we require to determine the square root of 8·5. We have to set the 1 of slide (either end) under 8·5 (850) on A, and take the opposite reading on D for the square root. But there are two 850 readings on scale A, one on the first half and another on the second, and while the opposite reading on D in the one case is 2915, in the other it is 922. Looking further into the matter, we find that

$\sqrt{\cdot 085}$	=	·2915	while	$\sqrt{\cdot 85}$	=	·922
$\sqrt{8{\cdot}5}$	=	2·915	„	$\sqrt{85}$	=	9·22
$\sqrt{850}$	=	29·15	„	$\sqrt{8500}$	=	92·2
$\sqrt{85000}$	=	291·5	„	$\sqrt{850000}$	=	922

and so on. It appears, therefore, that when extracting the square root we must either adopt a restricted reading of the scale A—using the first half of the scale for numbers from 1 to 10, 100 to 1,000, 10,000 to 100,000, $\frac{1}{100}$ or (or ·01) to $\frac{1}{10}$, etc., and the second or right-hand half of the scale for numbers between these (10 to 100, 1,000 to 10,000, $\frac{1}{1000}$ to $\frac{1}{100}$, $\frac{1}{10}$ to 1, etc.)—or else we must content ourselves with taking *both* the possible readings, and finding, by inspection, which is the one we require.

To obtain the cube root of a number the slide is *inverted*, scale C being brought into contact with scale A. Having done this, the rule is:—Set 1 of slide under the given number on A, and find where *exactly similar* readings on A and C *coincide*. There are three possible readings, which of these is the one required being easily determined by inspection. Example:—Determine the values of $\sqrt[3]{1{\cdot}25}$, $\sqrt[3]{12{\cdot}5}$, $\sqrt[3]{125}$, $\sqrt[3]{1250}$, $\sqrt[3]{12500}$, and $\sqrt[3]{125000}$. Having reversed the slide so as to bring scales C and A together, set the left-hand 1 of C un[illegible] 125 on the first half of scale A; then looking along the scales we find that 5 (or 500) on A coincides with 5 (or 500) on C, and also that 232 on A coincides with 232 on C; 500 and 232 are therefore two out of the three possible readings. Now set the right-hand 1 of the slide under 125 on the second half of scale A, and we shall find that 1078 on A coincides with 1078 on C; also that 232 on A coincides with 232 on C. The last-mentioned reading we had already obtained with the first setting of the slide, so our three possible readings are 1078, 232, and 500. Answer:—$\sqrt[3]{1{\cdot}25} = 1{\cdot}078$; $\sqrt[3]{12{\cdot}5} = 2{\cdot}32$; $\sqrt[3]{125} = 5$; $\sqrt[3]{1250} = 10{\cdot}78$; $\sqrt[3]{12500} = 23{\cdot}2$; $\sqrt[3]{125000} = 50$.

The 4th root of a number is easily obtained by extracting the square root *of the square root* of the number.

Fractional Powers.—The simpler ones, $x^{\frac{n}{2}}$, $x^{\frac{n}{3}}$ and $x^{\frac{n}{4}}$, may be obtained by determining, first, the nth power of x, and then extracting the 2nd, 3rd, or 4th root as the case may require, but fractional powers in general and roots other than those mentioned are best obtained with the aid of logarithms.

se of Cursor.—The s or, or marker, is of the highest importance as an adjunct to the Rule. Its extraordinar is more especially evident in connection with the solving of problems or formulæ involvi multiplication and division; in such operations a great saving of time is effected by it liability to small errors and slight inaccuracies is reduced to a minimum. Usually, whe with a Rule provided with a cursor, it is advisable to resolve the problem under consi into the form of a compound fraction, leaving all calculating until the last. Supposing we ng, we find in this way that $x = \frac{2{\cdot}2 \times 25 \times 42 \times 180}{12 \times 9 \times 56}$ we proceed to multiply 2·2 by 25, and instead of taking a reading for the result and setting it down, or remembering it for the next setting, we simply *mark it on the scale itself by setting the cursor to it.* We then set the 1 of the slide *to the cursor* and multiply our first result by 42; set the cursor to the new result, and then proceed to multiply by 180 or to divide by 12, and so on. It is, of course, quite immaterial (so far as the final result is concerned) in what order we take the numbers in multiplying and dividing, but it is rather quicker to multiply and divide *alternately*.

