NESTLER

Polymath Duplex

Multimath Duplex

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Index

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Instructions for Slide Rule
POLYMATH DUPLEX

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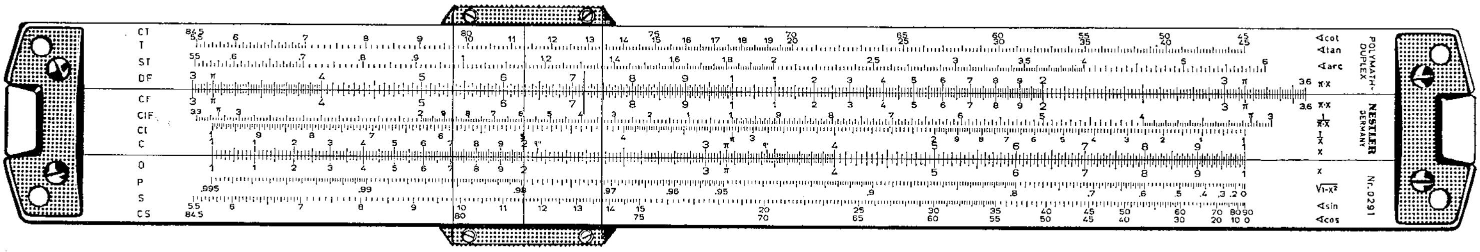
The Nestler Slide Rule — Polymath-Duplex No. 0291

1. The arrangement of scales

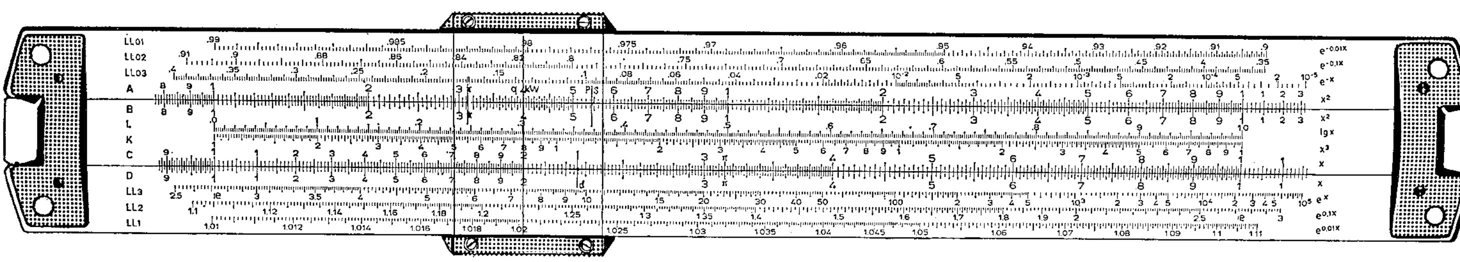
Angular functions side

CT	cotangent scale increasing from 45° to 84.5°	∢ cot	Upper body
T	tangent scale from 5.5° to 45°	∢ tan	
ST	scales for small angles in arc measure from 0.55° to 6°	∢ arc	
DF	π folded fundamental scale	$\pi \cdot x$	
CF	π folded fundamental scale	$\pi \cdot x$	Slide
CIF	π folded reciprocal scale	$1/\pi \cdot x$	
CI	reciprocal fundamental scale	$1/x$	
C	fundamental scale	x	
D	fundamental scale	x	Lower body
P	Pythagorean scale	$\sqrt{1-x^2}$	
S	sine scale from 5.5° to 90°	∢ sin	
CS	cosine scale increasing from 0° to 84.5°	∢ cos	



Exponential functions side

LL 01	Exponential scale	0.99 to 0.90	$e^{-0{,}01x}$	Upper body
LL 02		0.91 to 0.35	$e^{-0{,}1x}$	
LL 03		0.40 to 0.00001	e^{-x}	
A	Square scale		x^2	
B	Square scale		x^2	Slide
L	Mantissa scale		log	
K	Cubic scale		x^3	
C	Fundamental scale		x	
D	Fundamental scale		x	Lower body
LL 3	Exponential scale	2.50 to 100.000	e^{x}	
LL 2		1.10 to 3.00	$e^{0{,}1x}$	
LL 1		1.01 to 1.11	$e^{0{,}01x}$	



2. The Principle of the Slide Rule

The slide rule owes its existence to the logarithm. By the use of the logarithm of the number in place of the number itself, every arithmetic form is replaced by a different form. Multiplication becomes addition; division becomes subtraction.

In multiplication, the logarithms of the factors are added one after the other as geometrical line-lenghts. The sum of both lengths, the single total length, is the logarithm of the product.

$a \times b = c$ multiplication $2 \times 3 = 6$

$\log a + \log b = \log c$

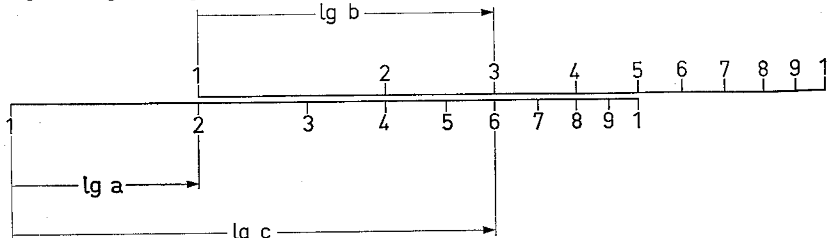


Fig. 3

In division, from the length of line representing "log of the dividend" subtract the length "log of the divisor". The remainder, the difference of the two lengths, is the logarithm of the quotient.

$a \div b = c$ division $6 \div 2 = 3$

$\log a - \log b = \log c$

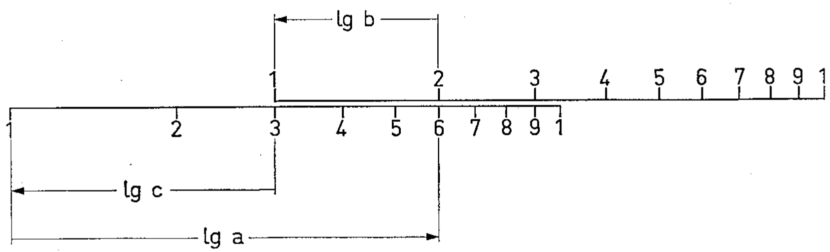


Fig. 4

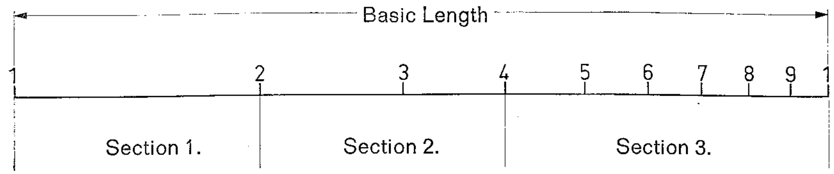
3. Setting and Reading of the Scales C, D, CF, DF, CI, CIF

With the slide rule, you want to calculate rapidly and accurately. For that purpose, the correct setting and reading of the scales must be understood. The most frequently used fundamental scales C and D are shown in Fig. 5 with the primary graduations and in the three following illustrations are shown with individual sections noted and explained.

Notice that the slide rule does not provide fractional parts of a number, but only whole numbers. The second graduation can refer to 2 as well as 20, 200, 2000, 0.2, 0.02, 0.002, and so on. With this understanding, any desired number can be set on a scale, going from 1, the left-hand index, to 1, the right-hand index. It is therefore better to speak of the integers without decimal- and place-value.

Example: 0.0238
2–3–8 two – three – eight
not: zero decimal-point zero two three eight.

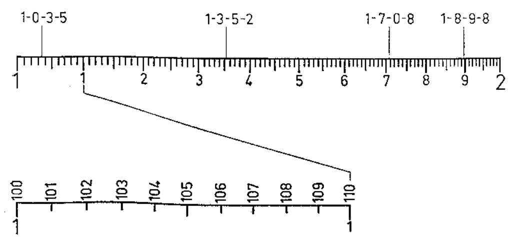
27500
2–7–5 two – seven – five
not: twenty-seven thousand five hundred.



Primary graduations and sections — Fig. 5

According to Fig. 5, there are three different sections to distinguish. The first section, from 1 to 2, is divided into ten parts by shorter graduation lines. These shorter graduations are once more sub-divided into ten parts. This section is similar to the divisions on a millimeter ruler. As Fig. 6 shows, reading of three exact numbers can be done directly on the graduations; a fourth number can be assumed by estimation.

The precise graduations and the fine hairline of the runner allow subdivision between the tertiary graduations. This makes it possible for the fourth digit . . . 2 . . . 4 . . . 5 . . . 6 . . . 8 to be read.

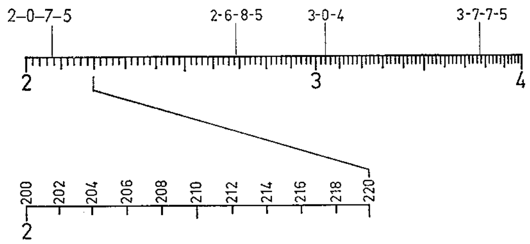


Read three digits exactly. Estimate the fourth digit.

Section 1 to 2 on the fundamental scales — Fig. 6

The second section, from 2 to 4, is between the primary graduations which are again divided into ten parts. In this section, however, the smaller spaces between the secondary graduations are divided into five parts. You can again read three whole digits, but the last digit must be even: 202, 204, 206, and so on. If the last digit is odd, it must be positioned between two graduations. Like the fourth digit, the fifth digit can also be estimated.

Example: 2–0–7–5; 2–6–8–5; 3–0–4; 3–7–7–5.

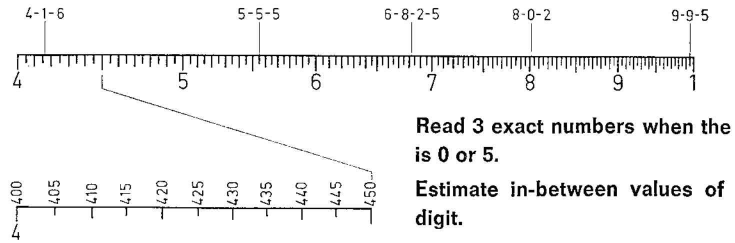


Read 3 exact numbers when the last even digit is even. Estimate numbers like 0005.

Section 2 to 4 of the fundamental scales. Fig. 7

The third section, from 4 to 1, is shown in Fig. 8. The primary graduations are separated into ten parts; the secondary graduations bisect the primaries so that numbers with 0 and 5 at the end – 4–0–0; 4–0–5; 4–1–0; 4–1–5; and so on – can be read exactly. The in-between values 4–0–1; 4–0–2; 4–0–3; 4–0–4 and so on; or 4–0–7–5 and so on, are determined by estimation.

Example: 4–1–6; 5–5–5; 6–8–2–5; 8–0–2; 9–9–5.



Read 3 exact numbers when the last digit is 0 or 5.

Estimate in-between values of the third digit.

Section 4 to 1 of the fundamental scales. Fig. 8

Scales numbered in red increase from right to left. These are the reciprocal, or inverse, scales. On the right- and left-hand ends of certain scales: A, B, C, and D, some divisions are colored red. Their purpose is to add convenience by extending the scales to some extent.

4. **Schematic Representation of an Arithmetic Example**

Each example is represented by a simple graphical model so that the sequence of settings is clear. The scales are shown as parallel lines in the colors used on the slide rule itself. The slide is signified by green stripes. At the left end of each indicated scale is entered its international symbol; at the right end, its mathematical symbol.

The vertical line with the crossbar on both ends represents the hairline of the runner. The arrow at the top of the hairline points in the direction in which the runner is to be moved. The number above the hairline shows the sequence to be applied. Crossed lines between two models means that the slide rule should be turned over to the other side. Each model represents only the scales which are needed for the example being illustrated.

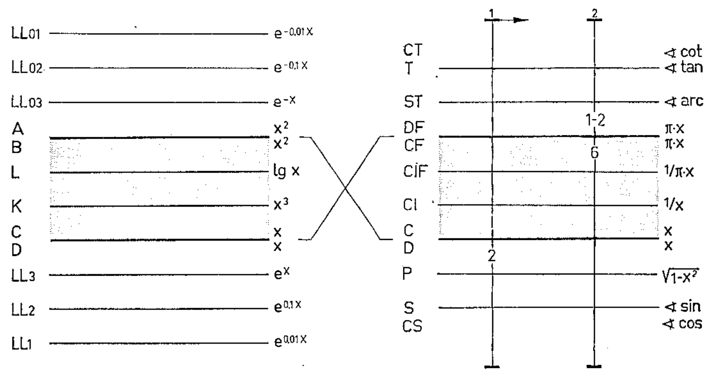


Fig. 9

5. **Multiplication**

$$\mathbf{a \cdot b = c}$$

By using the logarithm of the number instead of the number itself, multiplication becomes addition. As you join together the lengths of the lines for the corresponding logarithmic factors, multiplication is carried out.

