

## HOW TO ADJUST YOUR SLIDE RULE

Each rule is accurately adjusted before it leaves the factory. However, handling during shipment, dropping the rule, or a series of jars may loosen the adjusting screws and throw the scales out of alignment. Follow these simple directions for slide rule adjustment.

### CURSOR WINDOW HAIRLINE ADJUSTMENT

Line up the hairline on one side of the rule at a time.

1. Lay rule on flat surface and loosen adjusting screws in end plates.
2. Line up C index with D index. Then align DF (or A) index with CF (or B) index.
3. Tighten screws in end plates.
4. Loosen cursor window screws. Slip a narrow strip of thin cardboard (or 3 or 4 narrow strips of paper) under center of window.
5. Align hairline with D and DF (or D and A) indices, and tighten cursor window screws. Check to see that window surfaces do not touch or rub against rule surfaces.

*Note: The narrow strip of cardboard under the window will prevent possible distortion or "bowing in" of the window when screws are tightened. "Bowing in" may cause rubbing of window against rule surface with resultant wear or scratches.*

Line up hairline on reverse side of rule.

1. Loosen all 4 cursor window screws.
2. Place narrow strip of thin cardboard under window to prevent "Bowing in" when screws are tightened.

## HOW TO KEEP YOUR SLIDE RULE IN CONDITION

**OPERATION** • Always hold your rule between thumb and forefinger at the ENDS of the rule. This will insure free, smooth movement of the slider. Holding your rule at the center tends to bind the slider and hinder its free movement.

**CLEANING** • Wash surface of the rule with a non-abrasive soap and water when cleaning the scales. If the Cursor Window becomes dulled clean and brighten the surfaces with a small rag and tooth powder.

**LUBRICATION** • The metal edges of your slide rule will require lubrication from time to time. To lubricate, put a little white petroleum jelly (White Vaseline)

3. Align hairline and indices on first side of rule, then turn rule over carefully to avoid moving cursor.
4. Align hairline with indices and tighten cursor screws.
5. Check to see that window surfaces do not touch surfaces of rule during operation.

**SLIDER TENSION ADJUSTMENT** • Loosen adjustment screws on end brackets; regulate tension of slider, tighten the screws using care not to misalign the scales. The adjustment needed may be a fraction of a thousandth of an inch, and several tries may be necessary to get perfect slider action.

**SCALE LINE-UP ADJUSTMENTS** • (1) Move slider until indices of C and D scales coincide. (2) Move cursor to one end. (3) Place rule on flat surface with face uppermost. (4) Loosen end plate adjusting screw slightly. (5) Adjust upper portion of rule until graduations on DF scale coincide with corresponding graduations on CF scale. (6) Tighten screws in end plates.

**REPLACEABLE ADJUSTING SCREWS** • All Pickett All-Metal rules are equipped with Telescopic Adjusting Screws. In adjusting your rule, if you should strip the threads on one of the Adjusting Screws, simply "push out" the female portion of the screw and replace with a new screw obtainable from your dealer, or from the factory. We do not recommend replacing only the male or female portion of the screw.

on the edges and move the slider back and forth several times. Wipe off any excess lubricant. Do not use ordinary oil as it may eventually discolor rule surfaces.

**LEATHER CASE CARE** • Your Leather Slide Rule Case is made of the finest top-grain, genuine California Saddle Leather. This leather is slow-tanned using the natural tanbark from the rare Lithocarpus Oak which grows only in California. It polishes more and more with use and age.

To clean your case and to keep the leather pliable and in perfect condition, rub in a good harness soap such as Probert's Harness Soap.

# M 520-700

## U S A F TECHNICAL MANUAL

# HOW TO USE THE U.S. AIR FORCE AERIAL PHOTO SLIDE RULE

MODEL 520-TYPE-A1 (10-INCH)  
MODEL 700-TYPE-A2 (6-INCH)



PICKETT, INC.  
Chicago, Ill. / Santa Barbara, Calif.

AIR MATERIEL COMMAND • JULY 1951

## PREFACE

This manual is neither a textbook in mathematics nor a treatise on the slide rule. Its purpose is to provide sufficient basic instructions so that the user can easily discern the purposes for which the rule was designed, and with a minimum of effort, learn to use the rule quickly, accurately, and with confidence. An endeavor has been made to keep the manual useful and usable by making it compact, easily carried and easily studied. Those users who have more complicated problems to solve and who desire further information should refer to standard texts in both mathematics and slide rule theory.

