materials will often be determined by such factors as the rate of aging or deterioration under the action of oil, water, solvents, ozone, and range of operating temperatures.

Although felt is not effective in vibration isolation for exciting frequencies below 40 to 50 c.p.s., at higher frequencies and in the audible range it produces effective isolation and the thickness, pressure, and type of felt are not at all critical.

The authors wish to express their appreciation to the Western Felt Works for permission to publish the results of the tests and to Dr. H. A. Leedy for initiating the investigation and encouraging the work throughout the project.

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A Slide Rule for the Study of Music and Musical Acoustics†

L. E. WADDINGTON*
Miles Laboratories, Inc., Elkhart, Indiana
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Musicians are seldom concerned with the mathematical background of their art, but an understanding of the underlying physical principles of music can be helpful in the study of music and in the considerations of problems related to musical instrument design. Musical data and numerical standards of the physics of music are readily adaptable to slide-rule presentation, since they involve relationships which are the same for any key. This rule adjusts relative vibration rates, degrees of scale, intervals, chord structures, scale indications, and transposition data, against a base of the piano keyboard. It employs and relates several standard systems of frequency level specification.

THE theory of music is commonly thought of in terms of scales, intervals, and harmonic relationships. People who have studied theory have learned all of those fundamentals and supposedly have them in mind for instant use. On the other hand, the physics of music is considered in terms of numbers applied to those same musical relationships, and the physicist can perform all kinds of musical calculations, often without any understanding of harmony as a musical concept. The need for a reference showing the physical and the harmonic relationships encountered in music prompted the preparation of this slide rule.

The Acoustical Society of America, through an appointive committee on music, is encouraging improvements in musical instruments through research in musical matters, where the science of acoustics and the art of music possess common interests.

Although many members of this Society are highly trained in music as an art as well as in acoustics as a science, it may be well, through the discussion of this slide rule, to reaffirm some of the acoustical standards which have been previously presented to the Society, and to tie them in with some of the fundamentals of music and musical instruments. These data are readily adaptable to slide-rule presentation since they involve many relationships which hold true regardless of key in which they are considered. A simple guide such as this can serve as a reference for the acoustician as well as the musician when studies are pursued in either direction. (Figure 1.) (Sections of the rule are shown in this and succeeding views for clarity of presentation.)

The face of this rule has a lower rail on which is imprinted a piano keyboard consisting of the standard $7\frac{1}{2}$ octaves, identified by the black and white keys and designated by letter notation (A, B, C, etc.), standard piano key numbers (there are 88 keys on the piano), and by a

† This is an adaptation of a paper on this subject which was recorded on disks with appropriate musical background and demonstration sound effects, and illustrated with 26 Kodachrome slides. It was presented at the Acoustical Society meeting in New York, May 9, 1947.

* Formerly Design Engineer, C. G. Conn, Ltd., Elkhart, Indiana.

subscript notation which will be further described. The top rail is equally divided to correspond to the lower rail, and the divisions are identified by chromatic notation as well as by an additional frequency level designation known as semitone count. Subscript numbers and semitone count are printed in red.

Our standard of musical pitch, having moved up and down the scale since the 17th century, is now fixed at A-440, this having been adopted by A.F. of M. in 1917, accepted by Music Industries Chamber of Commerce in 1925, and approved by American Standards Association in its standards on acoustical terminology from 1936 to date. Moreover, steps have been taken to make this the international standard.[1] In the equally tempered scale based upon the American standard of $A_4=440$ cycles per second, there is a "C" which is just about at the threshold of hearing. It is convenient to use this as a reference frequency as proposed by Fletcher[2] and, following Young,[3] to call it C_0. The tones of the first complete octave of the piano keyboard starting with the lowest C are thus identified by the subscript one (1), and the three tones below that octave drop into the frequency range of the octave identified by the subscript zero (0) (Fig. 2). Using C_0 as a reference for counting semitones and remembering that there are 12 semitones in any octave, then C_1 can be designated as number 12. Similarly, Middle C_4, four octaves above C_0 bears the number 48 (equal to 4×12) as a semitone count for frequency-level determination.

The frequencies of all of the equally tempered scale tones within the frequency range of the piano keyboard are printed in the groove, and indicated by either end of the slide (A_2 is indicated as 110.0 cycles per second. Three octaves above is A_5 equal to 880.00) (Fig. 3).