For the examples, the fundamental scales C and D are used.

Example: $2.36 \times 3.6 = 8.5$

Calculation: With the help of the hairline, place C–1, the left hand index, over D 2–3–6, shift the hairline to C 3–6 and read the result 8–5 below the hairline on D.

The second example, 2.36 × 19.7 = 46.5, Fig. 10, is done in the same manner. The decimal point must be placed by estimation, since the slide rule gives only whole numbers, not fractional parts of numbers.

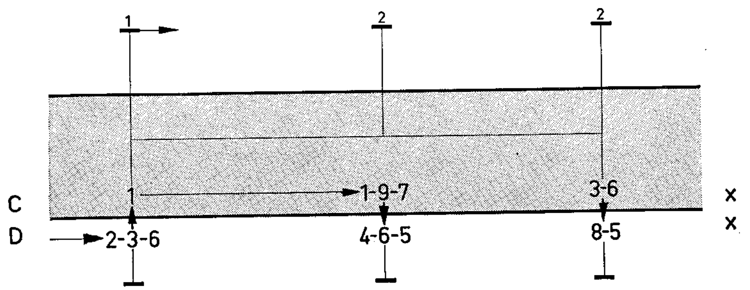


Fig. 10

If the multiplication 2.36 × 5 = 11.8, Fig 11, is to be carried out, a different slide-position must be selected. In this case, the slide must extend toward the left; that is, the right-hand index, C-1, must be placed over D 2–3–6 so that the result may be found on D within the limits of the slide rule length.

Example: 2.36 × 5 = 11.8 2.36 × 8.05 = 19

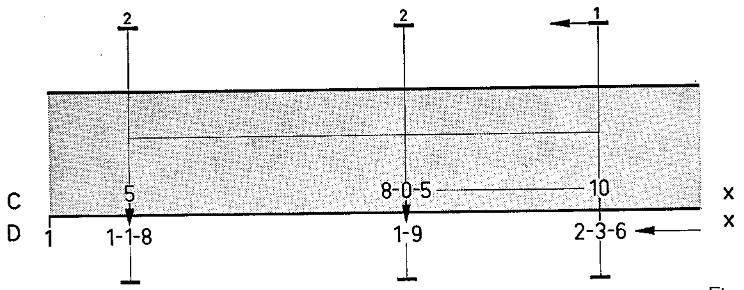


Fig. 11

Calculation: With the help of the hairline place C–1, the right-hand index, over D 2–3–6, shift the hairline to C5 and read the result 1–1–8 below the hairline on D.

After some practice, you will know which slide position to select. In the beginning, however, you may memorize the following rule:

If the product of the first whole numbers from *a* and *b* is less than 10, then the first factor must be set with the left-hand C–1 (the slide extends towards the right); if the product is greater than 10, the first factor must be set with the right-hand C–1 (the slide extends towards the left).

The first whole numbers from *a* and *b* are 2 and 3 in the first example; 2 × 3 = 6 can be placed with C 1 to the left.

The square scales A and B can also be used for multipliaction. Here we avoid the problem of having the slide extend too far to the right or left. The A and B cycles are half the length of the corresponding C and D scales; consequently, each A and B scale has two cycles. Note, however, the accuracy of calculations on A and B is less than that on C and D.

The "folded" scales CF and DF have the advantage of the same accuracy as have the C and D scales (see Sec. 10).

6. **Division**

$$\boxed{\frac{a}{b} = c}$$

Division is the inverse operation of multiplication. By using the logarithm of the number instead of the number itself, division becomes subtraction. From the line log *a* subtract the line log *b* and obtain the line log *c* as the quotient. Where multiplication and division require only that numbers be set in, it is logarithms that determine the arrangement of the scales of the slide rule to perform the respective operations.

Example: 71.5 ÷ 2.86 = 25 85 ÷ 34 = 2.5

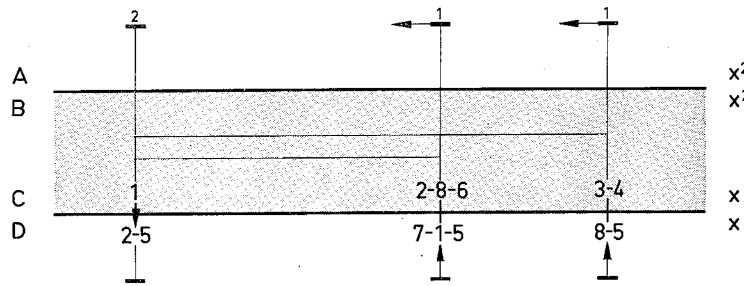


Fig. 12

Calculation: With the help of the hairline, place the dividend 7–1–5 on D and the divisor 2–8–6 opposite on C; shift the hairline over to C–1 and read as the result the quotient 2–5 on D.

Example: 1.075 ÷ 1.72 = 0.625 230 ÷ 36.8 = 6.25

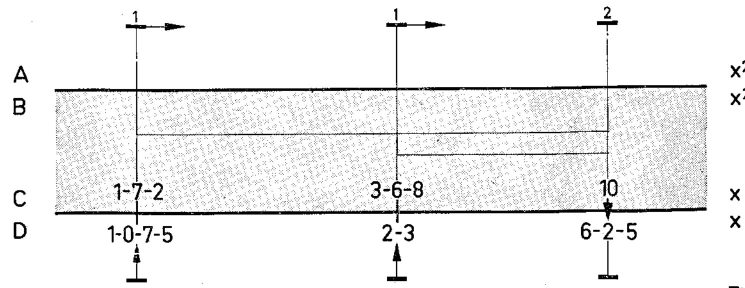


Fig. 13

The position of the slide is the same as in the multiplication 25 × 2.86, only the sequence of settings is reversed.

If the dividend's first digit is less than the divisor's first digit, the slide must be moved toward the left and the result read off under the right-hand C–1. In contrast to multiplication, however, no consideration need be given to the matter in division, for the slide rule is automatically moved in the proper direction when the dividend and the divisor are correctly set!

The procedures are the same as before. Division can also be performed on the square scales A and B. Again, because of shorter cycles, the accuracy of scales A and B is less than that of C and D.

The same accuracy as with C and D can be obtained with the folded scales CF and DF (see section 10).

7. Combined multiplication and division

$$\boxed{\frac{a \cdot b}{c}}$$

With expressions of the form $\frac{a \times b}{c}$ always begin with division and then follow alternately with multiplication and division.

Example: $\frac{450 \times 2}{18} = 50.$

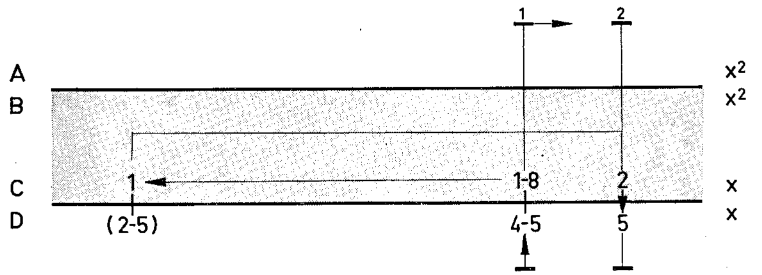


Fig. 14

Calculation: With the help of the hairline place the value of c (1–8) on scale C over the value of a (4–5) on scale D. Shift the hairline to the value of b (2) on C and read off the result under b on scale D.

The procedure is the same when more factors are in the numerator and denominator. The intermediate results need not be evaluated while the calculation is in progress.

The fraction $\frac{a \times b \times c \times d}{e \times f \times g}$ is calculated according to the following scheme

$a \searrow e \nearrow b \searrow f \nearrow c \searrow g \nearrow d$ and so on.

Example: $\frac{3.8 \times 6.27 \times 9.35}{7.2 \times 0.37} = 8\text{–}3\text{–}6 = 83.6$

By approximating the answer, the proper placement of the decimal point can be determined.

8. Tables and proportions

$$\boxed{\frac{a}{b} = \frac{c}{d} = \frac{e}{f}}$$

With every setting of the slide, the slide rule forms a table

on scales C and D, where all other opposing sets of numbers are always proportional. Every multiplication – or division – setting supplies not only the desired value, but also an entire range of pairs of corresponding values. For example, if C–1, an index of the C scale, is placed over the 1–5 of the D scale, a multiplication table is formed with the constant factor 1–5; at the same time, a division table is formed with the constant quotient 1–5.

Numbers in pairs of values in the proportion 1 : 1–5 are opposite each other. The dividing line between the body scale and the slide scale symbolizes the fraction-bar.

Example: Table for mm and inches
1 inch = 25.4 mm — 940 mm = 37 inches
24 inches = 610 mm

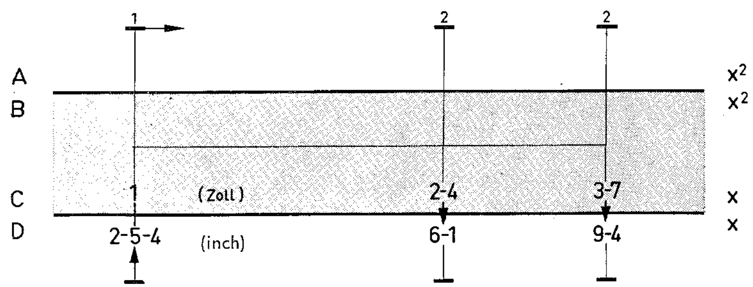


Fig. 15

Calculation: The given equality is set up with C–1 over D and, with the help of the hairline, the required proportion is estabilished.

As the example shows, with this setting the metric system can be converted into the English system and vice versa. Here pairs of numbers opposite each other are in the proportion 1 : 2–5–4.

Example:
$8.8 : \pi = 5.6 : x$
$x = 2$

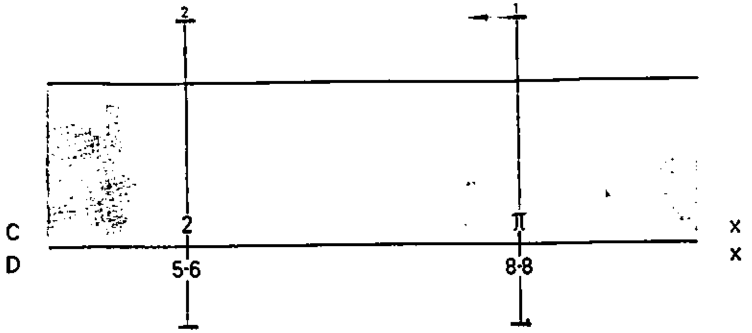


Fig. 16

A proportion of the form $a : b = c : x$ involves the same proportionsolving procedure: a and b are set opposite each other and opposite c the desired value x is read off.

The folded scales CF and DF offer the advantage of providing complete pairs of values without resetting the slide (see section 10).

9. The reciprocal scale CI

$\boxed{\frac{1}{a}}$

The CI scale (the inverse of C) corresponds entirely in its subdivisions to the fundamental scales C and D. The numbers of the CI scale increase, however, from right to left and are therefore colored in red. This scale has various advantageous features.

a) For every value a on C, its reciprocal $\frac{1}{a}$ is found on the CI scale.

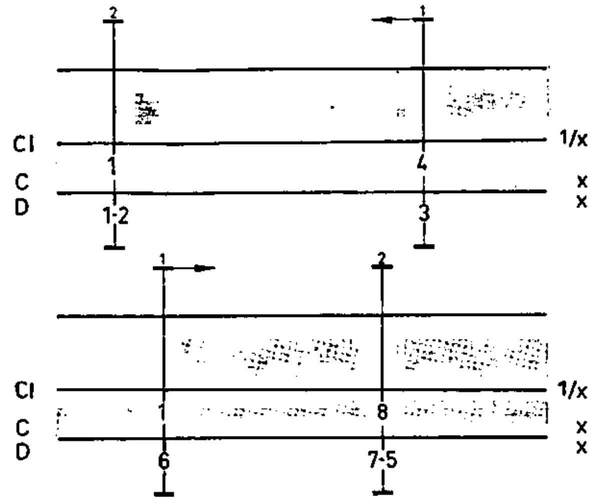
   The slide remains in the fundamental position, C–1 over D–1, for only the hairline is needed for the setting and reading.