## ACKNOWLEDGEMENT

The slide rule was designed by Amrom H. Katz, Aerial Photographic Laboratory, Air Materiel Command, Wright-Patterson Air Force Base, Dayton, O. The manual was prepared by H. E. Wilson, Pickett & Eckel, Inc., Chicago 3, Ill., and edited by Dr. E. J. Hills, Los Angeles City College, Los Angeles, Calif.

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# THE AERIAL PHOTO SLIDE RULE

## INTRODUCTION

The Aerial Photo Slide Rule was designed by Amrom H. Katz as a duplex double purpose rule with a series of scales on the front face for solving problems in vertical aerial photography and a series of scales on the back face for solving general problems common on a standard slide rule.

This rule was originally designed for the Photo-Interpreter, and others who must interpret measurements made on aerial photographs. In its present form, it is of great value to Photo-pilots, navigators, photo-officers and others concerned with planning and executing missions or using aerial photographs.\*\*

The scales for aerial photography are:

1. An altitude scale on the left bar reading in feet from 100 to 250,000.
2. A focal length scale on the left side of the slide reading in inches, from 1 to 240.

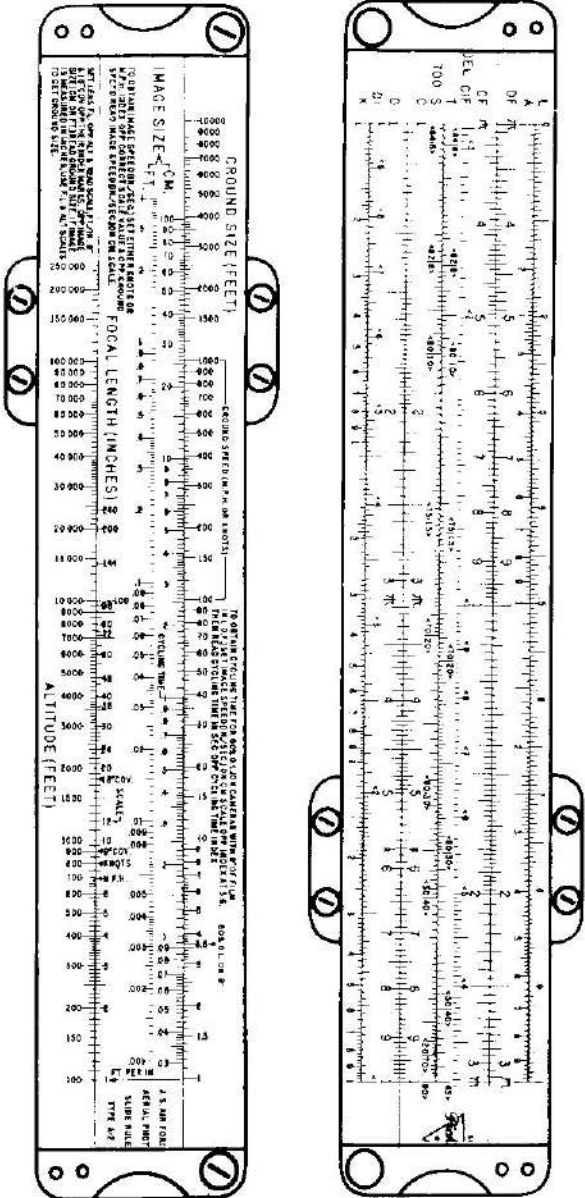
This scale is important because it carries a series of special scale marks which when related to the altitude scale give three or more measurements with one setting of the focal length and altitude scales. Special marks 9" cov. and 18" cov. give the width and length of ground area on the altitude scale. Scale mark 12 gives the representative fraction on the altitude scale for all settings on the focal length scale. Other index marks are M.P.H. at 6.93, Knots at 7.98 and Feet Per Inch at 1 (Feet Per Inch) at the bottom of the scale.