For purposes of reference and comparison, the Just major scale fractional ratios from a given starting tone, and the ratios between tones, are indicated on one space of the slide. (Fig. 4.) In another position, the ratios of the equally tempered scale are indicated for one octave in terms of relative vibration numbers. Remember one-

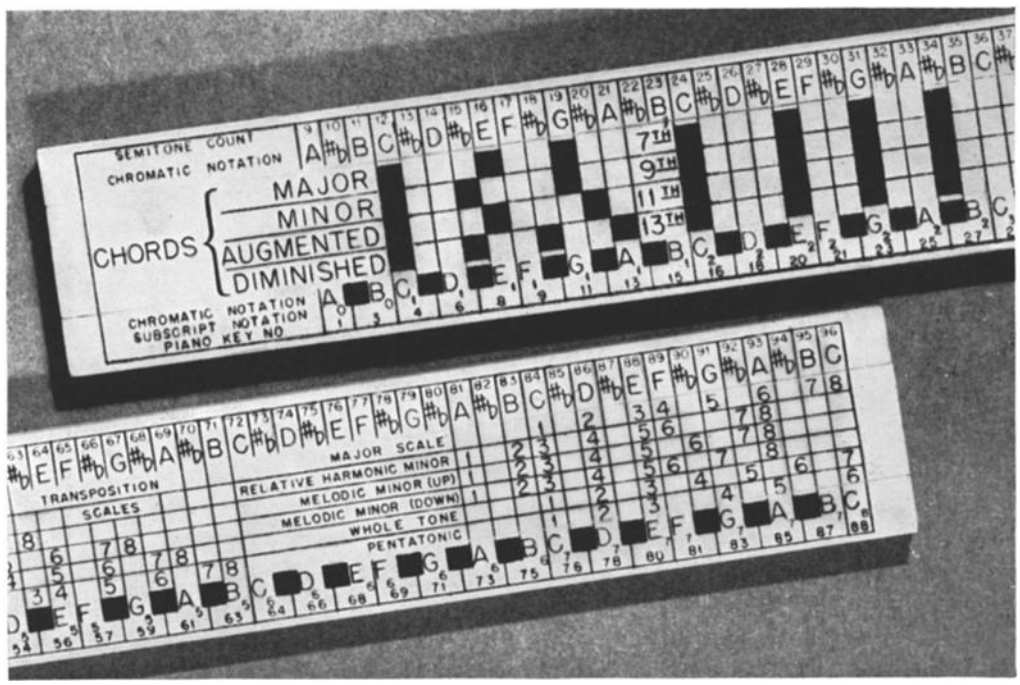

FIG. 1. Face of the rule with slide in normal position.

[1] Musique et Instruments 29, 237, 263, 283, 287 (1938); also British National Acoustics Committee Resolution of 1938.
[2] Fletcher, J. Acous. Soc. Am. 6, 59–69 (1934).
[3] R. W. Young, J. Acous. Soc. Am. 11, 134–139 (1939).