Example: $\frac{1}{6} = 0.166 \quad \frac{1}{4} = 0.25$

b) The reciprocal scale allows a division to be replaced by a multiplication as well as a multiplication to be replaced by a division.

   $a \times b = a \div \frac{1}{b}$ and $a \div b = a \times \frac{1}{b}$

On the CI scale, set the reciprocal $\frac{1}{b}$ instead of b.



Example:

$3 \times 4 = 3 \div \frac{1}{4} = 12.$

$60 \div 8 = 60 \times \frac{1}{8} = 7.5.$

Fig. 17

Rule: **By using the CI scale, arithmetic processes are inverted:**
Multiplication becomes division.
Division becomes multiplication.

If the example is calculated with the fundamental scales C and D, the slide must be set to project left or right depending on the numbers involved. When using the CI scale, you always work with the slide projecting correctly automatically.

c) The CI scale is advantageous in multiplication with more factors of the form $a \times b \ldots \times n$.

Example: $35 \times 0.7 \times 25 = 612.5.$

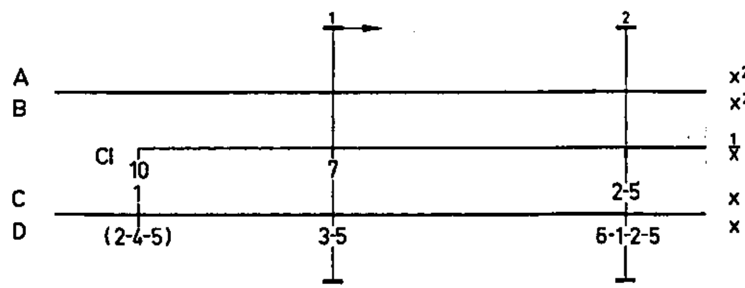


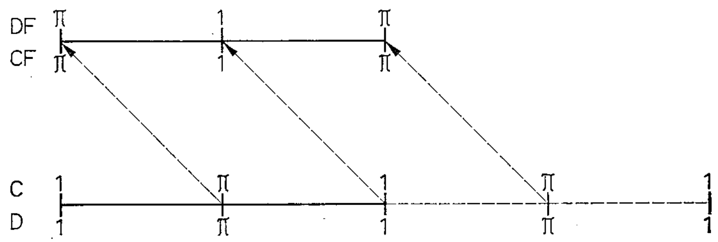
Fig. 18

Calculation: Place CI 7, with the help of the hairline, over D 3–5. Shift the hairline to C 2–5 and under it, read off the result 6–1–2–5 on D.

d) The combined multiplication and division is calculated with the reciprocal scale CI. As described in Section 8, begin with division and then follow alternately with multiplication and division.

10. **The folded scales CF, DF, CIF.**

a) These scales reduce the number of settings of the slide as well as serving to complete a common multiplication or division table-position (Sec. 5, above). Fig. 19 shows the displacement of the scales by the factor π.



Folded scales: displacement by π. Fig. 19

The scales CF, DF, and CIF correspond in their basic length and graduations (subdivisions) to the fundamental scales C, D, and CI. Note that CI and CIF are both reciprocal scales.

It is profitable to work both pairs of scales combined. As you go up from the lower to the upper pairs of scales, it is evident that CF represents the continuation of scale C just as DF represents the continuation of scale D. This is emphasized by the greencolored slide.

The principle is: **Same color – same scale.** That is, C and CF are on the green-colored slide as are also CI and CIF; but D and DF are on white.

b) When you form multiplication and division tables, you can see most clearly the advantages of the folded scales.

Example: Conversion of lengths by the drafting rule 1 ÷ 2.5 (reduction).

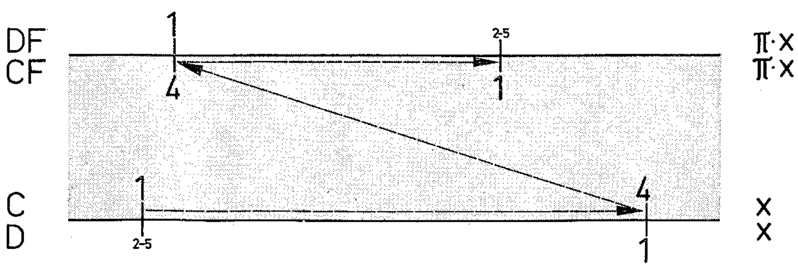


Fig. 20

As described in Section 8 – the formation of tables – set the proportion $\frac{C-1}{D2-5}$ and, with the help of the hairline, read off all pairs of values $\frac{C}{D}$ as far as C 4, where the table is discontinued because the slide extends beyond the body of the slide rule. For all remaining pairs of values, you could shift the slide and use the other index C–1. On the other hand, you can get all the remaining pairs of values by reading them on the $\frac{DF}{CF}$ folded scales without shifting the slide at all. The entire cycle can thus be read with a single setting of the slide. You use the proportion $\frac{C}{D} = \frac{CF}{DF}$

c) Furthermore, the combined use of the upper and the lower sets of scales is advantageous for both multiplication and division. In most cases, the inconvenient shifting of the slide becomes unnecessary.

In multiplication, if both factors are > 6, then the overlapping of the fundamental and the folded scales is not sufficient. It is useful to use the reciprocal scale CI (as described in Section 9) by replacing multiplication with a division using the reciprocal of a factor. The same holds true when in the division $a \div b$ the dividend $a >$ 3–6 and $a > b$; as for example, $40 \div 5 = 8$ is calculated $40 \times \frac{1}{5} = 8$ (see Section 9).

d) As Fig. 9 shows, the folded scales CF, DF, CIF are displaced by the value π. With the transition from C and D (lower) to CF and DF (upper), the CF and DF scales are C and D scales respectively multiplied by π. With the reversed transition (upper to lower) the C and D scales are CF and DF scales respectively divided by π. If π occurs in a multiplication problem, work the other factors on C and D and then deal last with the factor π by reading under the hairline on DF.

Example: Surface of an ellipse, $F = a \times b \times \pi$
with $a = 1.64$ cm, $b = 4.25$ cm, $F = 1.64 \times 4.25 \times \pi \doteq 21.9$ cm².

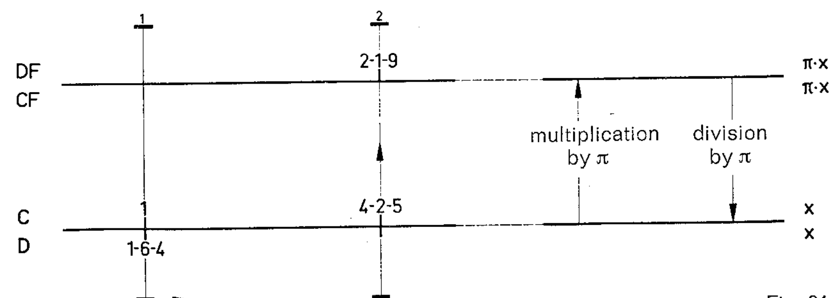


Fig. 21

e) The scale CIF functionally belongs to CF, DF just as CI belongs to C, D; therefore everything said in Section 9 concerning the reciprocal scale CI is also true for the displaced reciprocal scale CIF.

11. Squares and Square Roots

$\boxed{a^2, \sqrt{a}}$

For all numbers x on C and D, you can find the square x^2 on A and B. For all numbers x on A and B, you can find the square root $\sqrt{x}$ on C and D. The slide remains in the fundamental position; the reading is made with the help of the hairline.

Example: $2^2 = 4$; $5^2 = 25$; $8.06^2 = 65$
$\sqrt{2} = 1.414$; $\sqrt{10} = 3.16$; $\sqrt{39} = 6.24$

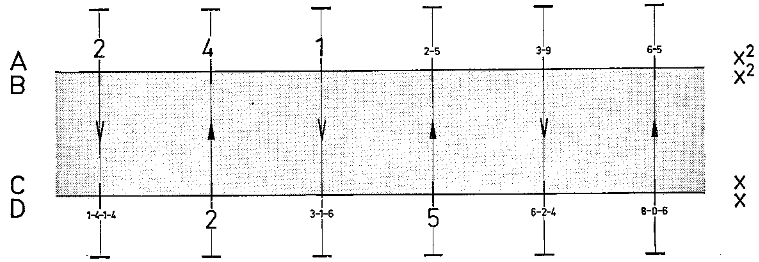


Fig. 22

Note the correct placement of the radicand on scales A and B. Numbers from 1 to 10, 100 to 1000 and so on, must be in the first logarithmic cycle, that is, in the left-hand half of the scale; numbers from 10 to 100, 1000 to 10000 and so on, must be placed in the second logarithmic cycle, that is, the right-hand half of the scale. In general: Radicands with an odd number of digits are located in the left half of the A scale.

Radicands with an even number of digits are located in the right half of the A scale.

12. Cubes and Cube Roots

$\boxed{a^3, \sqrt[3]{a}}$

The cube root scale K consists of three consecutive logarithmic cycles, each one-third the length of the basic C scale. The K scale is referred to the C scale and also to the D scale when the slide rule is closed in fundamental position. On the K scale you can find, with the help of the hairline, the cube x^3 for every number x on C and D.

Changing the point of view, you can find the cube root $\sqrt[3]{x}$ on C and D for every radicand x on K.

Example: $1.64^3 = 4.4$; $4^3 = 64$;
$\sqrt[3]{18} = 2.62$; $\sqrt[3]{180} = 5.65$

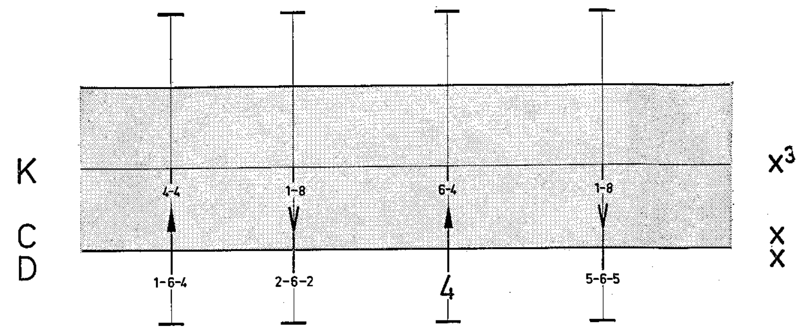


Fig. 23

Note the correct location of the radicand on scale K. One-digit numbers are located in the first, the left-hand cycle. Two-digit numbers are located in the middle cycle. Three-digit numbers are located in the third, the right-hand cycle. For larger or smaller numbers, mark off the digits in groups of three starting from the decimal point. Then, according to how many digits (1, 2, or 3) remain in the last group to the left of the decimal point, the number can be placed in the correct cycle.

B. Angular Functions

1. Trigonometric Scales

In order to calculate angular functions, you will find on the angular-function side of the slide rule the sine scale S, the tangent scale T, the sine and tangent scale ST, the red-numbered scales CS for the cosine, and CT for the cotangent; the CS and CT scales run from right to left, but they have the same scale divisions as S and T. You will find also here the Pythagorean scale P.

The trigonometric scales work with the fundamental scales C, D; the sine, cosine scales S, CS work additionally with the Pythagorean scale P; the tangent, cotangent scales T, CT also work with the reciprocal fundamental scale CI.

For every setting of an angle on the trigonometric scales, the angular function is given on C, D otherwise P or CI. Conversely, for every functional value you can find the corresponding angle. In order to avoid mistakes in reading, the slide remains in the fundamental (closed) position.

Note that the values of angles on the scales are decimally subdivided. The angles are presented in decimal form rather than in minutes and seconds.

The accuracy of the trigonometric scales corresponds to a three-place or at best a four-place table of angular functions. In relatively rare cases where you need more exact functional values, you must use a table with more columns. Also, slide rules only give function values for angles not greater than 90°. Functions of larger angles can be found by relating them to corresponding functions of angles of the first quadrant from 0° . . . 90°.

2. The Sine-Cosine Scale S, CS

sin α
cos α

The sine scale S is numbered in black and has the symbol "∢ sin" on its right side. It calculates the value of angles from 5.5° to 90°. In accordance with the relation cos α = sin (90° $-\alpha$), the cosine scale CS increases from right to left and is therefore numbered in red. It calculates the value of angels from 0° to 84.5°; on its right side is the symbol "∢ cos". The subdividions are common to both scales.