In cursors made with a glass (or transparent celluloid) face, a line is provided to set by; in metal cursors carefully-adjusted chisel-shaped projections are provided for the same purpose. The latter class are less liable to damage and are also rather quicker in use.

The Rule as a Table of Logarithms.—When a scale for obtaining the logarithms of numbers is provided, it is to be found either in the middle at the back of the slide, or on the back of the Rule itself. In either case it is simply a scale in which 25 centimetres = 1·00, corresponding to the plotting scale used in determining scales C and D. When on the back of the slide it is used in conjunction with scale D; thus, to obtain log. 2, set the *left-hand* 1 of the slide over 2 (or 200) on D, then turn the Rule over and from the scale of equal parts on the back of the slide read the required *mantissa* of the logarithm (=·301) opposite the index line provided in the recess at the end of the Rule. The index of the logarithm (in this case = 0) is of course found by the usual rules.

Where the scale for obtaining logarithms is on the back of the Rule itself, a scale exactly similar to scale D is placed in close proximity to it; the result being a graphical table of logarithms and anti-logarithms, from which either are read off at will with *no setting whatever* of the slide.

If the *hyperbolic* logarithm of a number is required, determine first its common logarithm as explained and then use the Rule to multiply the common logarithm by 2·3.

Sines and Tangents.—Scales for reading the sines and tangents of angles are often provided on the back of the slide, and are marked S and T respectively. The arrangement varies somewhat in different makes of the Rule—in the best and simplest design the sine of an angle is read on scale A, at the left-hand 1 of the slide, by simply setting the angle, on scale S, to the index mark in the recess at the back of Rule; while the tangent of an angle is read on scale D (also at the *left-hand* 1 of the slide) by setting the angle, on scale T, to the same mark. Conversely, an angle of which the sine or tangent is given can be read on S or T by setting the left-hand 1 of the slide to the given sine on A, or to the given tangent on D. Note that in reading the sines the left-hand 1 of scale A must be considered as ·01, and the right-hand 1 as 1·0; in reading the tangents the left-hand 1 of scale D must be regarded as ·1, and the right-hand 1 as 1·0.

The sine scale commences at 34′ 22·69″; the next division corresponds to 40′, the next to 50′, the next to 1°. The sine of any angle θ less than 1° may easily be determined by proportion, since sin. 1° : sin. θ :: 1 : θ. Direct tangent readings on the Rule begin at 5° 43′; to obtain the tangents of angles smaller than this we must remember that *for very small angles* tan. θ = sin. θ. This is true to four places for angles up to 2°, and from 2° to 4° the difference is exceedingly small; thus, sin. 4° = ·0698 and tan. 4° = ·0699. From 4° up to 5° 43′ the tangent may be obtained *very nearly* from the sine by multiplying the latter by $\frac{304}{303}$. The tangents of angles larger than 45° are given by the formula, $\tan.\ \phi = \frac{1}{\tan.\ (90^\circ - \phi)}$

The Rule as a Table of Metrical Equivalents, etc.—A metre = 39·37 inches; if therefore we set either 1 of scale B under 3937 on scale A we have before us a complete conversion table, inches to metric and metric to inches, from which we read that 11 mm. = ·43 ins., 50 ins. = 1·27 metres, 30·5 cm. = 12 ins., and so on. Again, suppose we require to convert the prices given in a French catalogue to English equivalents, and we have ascertained that the rate of exchange is such that 1 franc = 9·6 pence. Set 1 of B under 960 on A; then any reading *in francs* on B will have the equivalent *in pence* opposite on A. Or, again, suppose prices in marks (German money) with 25% discount have to be worked out to nett cost prices in English money, with the rate of exchange such that £1 = 20·40 mks. Here, seeing that we have 25 % discount, an article priced at 20·40 marks would actually cost us 15/- nett. Hence, if we set 204 on B under 150 on A, opposite the *list prices* in marks on B will be read the *nett cost prices* in shillings (and decimals of a shilling) on A.

Concluding Remarks.—The foregoing instructions have been written to illustrate the principle of the Rule, and to indicate how it may be made to take the place of decimal arithmetic and mathematical tables in our calculations, rather than to show its application to special cases. When, however, the general instructions here given have been thoroughly mastered, no difficulty whatever should arise in applying the Rule to the special calculations which occur in one's business or professional work. Practice alone is necessary to gain skill in using it and confidence in its trustworthiness, and if, at the outset, no opportunity of testing its accuracy is neglected, the Rule will soon prove itself a reliable and indispensable companion.