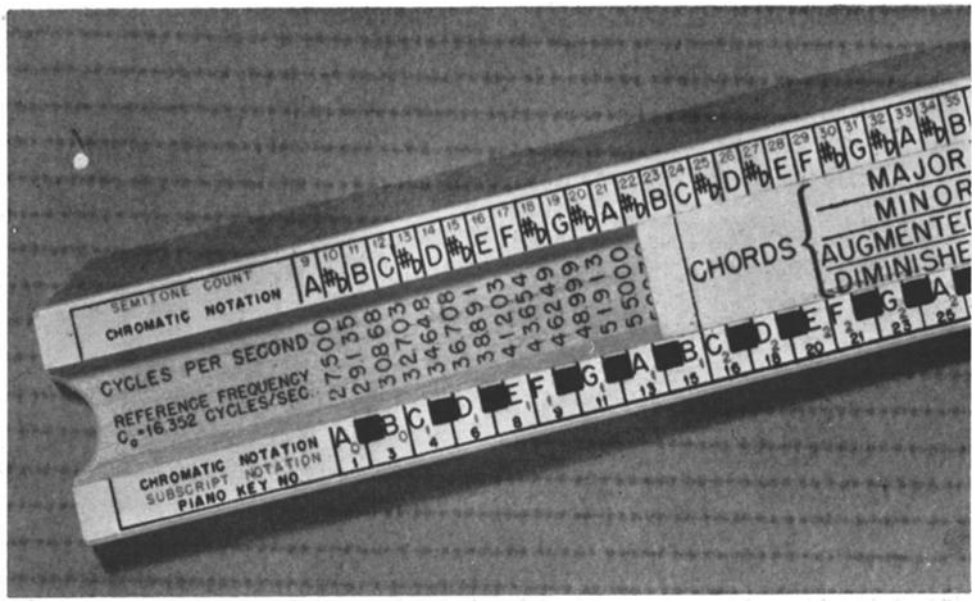

FIG. 2. Slide indicates first full octave of piano keyboard whose tones are designated by the subscript 1.

twelfth of an octave = one semitone = the $2^{1/12}$ $=1.059463$. This number multiplied by itself 12 times $=2$, or a relative frequency one octave above the basic tone (since each octave is exactly twice the frequency of the one below) (Fig. 5).

Bach said: "Music is the greatest of all sciences." D. C. Miller[4] continued, two decades ago:

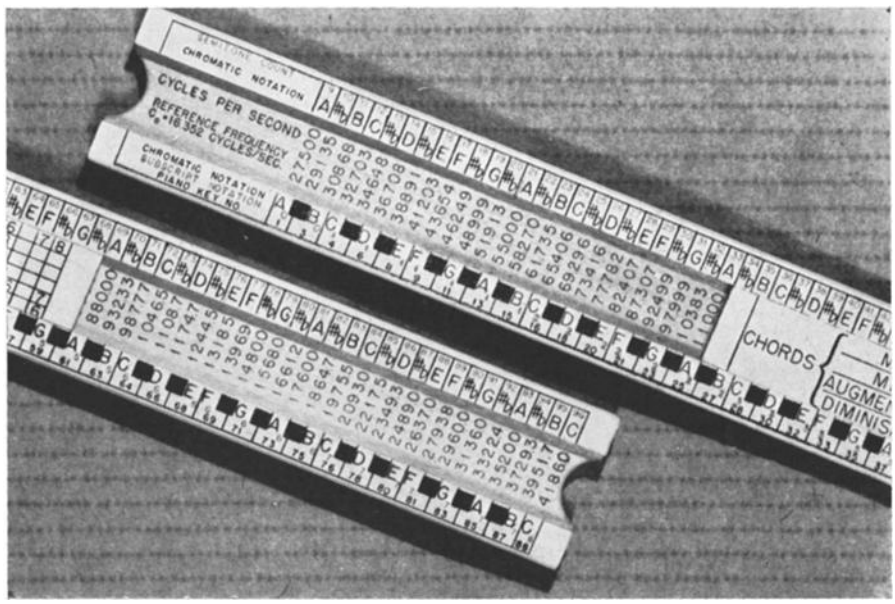

FIG. 3. One end of the slide indicates the A_2, semitone count 33, with a frequency of 110.00 cycles/sec. while another setting indicates the A_5, three octaves above with a frequency of 880.00 cycles/sec.

[4] Dayton C. Miller, *The Science of Musical Sounds* (The Macmillan Company, New York, 1926), p. 265.