The subdivision is set forth in 6 sections:

sin α	Graduation interval	cos α
5.5° to 7°	0.05°	84.5° to 83°
7° to 15°	0.1°	83° to 75°
15° to 35°	0.2°	75° to 55°
35° to 60°	0.5°	55° to 30°
60° to 80°	1.0°	30° to 10°
80° to 90°	2.0°	10° to 0°

The angle α, for a fundamental value x = sin α, cos α or the inverse of the functional value for an angle, is gotten by shifting the hairline. The slide remains in the fundamental position.

Example: sin 30° = 0.5   cos 70° = 0.342
sin 9° = 0.1564   cos 41° = 0.755

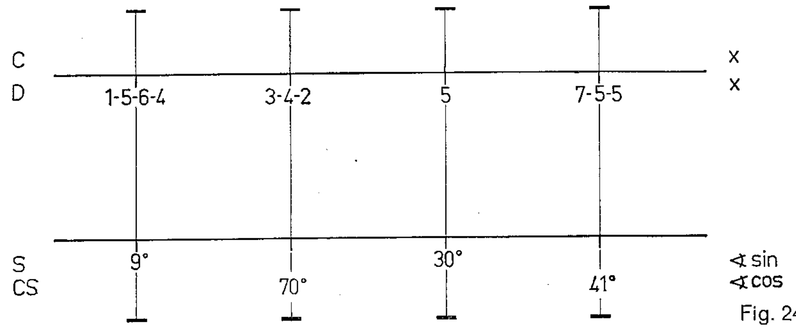


Fig. 24

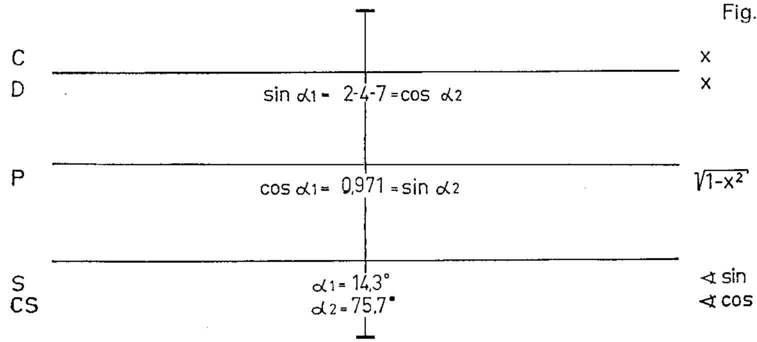
Calculation: Place the hairline above ∢ sin (black) or ∢ cos (red) on scale S, CS and read off the functional value x on scale D. When you are given the functional value and so are looking for the angle, you go from scale D to scale S, CS.

For sin > 45° and cos < 45° the accuracy is decreased. The use of the P scale is then useful in the following way:
For small angles (< 45°)
Scale S (black) with scale D (black) provides a sine table.
Scale S (black) with scale P (red) provides a cosine table.
For large angles (> 45°):
Scale CS (red) with scale P (red) provides a sine table.
Scale CS (red) with scale D (black) provides a cosine table.

Rule: **Like colors provide a sine table.**
Unlike colors provide a cosine table.

Example: α_1 = 14.3° (black) α_2 = 75.7° (red)
sin α_1 = 0.247 (black)   sin α_2 = 0.969 (red)
cos α_1 = 0.969 (red)   cos α_2 = 0.247 (black)

Fig. 25



By this illustration you can see that with one setting, the sines and cosines of an angle can be read without reading the angle from the sine to the cosine and vice versa. All the values are vertically aligned.

On scale S, CS, the angle setting is unique: the 7° graduation is only for 7° and not for common multiples or divisions of it. For the angles α on S, CS, the functional value x on C, D lies between 0.1 . . . 1.0. On scale P, the functional values are unique, as indicated in its symbol at its right side.

3. Sine Law

With the slide rule you can solve plane triangles with the sine law:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

With one slide setting, all pairs of values can be read since a proportion is thus established. The line between the slide and the body can be thought of as a fraction-bar, whereby the sides, a, b and c on scale C are opposite the angles α, β, and γ on scale S. With the setting of the known proportion, the two other proportions can also be read. For every side the angle opposite can now be read and vice versa.

For any triangle, a proportion (side : angle) must be given and also another angle or another side must be given. With $\alpha + \beta + \gamma = 180°$, the triangle can be completely determined.

The calculation involving the right triangle is of greater importance. With $\gamma = 90°$, $\sin\gamma = 1$ and $\sin\alpha = \sin(90° - \beta) = \cos\beta$ as well as $\sin\beta = \sin(90° - \alpha) = \cos\alpha$. Thus the sine law sor the right triangle is:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{1} \text{ and } \frac{a}{\cos\beta} = \frac{b}{\cos\alpha} = \frac{c}{1}$$

The triangle can be determined when two values are given.

Example: $b = 15.5$ cm, $\alpha = 70°$ Required: a, c, β $a = 42.6$ cm
$c = 45.3$ cm
$\beta = 90° - \alpha = 20°$

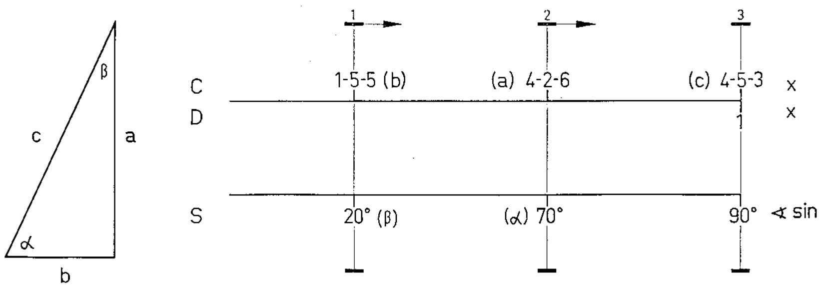


Fig. 26

Calculation: Get angle β and set the proportion $\frac{b}{\sin\beta}$ with scale C and scale S.
Hairline-settings: Above angle α on S and above 1 on D, find the lengths of a and c.

If both legs a and b are given, angle α must be calculated from the relationship $\tan\alpha = \frac{a}{b}$ and carried over to scale S. As shown in the last example, the proportion can now be set and the triangle thus determined.

4. The Tangent-Cotangent Scales T, CT.

tan α
cot α

The tangent scale black numbers cover the range of angles from 5.5° to 45°. The tangent scale (cotangent scale) red numbers continue the range from 45° to 84.5°.
The tangent and cotangent of the same angle are reciprocals, $\cot\alpha = \frac{1}{\tan\alpha}$.

The subdivision is set forth in 6 sections:

tan α	Graduation interval	cot α
5.5° to 7°	0.05°	84.5° to 83°
7° to 20°	0.1°	83° to 70°
20° to 45°	0.2°	70° to 45°
45° to 70°	0.2°	45° to 20°
70° to 83°	0.1°	20° to 7°
83° to 84.5°	0.05°	7° to 5.5°

The functional value $x = \tan\alpha$ of any angle from 5·5° to 45° (black), and conversely, the angle from a given functional value, can be found on scales C, D (black) from 0.1 to 1. For any angle from 45° to 84.5° (red), the functional value $x = \tan\alpha$ can be found on scale CI (red) from 1 to 10. This is explained by the relationship $\tan\alpha = \frac{1}{\tan(90° - \alpha)}$.

The cotangent is the reciprocal of the tangent. Consequently the functional value $x = \cot\alpha$ of any angle from 5.5° to 45° can be found on CI (red) from 10 to 1. For any angle from 45° to 84.5° (red) the functional value $x = \cot\alpha$ can be found on scales C, D (black) from 1 to 0.1.

The following summary may serve as a guide:
For small angles (< 45°):
Scale T (black) with scale D (black) provides a tangent table.
Scale T (black) with scale CI (red) provides a cotangent table.
For large angles (> 45°):
Scale CT (red) with scale CI (red) provides a tangent table.
Scale CT (red) with scale D (black) provides a cotangent table.

Rule: **Like colors provide a tangent table.**
Unlike colors provide a cotangent table.

Example: $\alpha_1 = 75^\circ$ (red) $\alpha_2 = 30^\circ$ (black)

$\tan \alpha_1 = 3.73$ (red) $\tan \alpha_2 = 0.577$ (black)

$\cot \alpha_1 = 0.268$ (black) $\cot \alpha_2 = 1.732$ (red)

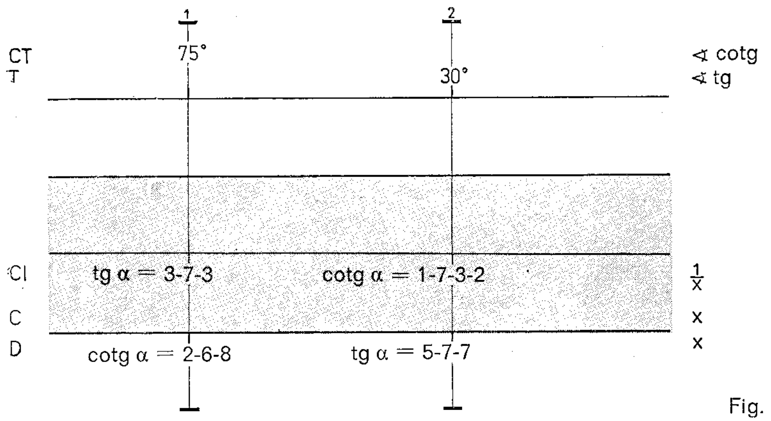


Fig. 27

With one setting, the tangent and cotangent of an angle can be read directly without reading the angle from the tangent to the cotangent and vice versa.

Note the color rule and the proper positioning of the decimal point on the scales C, D from 0.1 to 1 and on CI from 10 to 1 as shown in Fig. 27. The scales T, CT, as also the scales S, CS, have the actual values of the angles.

5. The ST Scale — Small Angles

arc a

The values of functions of small angles can be determined with the ST scale by the relationship: $\sin \alpha \simeq \text{arc}\, \alpha \simeq \tan \alpha$

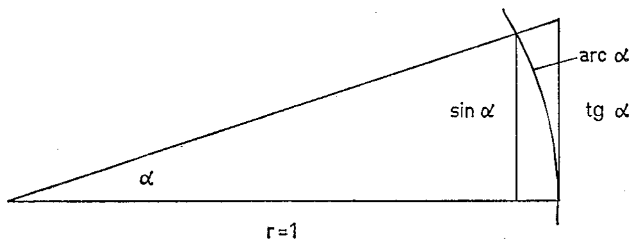


Fig. 28

With angles $< 6^\circ$ errors of approximation are insignificant with respect to slide rule accuracy. In Fig. 28, the angular functions are shown in the unit circle. There it can be seen that the difference between $\sin \alpha$ and $\tan \alpha$ decreases linearly with the angle.

The scale ST is divided into degree measure (°) and is used with the fundamental scales C, D; the range is from 0.55° to 6°. Four sections show the arrangement:

$\sin \alpha \simeq \text{arc}\, \alpha \simeq \tan \alpha$	Value of interval
0.55° to 0.7°	0.005°
0.7° to 2°	0.01°
2° to 4°	0.02°
4° to 6°	0.05°

On the ST scale, the decimal point location is correct for angles from 0.55° to 6°. Unlike the other angular function scales, decimal divisions (not multiples!) of the angles can be given from 0.55° to 6°. The functional value is read from scales C.

D. Note the decimal point location:

$\sphericalangle$ 0.55 to 6° gives functional values from $\simeq$ 0.01 to 0.1
$\sphericalangle$ 0.055 to 0.6° $\simeq$ 0.001 to 0.01
$\sphericalangle$ 0.005 to 0.06° $\simeq$ 0.0001 to 0.001
and so on.

By extending the relationships for $\alpha < 6^\circ$:
$\sin \alpha = \cos (90^\circ - \alpha) \simeq \tan \alpha = \cot (90^\circ - \alpha) \simeq \text{arc}\, \alpha$
the cosine and cotangent of angles $> 84.5^\circ$ can also be calculated.

Example:
$\sin 2^\circ = \cos 88^\circ \simeq \tan 2^\circ = \cot 88^\circ \simeq \text{arc}\, 2^\circ = 0.0349$
$\sin 0.01^\circ = \cos 89.99^\circ \simeq \tan 0.01^\circ = \cos 89.99^\circ \simeq \text{arc}\, 0.01^\circ = 0.0001745$

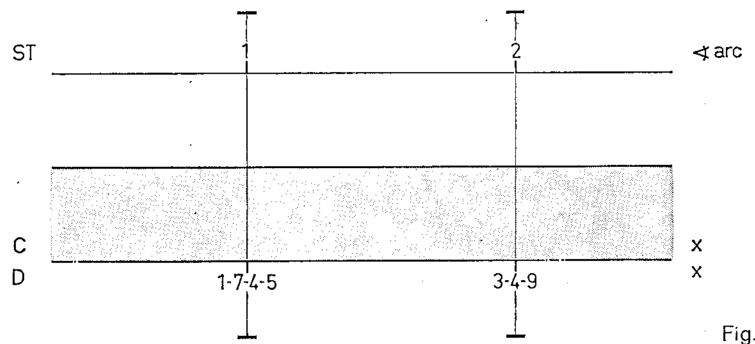


Fig. 29

Calculation: On scale ST, for the sine-tangent or the arc function of small angles, locate the angle and, for the cosine or cotangent function of large angles, the complementary angle. Read the functional value on scales C, D under the hairline. The decimal point location is determined by estimation.