3. In the center of the slide, is an image size scale on the slide calibrated in feet from .001 to 4.
4. A second image scale on the slide reading in centimeters from .03 to 100. This scale has a special index mark at 1 for determining cycling time when referred to the Ground size scale.
5. The Ground Size (feet) Scale. This scale reads in feet from 1 to 10,000 and M.P.H. or Knots from 100 to 1000. It has a special index mark at 3.6, which is used for calculating cycling time with a 60% overlap on 9" film.

The standard scales found on the reverse side of the rule consist of an L, an A, and a DF scale on the upper bar and a CF, a CIF, a T, S and C on the slide and a D, a DI and a K on the lower bar. These are used for multiplication and division of numbers, for finding the logarithm of a number, for finding the squares and square roots of numbers, for proportion problems, for finding the cubes and cube roots of numbers, and for solving triangles and other problems that require the use of sines, cosines, tangents and cotangents of angles. This particular arrangement of scales was specially devised, and is the most powerful and useful one.

\*\*It should be noted at this point that the problems associated with oblique photography are much more difficult than the corresponding problems in vertical photography. This slide rule can be used in conjunction with other aids for oblique photography; these are now under development by the Photographic Laboratory (Wright Air Development Center, Wright-Patterson Air Force Base) and will be available soon. The interested reader is referred to the following article, which describes the development of the Aerial Photo Slide Rule, oblique computers, and other aids.

Katz, Amrom H., "Contributions to the Theory and Mechanics of Photo-Interpretation From Vertical and Oblique Photographs." PHOTOGRAMMETRIC ENGINEERING, June, 1950, pp. 339-386.



Since the scales on either side of the rule have no direct relation to those on the opposite side, instructions for the use of the scales on the opposite sides have been prepared in separate parts.

Part I contains the instructions for the scales used to solve problems in vertical aerial photography. It will be noted, however, that complete (but concise) instructions for use of the Aerial Photo side of the rule appear on the face of the rule. These instructions alone have sufficed for everyone who got any early model of this slide rule (before this Manual was prepared). Average time to learn all the uses of the rule was about ten minutes. Part II contains the instructions for the scales used to solve general problems common to the slide rule.

## THE AERIAL PHOTO SLIDE RULE—PART I

### THE SCALES

The scales on the Aerial Photo side of the rule can be used to solve various problems common to vertical aerial photography, since many problems can be worked forward and in reverse. All types of problems are easy to solve and any operator familiar with the terms used should be able to learn the full use of the rule in a few hours of concentrated study. The important problems to be solved may be divided into four or more groups as follows:

#### I—PRIMARY VALUES

- (a) Scale of photography
- (b) Area covered
- (c) Ground feet per inch of photograph
- (d) Image speed
- (e) Cycling time

#### II—PHOTO INTERPRETATION

- (a) Image size ground size calculations

#### III—FUNCTIONAL VALUE OF THE RULE IN PLANNING AERIAL PHOTOGRAPHY PROBLEMS

- (a) Minimum altitude to obtain 60% overlap on 9" film.
- (b) Number of flights
- (c) Flight line spacing
- (d) Exposures required

#### IV—SOLVING PILOTAGE PROBLEMS

Some pilots who have tested the rule report that it facilitates pilotage problems. Using the left hand scales—distance, time and ground speed values are quickly and easily read.

#### V—CONVERSION OF KNOTS TO MILES PER HOUR

The two indices—knots and M.P.H. on the focal length scale set up ratios for every position on the slide. For example, when the index for M.P.H. is set at 400, the index for knots is at 460. Thus 400 knots=460 miles per hour (approximately).

## READING THE SCALES

**NOTE:** When the Aerial Photo side of the slide rule is used, the rule must be held vertically. You will note that the numbers are printed horizontally. This permits the use and printing of the large numbers encountered in aerial photography.