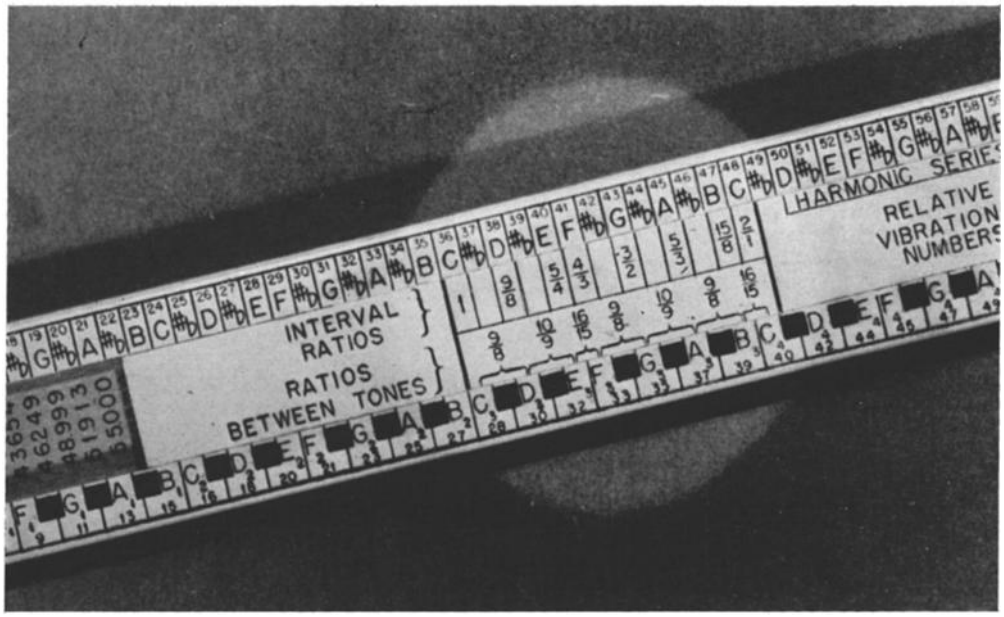

FIG. 4. Just major scale fractional ratios based on C.

"While this may not be true today, yet the construction of the equally tempered scale is clearly scientific, and it is no doubt true that the relations of the major and minor scales, and the nature of chords and their various forms and progressions, as well as various other fundamental principles can be explained better by science than precept."

Another section of the slide shows that divisions of the octave can be indicated as degrees of

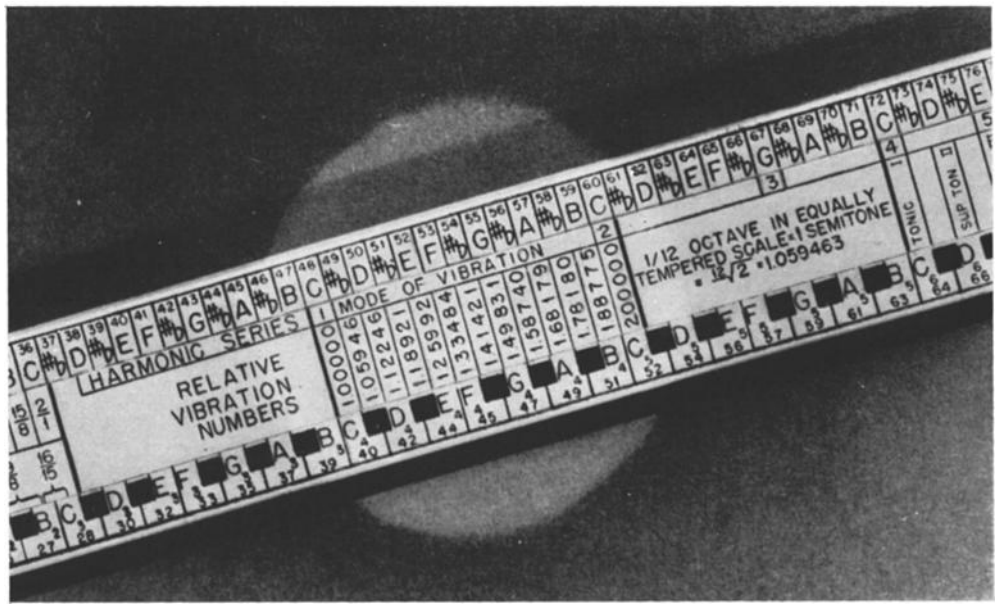

FIG. 5. Equally tempered scale fractional ratios, or relative vibration numbers, based on C. Also the harmonic series, showing first five modes of vibrations.