From $\cot \alpha = \frac{1}{\tan \alpha}$, the cotangent of small angles can be determined. Because the values are reciprocal, the functional value is read on scale CI. For angles from 0.55° to 6°, the value of the cotangent lies approximately between 100 to 10. The value of the cotangent of decimal divisions of these small angles increases correspondingly.

5.1 **Angle Measure in Radians**

From the relationship angle $\alpha = \frac{\pi}{180} \times \alpha = (0.01745)\,\alpha$, it is readily recognized that ST is a scale displaced from the fundamental D scale by a factor of approximately $\frac{\pi}{180}$. It is then a simple matter to convert degree measure to radian measure and conversely from radians to degrees. In such conversions only the constant factor $\frac{\pi}{180}$ appears. The ST scale is not only for the indicated angle but also for multiples and divisions of the angle. Note the proper positioning of decimal points.

Example:

on ST	on D	
$\sphericalangle 0.35^\circ$	$= 0.0061$	rad
$\sphericalangle 3.5^\circ$	$= 0.061$	rad
$\sphericalangle 35^\circ$	$= 0.61$	rad
$\sphericalangle 350^\circ$	$= 6.1$	rad ($360^\circ = 2\,\pi = 6.28$ rad)

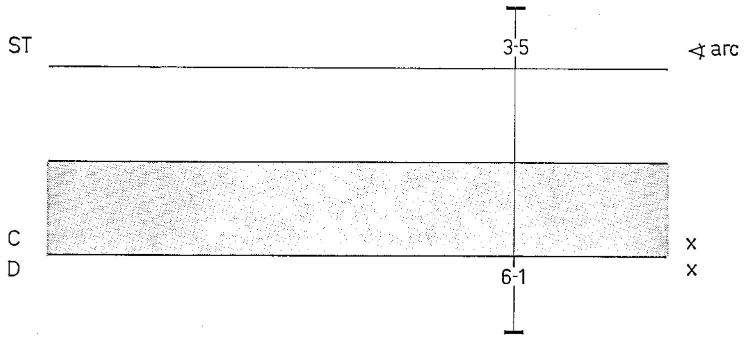


Fig. 30

Calculation: Set the angle on the ST scale. On scales C, D with the help of the hairline, read the angle's radian measure. Decimal point location is determined by estimation, as illustrated in 5–1 above.

5.2 **The marks ϱ' and ϱ''**

Very small angles are given in minutes and seconds. With the help of the marks ϱ' and ϱ'' on scale C, these angles can be converted to radians.

For the marks:

$$\frac{180}{\pi} = \varrho^\circ = 57.3 \text{ in degrees}$$

$$\frac{180}{\pi} \times 60 = \varrho' = 3438 \text{ in minutes}$$

$$\frac{180}{\pi} \times 60 \times 60 = \varrho'' = 206265 \text{ in seconds.}$$

Thus, $\sphericalangle \alpha = \frac{\alpha^\circ}{\varrho^\circ} = \frac{\alpha'}{\varrho'} = \frac{\alpha''}{\varrho''}$

and because $\sin \alpha \cong \text{arc}\,\alpha \cong \tan \alpha$, the sine and cotangent of the angles can be found on scale D. With the help of the hairline, the angular value, which is in decimal form, is read on scale ST.

Example: $\sin 18' \cong \tan 18' \cong \sphericalangle 18' = \frac{18'}{\varrho'} = 0.00524$ (on D) an then

$18' = 0.3^\circ$ (on ST).

$\sin 36'' \cong \tan 36'' \cong \sphericalangle 36'' = \frac{36''}{\varrho''} = 0.0001745$ (on D) and then

$36'' = 0.01^\circ$ (on ST).

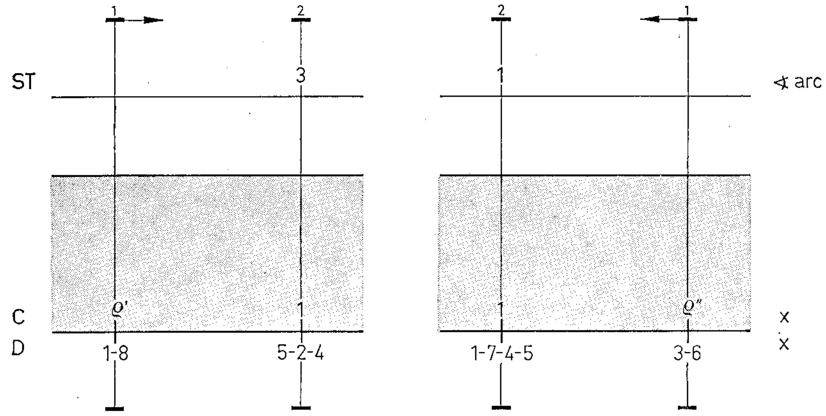


Fig. 31

Calculation: Place the mark ϱ' and ϱ'' of the C scale opposite the angular value, which is in minutes or seconds, of the D scale. Under C1 find the functional value and, with the help of the hairline, find the decimal angular value on scale ST. Determine the decimal point location by estimation.

6. **The Pythagorean Scale P**

$\boxed{\sqrt{1-x^2}}$

This scale corresponds to the function $y = \sqrt{1-0.1\,x^2}$. Since it increases from right to left, the numbers are colored red. The scale ranges from 0·995 to 0; the length of the scale is made up of nine differently subdivided sections.

The P scale is used with the D scale. Because of the symmetrical relationship $x^2 + y^2 = 1$, it does not matter whether x or y is given on scale D or P. The decimal point location is correct. In connection with scale P, scale D must be read from 0.1 to 1. In connection with scales S, CS and D, the P scale is used to obtain the functional values from the sines and the cosines (see Section 2). According to the Pythagorean trigonometric identity:

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} \text{ and also } \cos\alpha = \sqrt{1-\sin^2\alpha}$$

This means: the sine and cosine of an angle on scales S, CS lie opposite the scales C, D, and P. Recall the color rule from Section 2.

The second side of a right triangle can be determined with the help of scale P if the hypotenuse and another side are known.

If $c^2 = a^2 + b^2$, then $a^2 = c^2 - b^2$

and $a^2 = c^2\left(1 - \frac{b^2}{c^2}\right)$

$$\frac{a}{c} = \sqrt{1 - \left(\frac{b}{c}\right)^2}$$

Here $\frac{a}{c} = y$ and $\frac{b}{c} = x$ correspond to the terms in the equation for the Pythagorean scale P.

Example: Given $a = 26$ cm $\quad c = 80$ cm $\quad$ Required: $b, \measuredangle\alpha, \measuredangle\beta$

$\frac{b}{c} = x$; $b = x \times c = 0.9458 \times 80 = 78.52$ cm

$\measuredangle\alpha = 18.95°$

$\measuredangle\beta = 71.05°$

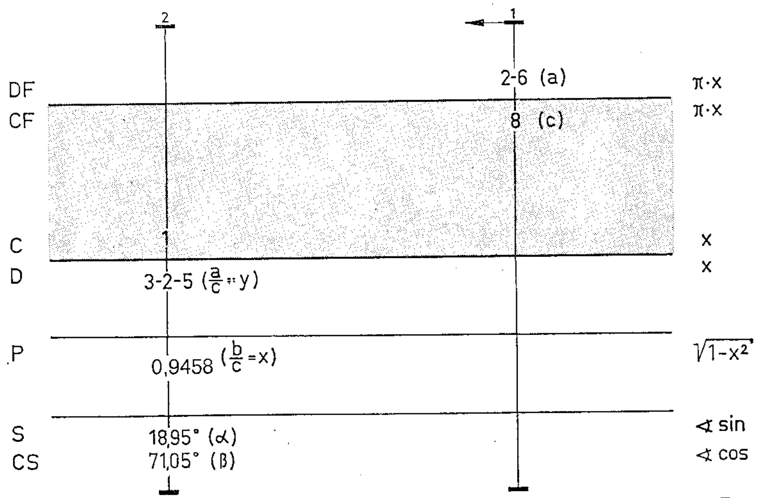


Fig. 32

Calculation: Carry out the division $\frac{a}{c}$ with scales C, D, or CF, DF. Shift the hairline to index C–1. Under the hairline on scale P, the fraction $\frac{b}{c}$ can be found; and on scales S, CS the angles α and β can be found. Retain b; the values obtained from scale P must still be multiplied by c. (This multiplication is not shown in Fig. 32.)

C. Exponential and logarithmic functions

1. Exponential Scales: LL

The exponential functions side of the slide rule has two main parts. The double-logarithmic (log log) scales LL1, LL2, LL3 are numbered in black from 1.01 to 10^5. The logarithmic scales LL01, LL02, LL03 are numbered in red from 0.99 to 10^{-5}. Because of the two aspects of the LL-logarithmic scales there exists no problem of decimal point location. The number 5 represents only 5, not multiplied or divided by positive or negative powers of 10. Thus this number does not represent 50, 500, 0.5 or 0.05. All LL-scale numbers are exactly as they are given.

The LL scales serve to calculate

any power of the form $y = a^x$

any root of the form $a = \sqrt[x]{y}$

and any logarithm of the form $x = \log_a y$.

The fundamental scales C, D are used with the LL scales. A setting of a value x on C, D yields the values e^x, e^{-x} on the LL scales. These values are in accord with the mathematical symbols shown at the right-hand end of the scales.
Corresponding to the reciprocal relationships,

$$e^x = \frac{1}{e^{-x}} \text{ and } e^{-x} = \frac{1}{e^x}$$

there are positive powers (numbered in black) and negative powers (numbered in red) on correspondingly numbered LL scales which are reciprocal to each other. With the help of the hairline, reciprocal values with their proper decimal points can be formed. Furthermore, setting of numbers from 1.01 (on LL1) through 3 (on LL2) is more accurate than on the reciprocal CI and CIF scales. In the LL-System, LL1 and LL01 are reciprocal to each other, as are also LL2 and LL02, and also LL3 and LL03.

By giving separate attention to decimal point location, reciprocal numbers from 10101 to 9900 can be formed. Such numbers are accurate to five places.

The exponential scales are continuous functional scales in sets of three, with a total length of about 75 cm. The graduations represent the logarithms of the logarithms of the fundamental D scale, but they are marked with the numbers themselves (rather than the logarithms). The general idea of the relationships of the fundamental scale with the corresponding values on the LL scales can be seen in Fig. 33. Here the range of the positive and of the negative LL-scales can be examined in detail.

The basic length is the length of the fundamental scales C, D. In the illustration, Fig. 33, this length is shown and repeated twice toward the left and marked with proper decimal points, moreover, so that the correspondence of the fundamental scale values with appropriate LL-scale values may be more readily recognized. The corresponding sections are arranged vertically above and below one another in three sections so that they can be analyzed.

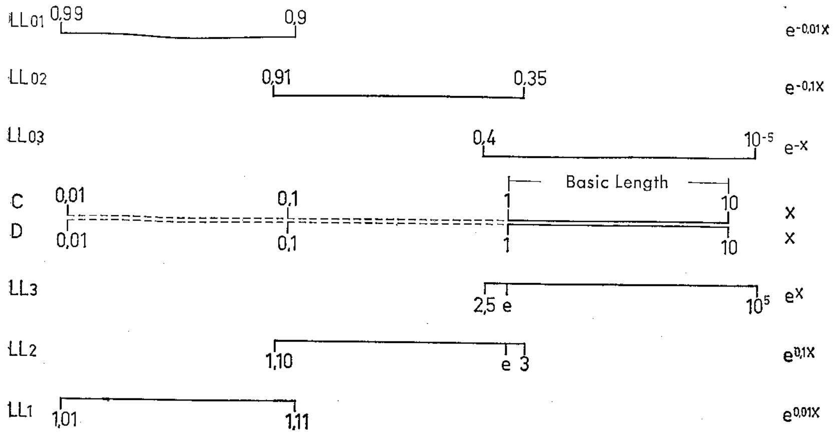


Fig. 33

The arrangement of the sections is such that when you go from an outer to an inner LL scale – say, from LL01 to LL02 or from LL1 to LL2 – you calculate the 10th power. Going from LL02 to LL03 or from LL2 to LL3 again you get the 10th power. But when you go from an outermost to an innermost scale – say, from LL01 to LL03 or from LL1 to LL3 – you calculate the 100th power. On the other hand, going from innermost to an outermost scale gives the 100th root. Two examples, one general and one numerical are given in Fig. 34.