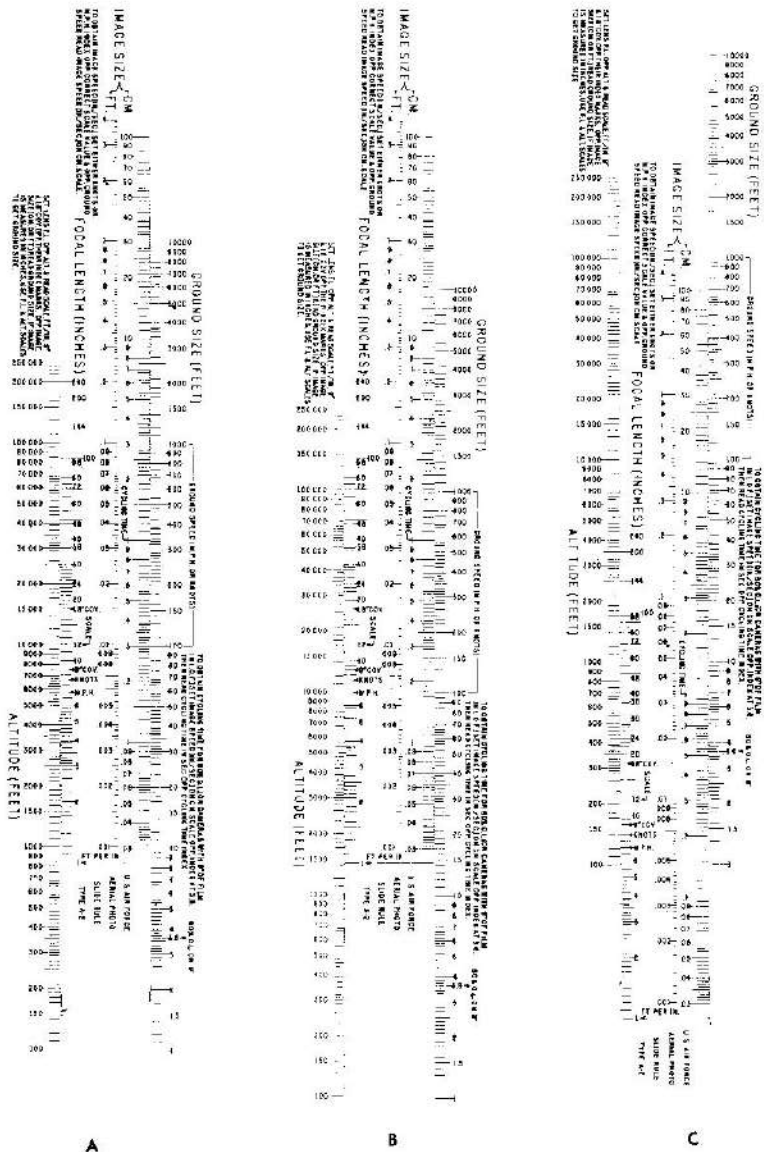


Fig. 2

Photographs of the photo-interpreter's slide rule in several settings are shown in Fig. 2. Fig. 2a represents a 36" camera, with 9" x 18" format flown at 30,000 feet. The slide setting is 36" on the focal length scale opposite 30,000 feet on the altitude scale. Opposite the index mark 12 on the focal length scale, the scale or representative fraction (R.F.) of the photograph is read — 1:10,000. This particular setting of the slide rule enables the immediate reading of several other quantities of interest. Opposite 1 (Feet Per inch) on the focal length scale, the ground feet per inch of photograph is read. The reading is 833 feet per inch. Without moving the slide the complete set of pairs of values — ground size and image size — can be read for any possible image size measurement on the photograph. For example, an 0.03 CM image length corresponds to a 98 ft. ground object.

This setting also gives the coverage of the 9" x 18" negative on the altitude scale opposite 9" cov. and 18" cov. on the focal length scale; read width 7500 feet x length 15,000 feet ground coverage.

Fig. 2b, shows a setting that permits the reading of image speed directly. Assume a ground speed of 300 M.P.H. With the previous setting, move the slide so that the index mark M.P.H. is opposite 10,000 on the scale value (R.F.). Move the indicator to 300 on the ground speed scale and read image speed as .525 inches per second on the centimeter image size scale. Were the ground speed given in knots, the "knots" index would have been set opposite the scale value of 10,000.

Fig. 2c, shows a third setting of the same problem for determining cycling time for 60% overlap. With the setting shown in Fig. 2b, move the slide downward so that .525 on the image size scale is opposite the 3.6 index mark on the ground size scale. Then opposite the index mark cycling time on the image size scale, read cycling time between photographs as 6.9 seconds (on the ground size scale).