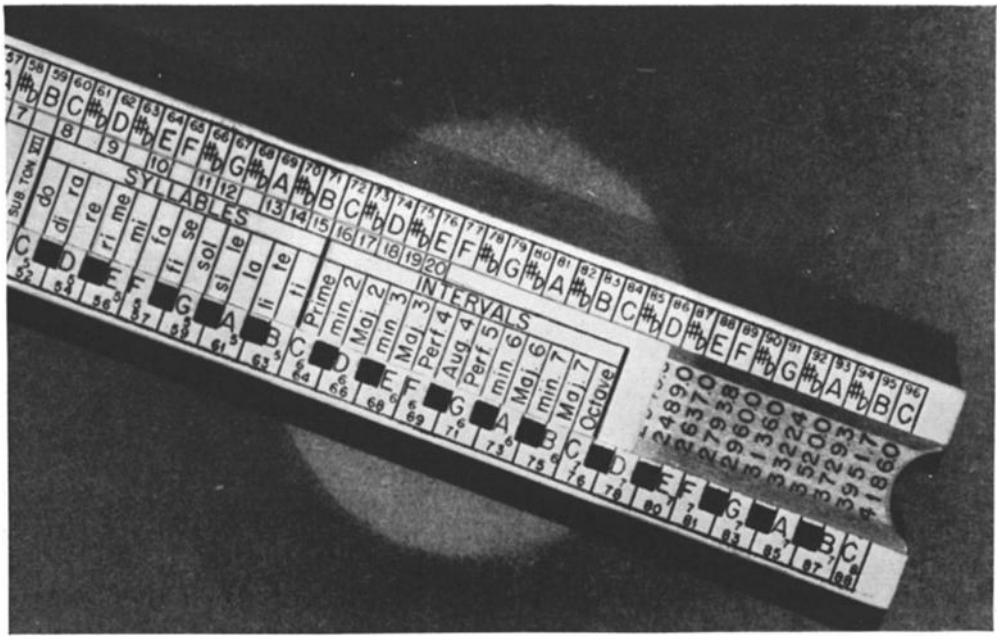

FIG. 6. Intervals of scale indicated for the C scale.

scale in which the scale is spelled by names, Tonic, Super Tonic, Mediant, Sub Dominant, Dominant, Super Dominant, and Sub Tonic. Still another approach outlines the divisions of the octave when indicated as Intervals of Scale from Prime, minor 2, Maj. 2, min. 3, Maj. 3, Perf. 4, Aug. 4, Perf. 5, min. 6, Maj. 6, min. 7, Maj. 7 and through the Octave (Fig. 6). And finally the slide

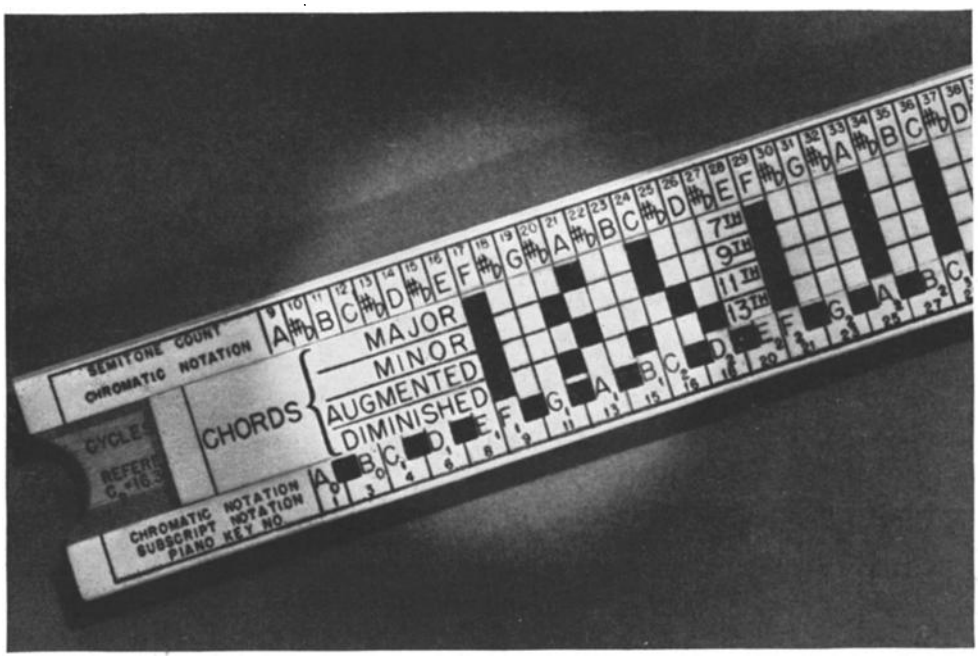

FIG. 7. The major, minor, augmented, and diminished chords indicated in the key of F.