$a = 5$ $b = 0.4$

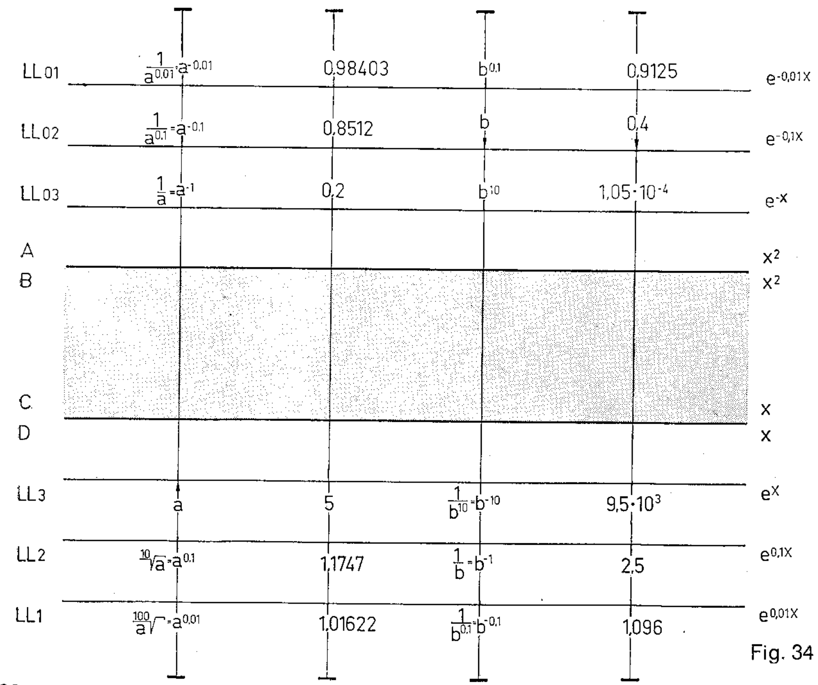


Fig. 34

Calculations involving 10th and 100th powers and roots are rare. The illustration shows the interrelationship of the exponential scales: the three sections of the log-log functional scales.

2. Powers of the form $\boxed{y = a^x}$

Calculations of powers with any base and exponent can be carried out as a multiplication. The green arrow in Fig. 35 shows how this can be done.

Example: $3^4 = 81$ $3^{-4} = 0.0123$

$3^{0.4} = 1.552$ $3^{-0.4} = 0.6445$

$3^{0.04} = 1.0449$ $3^{-0.04} = 0.957$

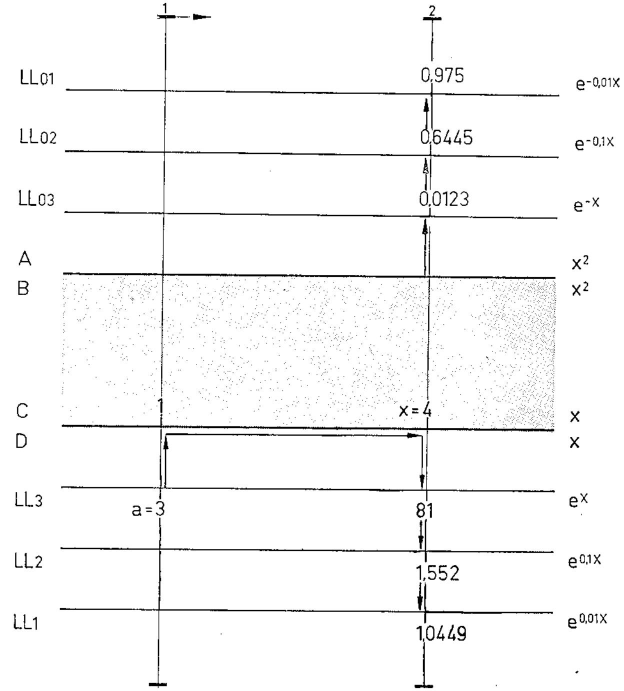


Fig. 35

Calculation: With the help of the hairline, set the left-hand index C–1 over the base $a = 3$ which is on the LL3 scale (position 1). Shift the hairline over the exponent $x = 4$ on C (position 2). On LL3, read the power: $y = 81$. On the corresponding LL scales, read the decimal variants. Set the base a in the range $10^{-5} < a < 0.99$. The powers $y = a^x$ can be found with positive exponents x on the scales from LL01 to LL03. The powers y with negative exponents can be found on the scales from LL3 to LL1.

Example:

$0.9^{0.4} = 0.9587$
$0.9^{4} = 0.656$
$0.9^{40} = 0.0148$

$0.9^{-0.4} = 1.043$
$0.9^{-4} = 1.524$
$0.9^{-40} = 67.5$

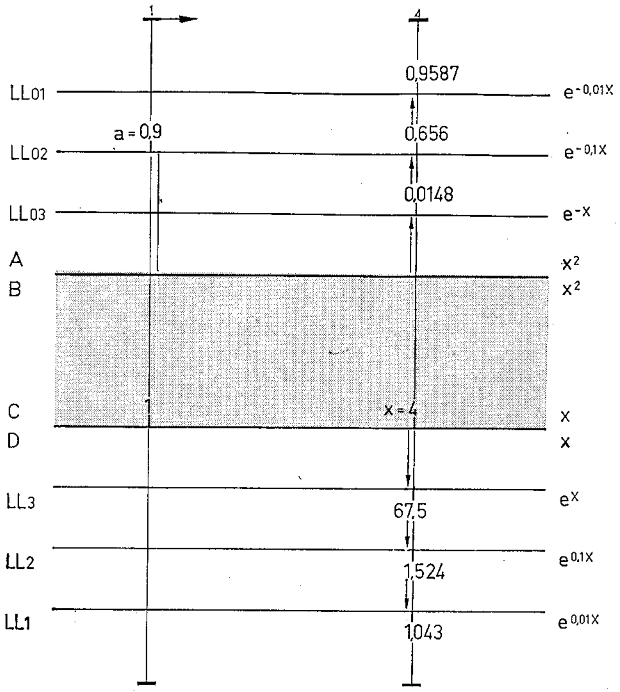


Fig. 36

There is no essential difference in procedure between the two sets of examples. In both examples, you can see: for powers with positive exponents, the setting of the base and the reading of the power were in the same scalar group; for powers with negative exponents, the base was set in one scalar group and the power was read in the other scalar group.

Rule: **Positive exponent – same scalar group and same color.**
Negative exponent – scalar group and color both change.

If the base a must be aligned with the right-hand index C–1, that is, with the slide projecting to the left, then the power must be read on the next higher scale. For example: suppose you have base a set on LL2 – $e^{0.1x}$, aligned with the right-hand index C–1, and the exponent lies between 1 and 10; then you must read the power y on LL3 – e^{x} (also see Fig. 37).

The slide rule constructs a table of powers for any selected base. With any base set on one of the LL scales and aligned with an index C–1, a particular power y, as well as continuous range of powers, can be read simply by resetting the hairline to any desired exponent on C. Therefore calculations of exponential functions of the form of $y = a^{x}$ are possible as long as the accuracy is sufficient.

Example: Let it be required to calculate a sequence of powers with base = 2 and exponents $x = 0.1 \ldots 10$

$2^{1} = 2;\ 2^{2} = 4;\ 2^{3} = 8;\ 2^{4} = 16 \ldots$
$2^{0.1} = 0.0718;\ 2^{0.2} = 1.1488;\ 2^{0.3} = 1.231;\ 2^{0.4} = 1.3195 \ldots$

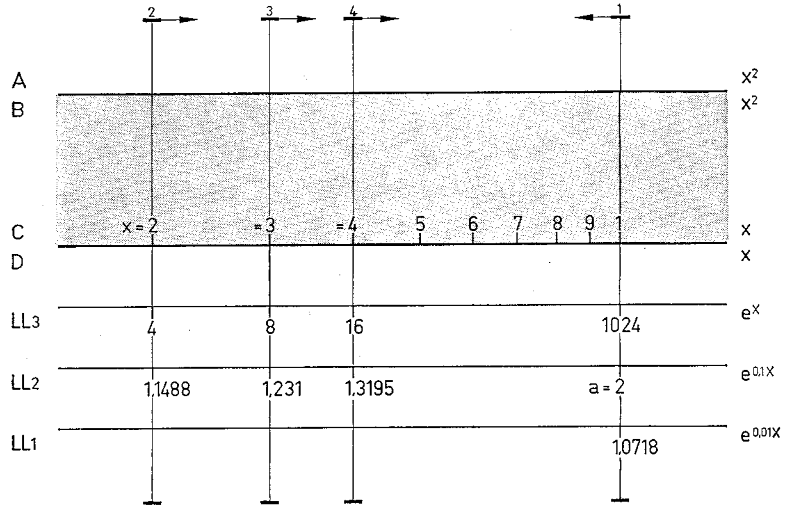


Fig. 37

With the alignment of a base with the right-hand index C–1, keep in mind that you read the power on the higher scale; for example, instead of $e^{0.1x}$, read e^{x}, and so on.

2.1 Powers of e

In the special case where base $a = e = 2.71828$ (the base of natural logarithms), C–1 must be aligned over e. But preferable work with the D scale since it is fixed in its relationship to e. Powers of e are calculated only with the setting of the hairline; in order to avoid errors in reading, the slide should be in the fundamental closed position.

The function of the form $y = e^{x}$ is called the natural exponential function; this function is of particular importance in natural science, technology, and engineering. The mathematical symbols at the right side of the slide rule indicate the correct decimal point location for exponents on e.

If the variable x is a composite term, and if a larger functional value is to be calculated, all values should be arranged in the form of a table; the variable x may then be determined. The functional value can then be calculated on the slide rule.

2.2 Special cases of the power $y = a^x$

Although certain limitations are present in exponential scales – values below 10^{-5} and above 10^5 and values between 0.99 and 1.01 are not available on the slide rule – nevertheless values of powers, bases, and exponents that happen to lie in these areas can be dealt with. In Fig. 38 the two extreme areas and the gap in the middle of the range are represented by green hatching on the line. For these three areas methods of close approximation are as follows.

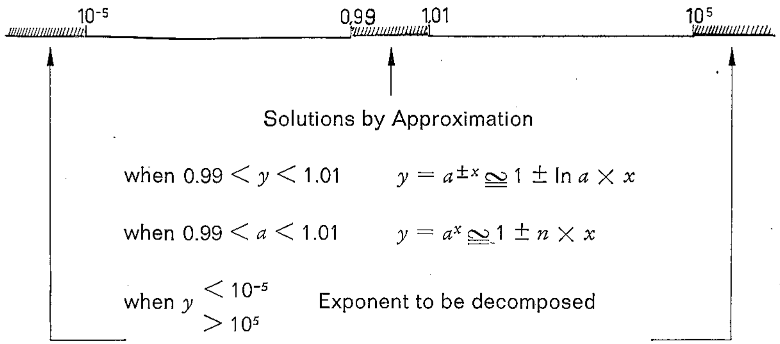


Fig. 38

If because of a very small exponent, the resulting power lies within the gap of the LL-scales, then the infinite series

$$y = a^{\pm x} = 1 \pm \frac{x}{1!} \ln a \pm \frac{x^2}{2!} \ln^2 a \pm \frac{x^3}{3!} \ln^3 a \pm \ldots$$

suggests that a good approximation in the form of

$$y = a^{\pm x} \cong 1 \pm x \times \ln a$$

is possible.

Example: $3^{0.004} \cong 1 + 1.1 \times 0.004 = 1.0044$
$3^{-0.004} \cong 1 - 1.1 \times 0.004 = 1 - 0.0044 = 0.9956.$

Calculation: Set C–1 over base $a = 3$ on LL3; in such a case C–1 is simultaneously set over $\ln a$ on scale D. Reset the hairline over the exponent $x = 4$ on scale C. The value under the hairline on scale D is now $\ln a \times x = 4.4$; decide on the decimal point location and, as previously indicated, add 1 and thus arrive at the correct result.