Note that the index at 3.6 is for cameras with 9 inches of film in line of flight. This is by far the most common type camera. For other cameras — a few of which are in use, and still others which may be developed — the index which should be used (instead of the mark at 3.6) is 40% of the film width in line of flight. For example, on a camera which uses 5" of film in line of flight — such as the K-24 — the index would be at 2.0 instead of 3.6.

### A. HOW TO FIND THE SCALE OF PHOTOGRAPHY

Set focal length in inches on the slide opposite altitude and on altitude scale, read scale (R.F.) opposite index mark scale at 12 on focal length scale. Examples:

Find scale (R.F.) with the following settings:

- (a) Focal length 36" and altitude 40,000 feet.  
Set 36" on the focal length scale opposite 40,000 on the altitude scale.  
Opposite index mark scale at 12, read scale as 13,330 (R.F.).
- (b) Focal length 24" and altitude 30,000 feet.  
Set 24" on the focal length scale opposite 30,000 on the altitude scale.  
Opposite the index mark scale at 12, read 15,000 (R.F.).



## B. HOW TO FIND AREA (COVERAGE):

Set focal length in inches on the Focal Length Scale opposite altitude and on the altitude scale read width opposite 9" cov. and length opposite 18" cov. on the altitude scale. If film sizes other than 9" and 18" are used — e.g. 2½", 4½", 5", 7", — read coverage opposite those values on the focal length scale.

- (a) Using the same settings as used for finding scale, a focal length of 36" and altitude of 40,000 feet gives coverage at 9" cov. as 10,000 feet width and 20,000 feet length at 18" cov. or 10,000' × 20,000' coverage.
- (b) Focal length 24" and altitude 30,000 feet gives coverage at 9" cov. as 11,500 feet width and at 18" cov. 22,500 feet length, or 11,500' × 22,500' coverage.

## C. HOW TO FIND IMAGE SPEED: (arising from forward aircraft speed):

in inches/second when ground speed is given in M.P.H. To find image speed, first find the scale of photography and move the slide upward until the special index mark M.P.H. on the Focal Length Scale is set opposite the scale of photography measurement. Then without moving the slide, set the hairline over flight speed in M.P.H. on the ground speed scale. Opposite ground speed — read image speed in inches/second on the image size scale. If ground speed is in knots, use the "knots" index instead of the "M.P.H."

### 1. Illustrations:

- (a) Find the image speed if the plane is flying 200 M.P.H. ground speed at 20,000 feet and the focal length is 36".  
Set 36" on the focal length scale opposite 20,000 feet on the altitude scale. Set hairline over scale mark 12 on the focal length scale and read the scale (R.F.) as 6,666 on the altitude scale. Move slide upward until index mark M.P.H. is under hairline opposite scale (R.F.) 6666. (Actually, one doesn't have to read the scale value, for it's held by the hairline. Without moving the slide, set hairline over 200 on the ground speed scale and read image speed .530 inches per second.
- (b) Find the image speed if the plane is flying 600 M.P.H. ground speed at 30,000 feet and the focal length is 36".  
Set 36" on the focal length scale opposite 30,000 feet on the altitude scale. Set hairline over scale mark 12 on the focal length scale and read the scale (R.F.) 10,000 on the altitude scale. Move the index mark M.P.H. under hairline. Without moving the slide, set hairline over 600 on the ground speed scale and read image speed 1.06 inches/second.
- (c) Find image speed if plane is flying 500 M.P.H. at 800 feet and focal length is 6".  
Set 6" on the focal length scale opposite 800 feet on the Altitude Scale. Move M.P.H. index, etc. — read 5.5 inches per second.
- (d) Do problems a, b, c if the ground speeds given (200, 600, 500) are in knots instead of M.P.H.  
Answers: (a) .610 inches/second. (b) 1.22 inches/second  
(c) 6.32 inches/second.

## D. HOW TO FIND CYCLING TIME FOR 60% OVERLAP ON 9" FILM:

Given the image speed on the image size scale, move the slide until the image size number is opposite the index mark 3.6 on the ground size scale. Without moving the slide, move hairline over the index mark cycling time on the image size scale and read cycling time opposite on the ground size scale.