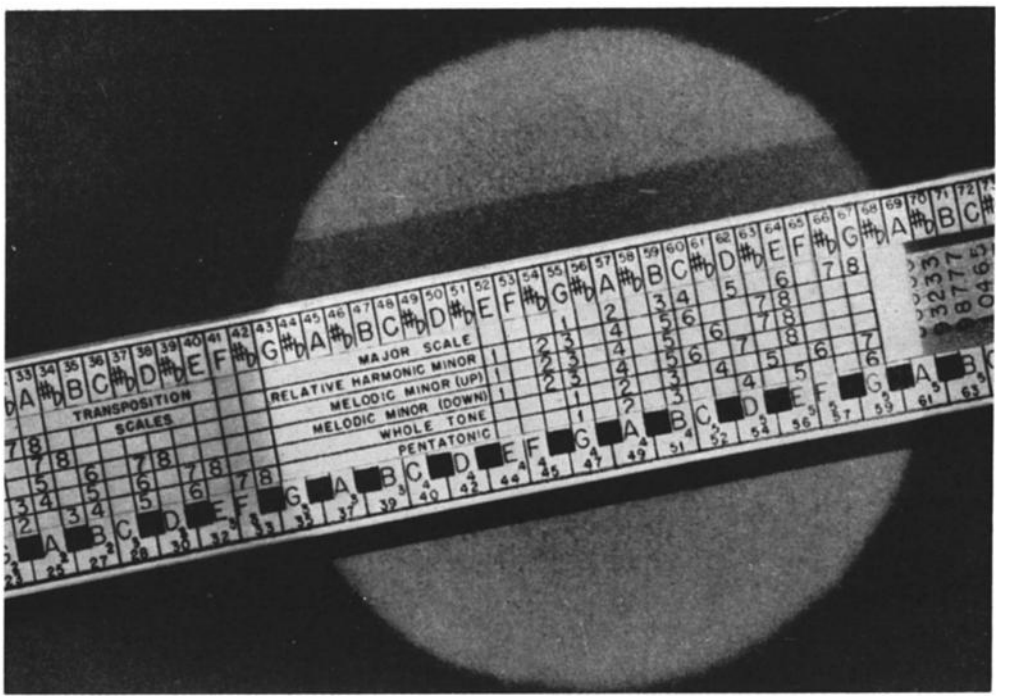

FIG. 8. Major and relative minor scale spelling is indicated for the key of F.

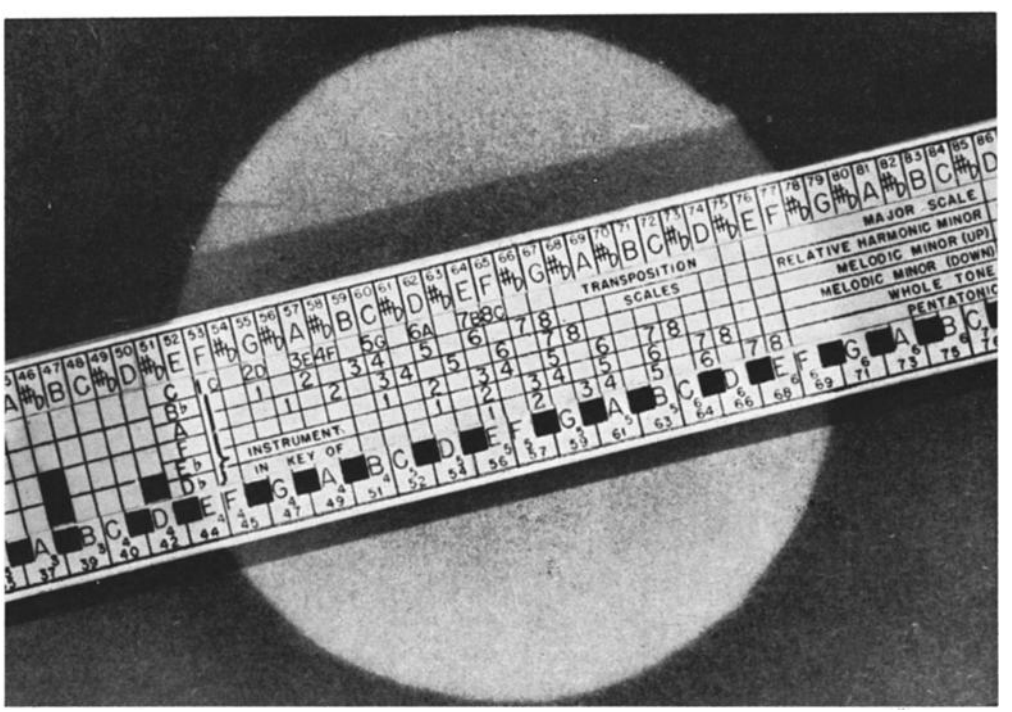

FIG. 9. With key note set on F, transposition of all tones of the scale is indicated for instruments built in common keys, by reference to the degrees of scale.