When base a lies within the gap of the LL-scales, the approximating solution can be found from the above mentioned equation

$$a^x = (1 \pm n)^x \cong 1 + n \times x$$

Here, n signifies the digits to the right of the decimal.

Example: $1.0038^{2.5} \cong 1 + 0.0038 \times 2.5 = 1.0095$
$1.0038^{-2.5} \cong 1 - 0.0038 \times 2.5 = 0.9905$
$1.0038^{25} \cong 1.0996$
$1.0038^{-25} \cong 0.9094$

Calculation: Set C–1 over $n = 3–8$ on scale D. Under $x = 2–5$ on C read the product $\times\ x$ and add 1 to that product.

If the exponent is larger, as in the 3rd and 4th examples, then the value of the power can be read directly on the properly selected LL-scale.

As a third possibility, represented in Fig. 38, the value of a power can exceed the range of the LL scales; its value can be smaller as well as larger than the coverage of the LL scales. Then it is necessary to decompose the exponent into a product and the power into factors. Accuracy is not very great. Calculations of such larger numerical values can be done with logarithms.

3. Roots of the form

$$\boxed{a = \sqrt[x]{y}}$$

Extracting a root is the inverse of raising to a power. Raising to a power on the exponential scales is analogous to multiplication with the fundamental scales; correspondingly, extraction of a root is analogous to division with the fundamental scale.

To get roots, reverse the procedure for getting powers.

Example:

$\sqrt[0,6]{64} = 1050$

$\sqrt[6]{64} = 2$

$\sqrt[60]{64} = 1{,}0718$

$\dfrac{1}{\sqrt[6]{64}} = 0{,}5$

$\dfrac{1}{\sqrt[0,6]{64}} = 9{,}75 \cdot 10^{-4}$

$\dfrac{1}{\sqrt[60]{64}} = 0{,}933$

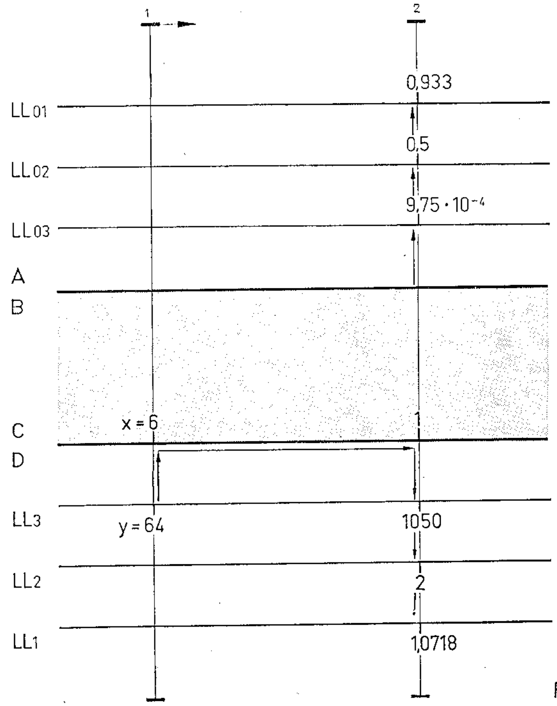


Fig. 39

Calculation: With the help of the hairline, set the exponential index x on scale C over the radicand y on the LL-scale. Under C–1, read the root on the corresponding LL-scale. When, as in the example, you must read under the right-hand index C–1, the value of the root can be found on the next lower LL-scale.

A root's index can be changed into a power. Consequently, there is a second means of solution. It is

$$a = \sqrt[x]{y} = y^{\frac{1}{x}} \qquad \sqrt[6]{64} = 64^{\frac{1}{6}} = 2$$

The exponent can be given as a reciprocal on the CI scale; in such a case the slide rule is turned over. The fraction can also be divided and expressed decimally. Then the resulting calculation is the same as for "Powers", previously described. That is, the root problem is converted to a power problem with the exponent expressed as a decimal.

$$a = \sqrt[x]{y^m} = y^{\frac{m}{x}} \qquad \sqrt[6]{16^3} = 16^{\frac{3}{6}} = 16^{0.5} = 4$$

4. Logarithms of the form $\boxed{x = {}^{a}\log y}$

The logarithm is a second inverse of a power. Any logarithm can therefore be calculated with the exponential scales. From the relationship

$y = a^x$ and $x = {}^{a}\log y$

it can be seen that the logarithm x and the exponent x, which are identical, may be determined.

Example: ${}^{3}\log 81 = 4.0$

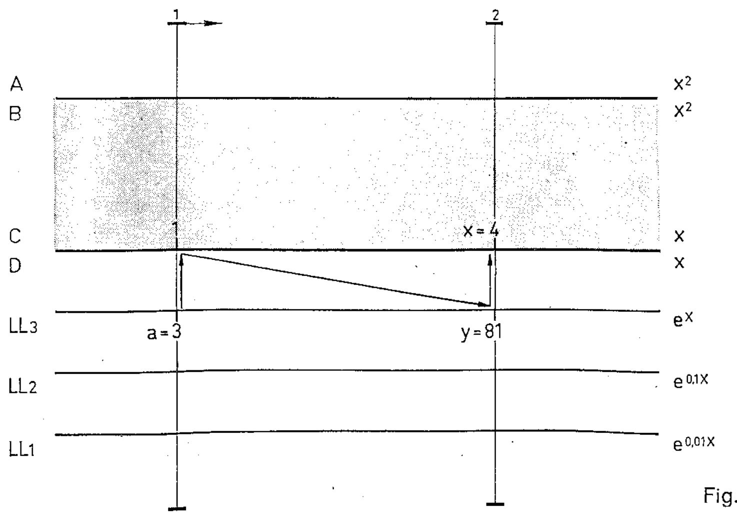


Fig. 40

Calculation: With the hairline, find the base $a = 3$ on the LL3 scale and set scale C–1 under the hairline (position 1). Shift the hairline over the number $y = 81$ on LL3 (position 2). Under the hairline, read the logarithm x

on scale C. The location of the decimal point of the logarithm is suggested by the following:

${}^{a}\log a = 1$; this means that for numbers $y >$ base a, the logarithm $x > 1$; for numbers $y <$ base a the logarithm $x < 1$.

Rule: $y > a$, log > 1 $\qquad$ $y < a$, log < 1.

The following observation will help you place the decimal point correctly in a logarithm: As you locate a number going from one LL-scale to the next lower one in the sequence LL 3 → LL 2 → LL 1 and similarly LL 03 → LL 02 → LL 01, the decimal point moves one place to the left in the logarithm.

When the base and the number are in the same scalar group (scales numbered in the same color), the logarithm is positive. When the base is in one scalar group and the number is in the other scaler group (one scale numbered in black and the other in red), the logarithm is negative.

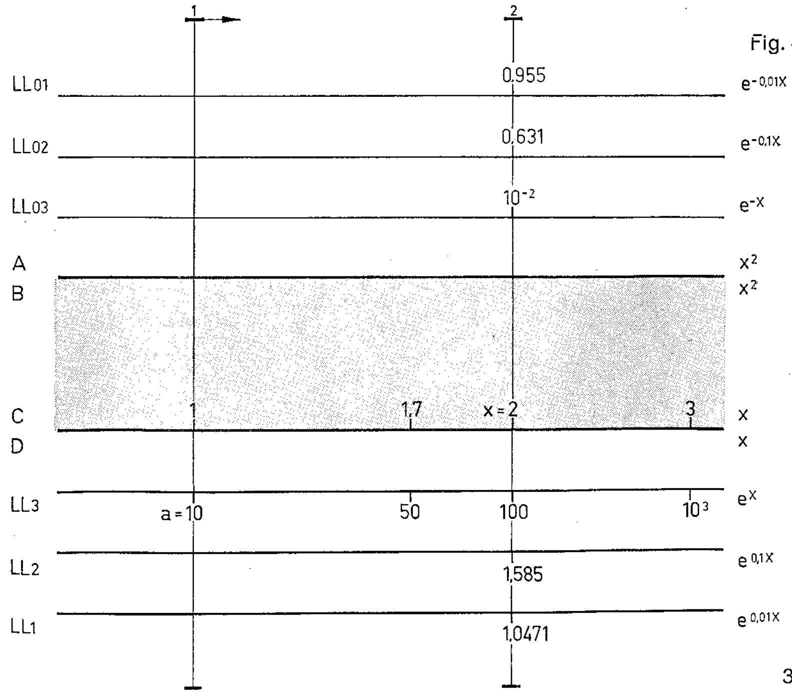
4.1 Common Logarithms: base 10.

Common (Briggsian) logarithms are to the base 10. The slide rule provides two possible procedures to find common logarithms.

The LL-scales, when used with the C scale, yield the complete base-10 logarithm; that is, characteristic and mantissa. The accuracy decreases with increasing (> 2.5) and decreasing (< 0.4) numbers. This method is, however, advantageous for numbers on the LL 2, LL 1, LL 02, LL 01 scales.

Example: log 100 = 2 $\qquad$ log 1.0471 = 0.02 $\qquad$ log 0.631 = – 0.2
log 1.585 = 0.2 $\qquad$ log 0.01 = log 10^{-2} = 2 $\qquad$ log 0.955 = – 0.02

Fig. 41



Calculation: To find common logarithms, set C−1 over base−10 on LL3 (position 1). With the hairline, find the number on the corresponding LL-scale. Then considering proper decimal point placement and previously stated rules, read on scale C the complete logarithm, that is, the characteristic and the mantissa both (Position 2, Fig. 31).

The other procedure is to use the L scale (marked "lg x" at the right-hand end) in conjunction with the C scale. For any number set on C, the mantissa of that number will be given directly above it on L. The characteristic that must be prefixed to the mantissa of the logarithm must be ascertained separately by well-known rules. For numbers > 2.5 and < 0.4, this is the procedure for finding common logarithms with greater accuracy.

Example: log 2 = 0.301 | log 0.2 = 0.301 − 1 = − 0.699
log 40 = 1.602 | log 0.04 = 0.602 − 2 = − 1.398
log 615 = 2.789 | log 0.00615 = 0.789 − 3 = − 2.211

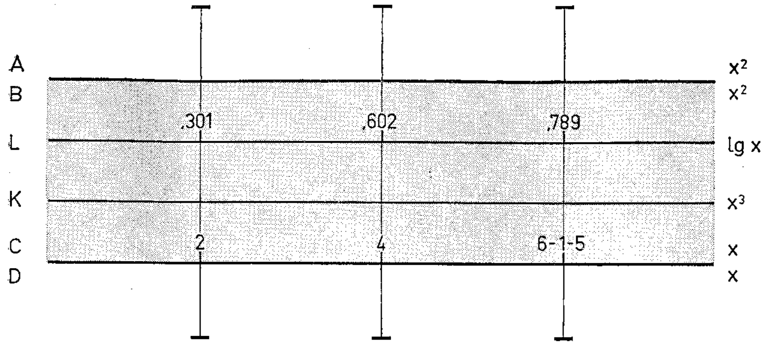


Fig. 42

Scale L corresponds to a three-place logarithm table. In order to avoid errors in reading – confusion with scale D – the slide should be in the fundamental closed position.

4.2 Natural Logarithms: base e.

These logarithms are to the base $e = 2.71828\ldots$ The natural logarithms are of great importance in natural science, technology, and engineering.