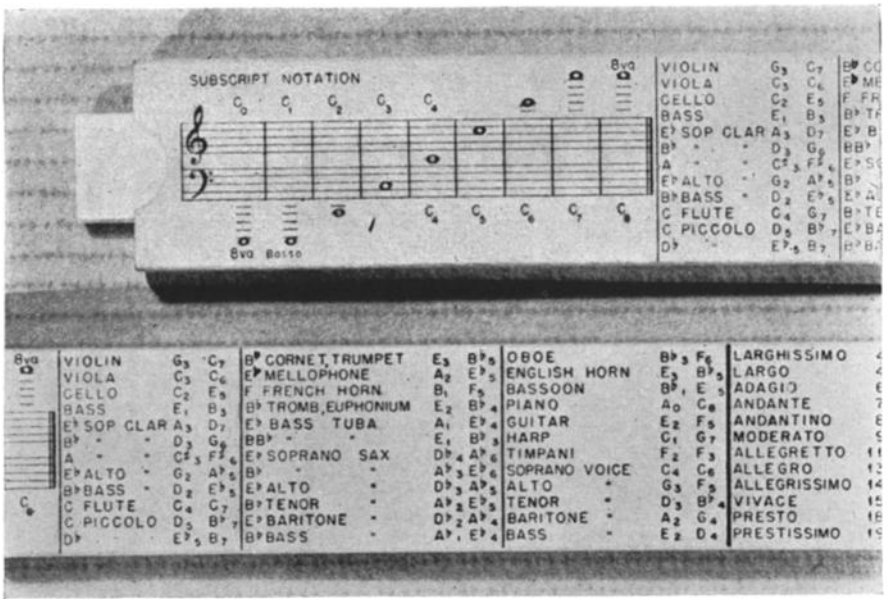

FIG. 10. Back of rule showing range chart and the relative position of subscript notation on the grand staff.

may be moved so the indicator rests on any preselected key tone, and based on the movable "do," syllables are indicated for that key as used in vocal music training, (do, re, mi, etc.) It is often convenient to relate these to the more familiar letter notation.

On the opposite side of the slide, which is reversible in its groove, are indicated tones composing several chords in all keys. (The tonic, or key note is indicated in red.) The slide is moved with the indicator set to F, and the tones composing F major chord are seen to be (FAC); F minor (FA♭C); F Aug. (FAC♯); and F Dim. (FA♭BD). Similar chord spelling may be de-