In this case, D−1 is fixed, vertically aligned with base e: consequently every number set on the D scale is the natural logarithm of numbers aligned with it on the LL scales, subject only to the rules for location of the decimal point. Taking the decimal points into consideration, the range of the natural logarithm is as follows

from 1 to 10 on scale LL 3
from 0.1 to 1 on scale LL 2
from 0.01 to 0.1 on scale LL 1
from − 1 to − 10 on scale LL 03
from − 0.1 to − 1 on scale LL 02
from − 0.01 to − 0.1 on scale LL 01

Example: ln 30 = 3.4 | ln 0.04 = − 3.22
ln 1.665 = 0.51 | ln 0.817 = − 0.202
ln 1.092 = 0.088 | ln 0.9864 = − 0.0137

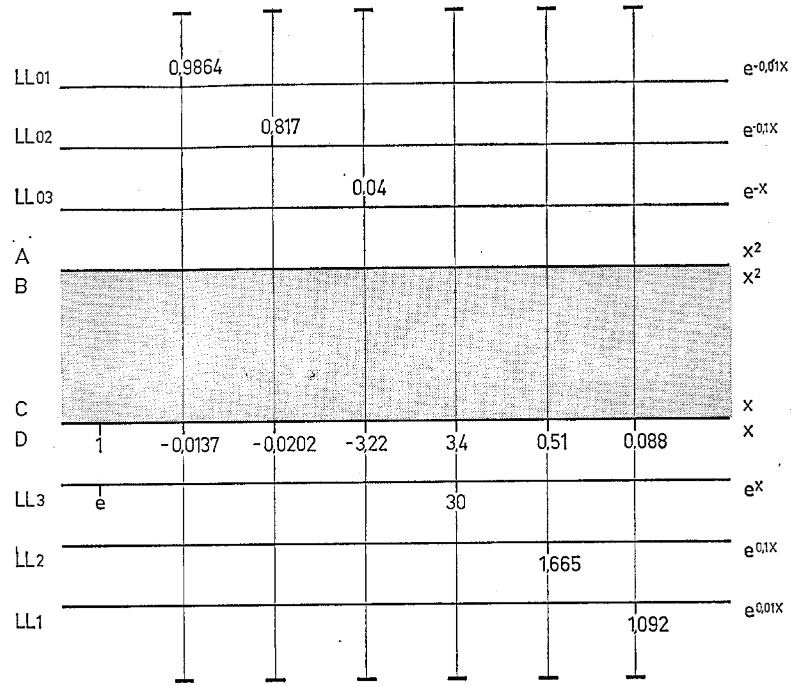


Fig. 43

The accuracy is greatest for numbers near the beginning of the LL 1 and LL 01 scales and greatly decreases toward the end of the LL 3 and LL 03 scales. In order to avoid errors in reading, the slids should be in the fundamental closed position.

5. Combinations of Functions

Every slide rule scale displays a particular functional value of the x of the fundamental scales C, D. On the exponential scales, this appears as $e^{\pm x}$. Keeping in mind that the settings are the actual values, you can pass from any scale whatsoevery directly to the exponential scales, avoiding the fundamental scales.

Such possibilites are exhibited in the following table in which all of the functions are shown in their relationships.

With C−1, set any base a on an LL scale; then the first 5 columns of the table serve not only for base e, but for any base. The selected value of x may then be set on the slide scales. Now, bypassing the fundamental C scale, go directly to the exponential scales.

Resulting function of x	A, B x^2	K x^3	L $\lg x$	CI $\frac{1}{x}$	CF,DF $\pi \cdot x$	S $\sphericalangle$sin	ST $\sphericalangle$arc	T $\sphericalangle$tan	P $\sqrt{1-x^2}$
x on the fundamental scale	$\sqrt{x}$	$\sqrt[3]{x}$	10^x	$\frac{1}{x}$	$\frac{x}{\pi}$	$\sin x$	arc x	$\tan x$	$\sqrt{1-x^2}$
x on the exponential scale	$e^{\pm\sqrt{x}}$	$e^{\pm\sqrt[3]{x}}$	e^{10^x}	$e^{\pm\frac{1}{x}}$	$e^{\pm\frac{x}{\pi}}$	$e^{\pm\sin x}$	$e^{\pm \mathrm{arc}\, x}$	$e^{\pm\tan x}$	$e^{\pm\sqrt{1-x^2}}$

Example: $4\sqrt[3]{6} = 7.28$ $\qquad 10^{\frac{1}{6}} = 1.1485$

6. Hyperbolic Functions

The hyperbolic functions are defined as:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

With a hairline set to any value x on scale D, the powers e^x and e^{-x} can be read on the exponential scales. The formation of the hyperbolic functional values can be done quickly and simply.

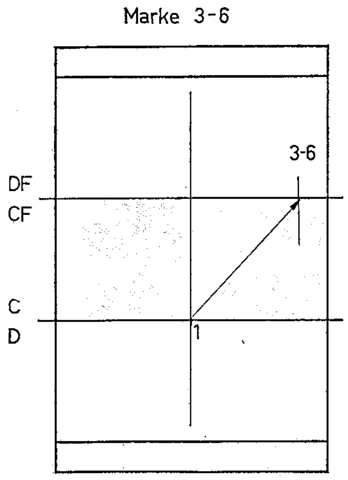
For $x = 0.01 \ldots 0.1$ use scale LL1 and LL01
$x = 0.1 \ldots 1$ use scale LL2 and LL02
$x = 1 \ldots 10$ use scale LL3 and LL03.

The greatest accuray of the functional value occurs when $x = 0.01$ and greater; but approaching $x = 10$, the accuracy decreases. Consequently, toward the end of the LL3 and LL03 scales, the accuracy is not very great.

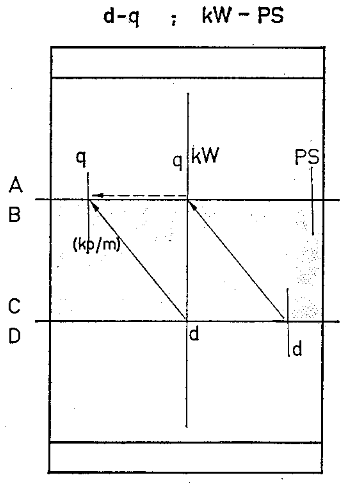
D. Miscellaneous

1. The Runner (Cursor)

The runner covers the full width of the slide rule, with a hairline on each side. These hairlines are so carefully adjusted to each other that when the slide rule is turned over from one side to the other, accurate related readings can be made. Refer to the face of the slide rule that has the π-folded scales CF and DF (see Fig. 44a). To the right of the hairline at the CF, DF level you will see a short red vertical mark above which is marked the factor 3–6. When the hairline is set at a value on the fundamental scales C, D, then the value under the red mark on CF, DF multiplies the C, D value by the constant factor 3–6, a conversion factor to get from degrees to seconds and so on. Going from the red mark on CF, DF to the hairline on C, D divides the mark value by 3–6. A few conversions that are easy to carry out are: 1° = 3600″
1 hour = 3600 seconds
1 meter/sec = 3.6 km/hr
1 year = 360 days



a.



b.

Fig. 44

On the LL side of the slide rule (Fig. 44b), the hairline has the mark KW and the line marked in red at the right has the mark PS above it. With these markings you can make conversions from KW to PS and from PS to KW in accordance with the equation:

0.746 KW = 1 PS

Conversions into the English measuring system, from KW to HP (horsepower) are done with the runner. For the equation 0.746 KW = 1 HP, the PS mark is replaced by the HP mark.

The lines marked in red at the upper left and at the lower right as in Fig. 44b – corresponding to the formula $q = d^2 \frac{\pi}{4}$ – are used to calculate the area q of a circle of a given diameter d. Set the lower-right red line as the diameter d on the D scale and read the area q under the hairline on the A scale. The same result can be obtained by using the hairline and the upper-left red line respectively on the indicated scales because the upper-left red line is displaced from the hairline by the factor $\frac{\pi}{4} = 0.785$.

The digits of $\frac{\pi}{4}$ are the same as those of the specific weight 7.85 of molten steel. Thus, in the cross-sectional area calculation, the weight per meter (kg/m) can be read at the left mark. Place B–1 under the mark so that the weight for any length can be calculated by multiplication involving the A and B scales.

Example: $d = 6$ mm; set the marked-line d over D–6; then the value $q = 28.3\ \text{mm}^2$, read on scale A under the hairline
Weight = 0.222 kg/m, read on scale A under the left marked-line kg/m.
Weight for 3.6 m = 0.8 kg, B–1 under the left marked line, read off of scale A above B-3–6.

2. The Treatment and Cleaning of the Slide Rule

Slide rules are precision tools which, because of their solid construction and reliable performance, sometimes receive rough treatment without losing their accuracy. Avoid exposing your slide rule, however, to temperatures above 70° C. After being exposed to these high temperatures, a return to normal temperature may deform the material to such an extent that the slide rule is no longer serviceable.

Occasionally, your slide rule should be cleaned. **Do not use any kind of corrosive chemicals or strong solutions.** If you use any of these substances, the scalar graduations may be obliterated. You should clean your slide rule with lukewarm water in which **a small amount** of the usual household cleanser is dissolved. After that, polish the slide rule with a dry, soft piece of wool or with a linen cloth. To make the slide move more easily, spread an extremely light coating of vaseline on the tongue and grooves of the slide.

The **lower side of the runner glass** can be cleaned with a piece of blodding paper which may be shifted between slide rule and runner.

3. Ruler with International and DIN Conversion Factors

With every slide rule you will find a strip of plastic. It may be used as a simple scale such as a draftman's scale or as a table of conversion factors.

The slide with the cm scale carries the conversion factors of the DIN-Standards (or Series) R 5, R 10, R 20, R 40, and a tabulation of multiples and divisions of units.

The other side with the inch scale also carries the conversion factors for the International Standards (or Series) E 6, E 12, E 24 in accordance with ISO (International Organization for Standardization), a table of decimal equivalents, and capital and lower case letters of the Greek alphabet.

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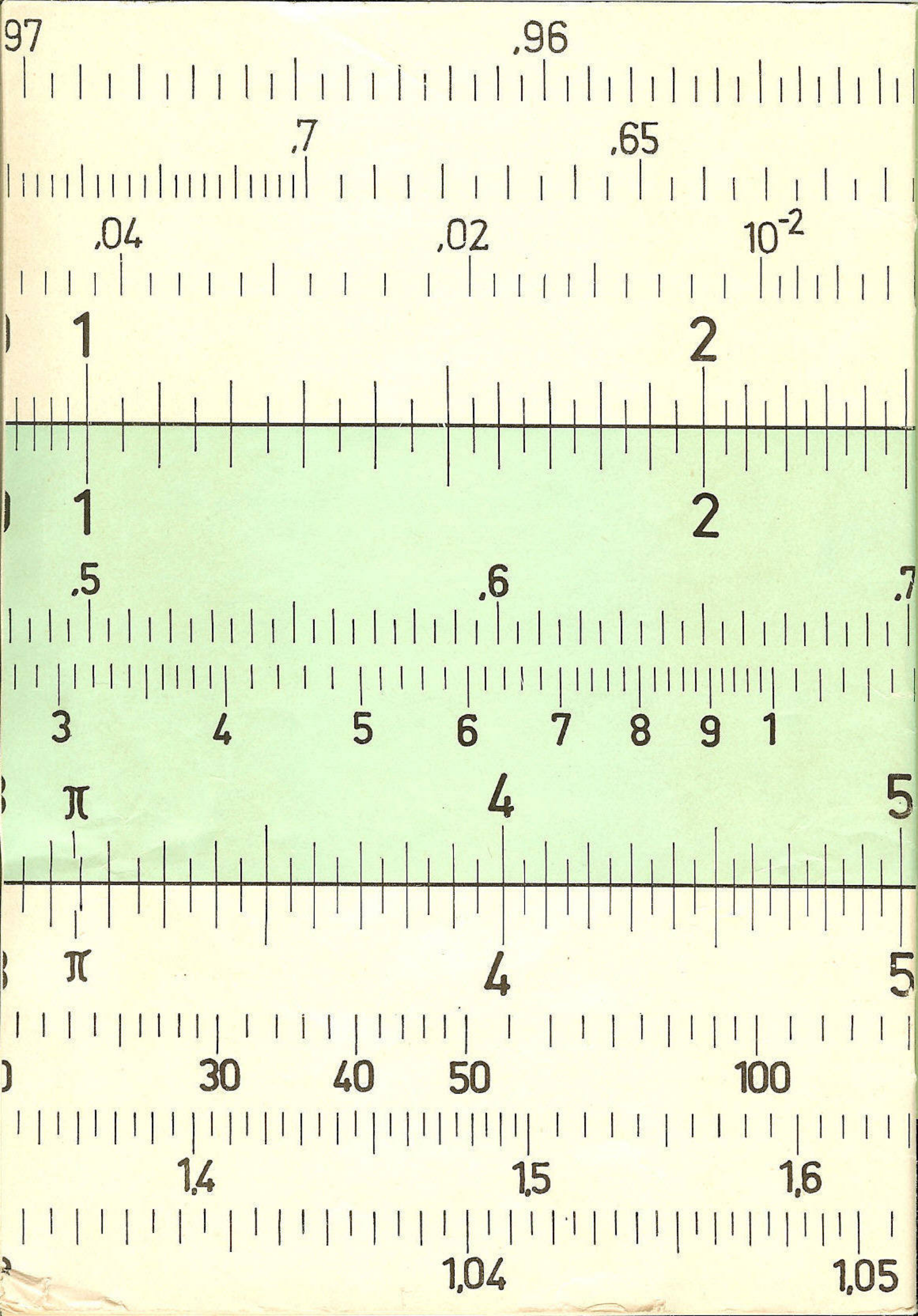
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