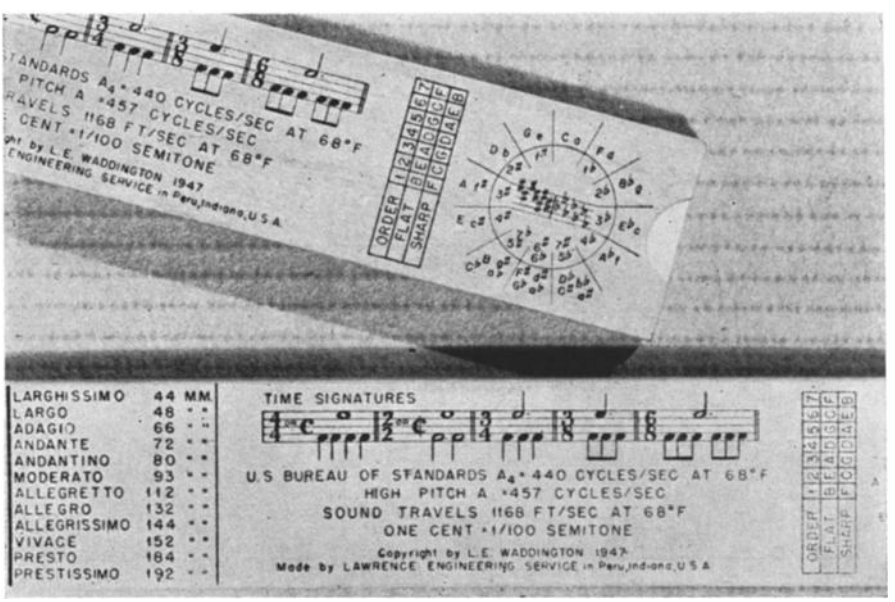

FIG. 11. Back of rule with Circle of Fifths, Metromome Markings, and Time Signatures.

termined for the 7th, 9th, 11th, and 13th chords, and all chords may be determined in any key by adjustments of the slide so that the indicator is set to the key note (Fig. 7).

If information about scales is desired, a section of the slide lists tones composing all major scales, relative harmonic and melodic minor (both up and down), in addition to whole tone and pentatonic scales. The key note of each of these is the first degree of the scale and is printed in red. It can be set to any desired key and the degrees of scale will indicate the proper composition of that scale. In Fig. 8 the key note is set at G and the major scale is spelled G A B C D E F♯ G. The relative minor starts a third lower on E and is spelled E F♯ G A B C D♯ E. The melodic minor is similarly spelled up and down. Also included are the whole tone spelling and the pentatonic as used by the Chinese.

Musicians are often required to play in other keys than that in which the music is written. For simple note transposition, the concert key degrees of scale are also named by letter notation, and the fundamental has only to be set to the desired key and all intervals of both scales will be properly aligned for transposition. For example, if it were wished to transpose in intervals equal to that from F to C, those two tones are lined up and automatically all other tones in the scale indicated by letter notation would fall in line (Fig. 9).

By use of the transposition scales, correct keys may be determined for all B♭, A, F, E♭, and D♭ instruments (the commonly used band and orchestra instruments) working from any given concert key. To demonstrate this, the slide is moved so that the red fundamental of the C instruments is set to F, supposing that a composition under consideration is written in the key of F. Now the red fundamental of the B♭ line indicates that the B♭ instruments would play in the key of C, and the E♭ instruments in the key of D. The same setting would indicate the transposition of any tone or series of tones, for with the fundamental of the C scale set to F, the C instruments would play a major scale in F as indicated by the degrees of scale, the B♭ instruments would play the scale of G, and so on. With this information on the movable slide, transpositions may be determined, from any given major or minor key to any desired major or minor key.

On the back of the movable slide, a way is provided for locating the tempered approximations to a harmonic series of sounds. These may be the modes of vibration from 1 through 20 nominally expected from brass instruments. For example, if a trumpet is of such length that its first mode is $B\flat_2$, then the other modes would produce $B\flat_3$, F_4, $B\flat_4$, D_5, F_5, etc. If tone quality is being considered, then the slide can be similarly used to locate the names of the tones nearest to the harmonic partials. (See Figs. 5 and 6 for the harmonic series showing modes of vibration based on C.)

Other miscellaneous bits of musical information have been added to the back of the rule. A range chart is included (Fig. 10) which shows the "as sounded" range of string, reed, and brass instruments, related to subscript notation. For example, the violin range is G_3 to C_7. This can be checked against the staff location of subscript notation, or against the piano keyboard on the face of the rule.

There will also be found a "Circle of Fifths" from which the names of major and relative minor scales may be seen along with the signatures (number of sharps or flats) of both, and the order of key progression in a clockwise direction. The names of sharps and flats and their order of addition may be determined from a table (Fig. 11).

Common tempo markings in terms of metronome markings are listed from Larghissimo through Prestissimo. (Adagio, for example, with metronome-mark of 66, means a rate of 66 beats per minute and is useful as the composer's indication of the standard time of a composition.) Time signatures are shown related to their basic unit of measurement, the eighth note, quarter note or half-note. And finally it is again pointed out that the standard pitch is based on the $A_4 = 440$ cycles per second.

Future acoustical research will explore the various fields of musical and scientific theory and will establish additional basic standards of expression. Musical instruments will be improved. If each worker in this field is aware of past progress in both the science and the art, the advances of the science can be applied to the art with resulting appreciation and enjoyment of music by increasing numbers of people.