### Illustrations:

- (a) Find the cycling time for 60% overlap when a plane is flying 275 M.P.H. at 25,000 feet and the focal length is 36" and the 9" × 18" format is oriented with 9" of film in line of flight.  
First show that the image speed is .582 CM on image size scale using method just shown. Now move slide downward until .582 is opposite the index mark 3.6 on the ground speed scale. Then set the hairline over the index mark cycling time on the image size scale and (on ground size scale) read cycling time approximately 6.2 seconds between photograph on ground size scale.
- (b) Find the cycling time for 60% overlap when a plane is flying at 460 M.P.H. ground speed at 38,000 feet and the focal length is 36".  
Proceed as above and find image speed .635 in./sec. on image size scale. Then set .635 image speed opposite index mark 3.6 on ground size scale. Without moving the slide, set hairline on index mark cycling time and read 5.7 seconds as the cycling time on ground size scale.
- (c) Find cycling time for 60% overlap for a 6" focal length 9" × 9" camera when in a plane flying 400 M.P.H. ground speed at 46,000 feet altitude.  
Proceed as above, and find cycling time to be 46.8 seconds.
- (d) Do problems above when ground speeds given are in knots instead of M.P.H.  
Answers: (a) 5.35 seconds. (b) 4.92 seconds. (c) 40.5 seconds.

NOTE: You will observe that in all cases, you can calculate the cycling time (or intervalometer setting) more accurately than need be. For example, there is neither need nor available mechanical settings for times such as 46.8 seconds, 5.35 seconds, etc. Either 46 or 47 seconds would be a suitable setting in the first case, and either 5 or 5.5 seconds would be all right in the second case.

### Cycling Times Less Than One Second:

As we approach very high ground speeds at low altitudes, very fast (less than one second) cycling times are required. Cameras with fast cycling times are being developed. Although these can't be read directly, by a *very simple, easily learned* trick, these cycling times may be readily calculated.

Observe that the cycling time index is at 1 on the centimeter scale. Were it moved to 10 on the same scale, the cycling time readings would be too high by a factor of 10. Hence if a problem arises wherein the cycling time index winds up *below* one second, read opposite 10 and divide the reading by 10.

*Example:*

A 6" 9x9 camera is flown at 500 M.P.H. at 750 feet altitude. What cycling time is needed for 60% overlap?

Going through the normal procedure an image speed of 5.85 inches per second is found. When this image speed is set opposite the 3.6 index, it will be noted that the cycling time index is below 1 second. Hence, at 10 (on the centimeter scale) read 6.15 seconds, which upon dividing by 10, yields a cycling time of .615 seconds.

Several minutes of practice on this technique will make you thoroughly familiar with the procedure.

**E. HOW TO FIND THE GROUND SIZE OF AN OBJECT FROM A PHOTOGRAPH:**

Set index mark scale 12 on Focal Length opposite scale of photography. Then set measurements of width and length on image size scale and read ground sizes on ground size scale.

Note that two complete image size scales are given, for measurements either in feet or centimeters. If altitude and focal length are given, the setting of focal length against altitude yields scale. Of course, if these two parameters are given, there is no need to ever read scale.

*Illustrations:*

(a) With scale 1/5000, given size of image as .066x.055 feet on photograph to find ground size.

Set hairline over scale index 12 opposite scale 5000 on the altitude scale and move hairline over .006 on image size feet scale. Read length of object 30' on ground size scale. Move hairline to .005 on image size scale and read width 25' on ground size scale.

(b) With scale 1/6200, given size of image as .002'x.0018'. Proceed as in previous problem and find answer 12.4'x11.2'.

(c) With scale 1/6400, given size of image as .05x.08 CM on photograph to find ground size.

Set scale at 6400 and move hairline over .05 on image size CM scale and opposite on ground size scale read width of object 105'. Move hairline to .08 on image size CM scale and read length as 168'.

(d) On a photograph made with a 48" lens from 43,000', a runway is measured as being .69'x.0177'.

Show that the dimensions of the runway are 7200'x190'.

*Use of Inches for Image Size Measurement:*

Most photointerpreters use either centimeters or feet as units in measuring image size. For the engineer or other layman who prefers to use inches, the lower part of the focal length scale has been subdivided into tenths of an inch. Just as in the previous cases of image size—object size determinations, set focal length against altitude, and, in this case opposite image size in inches (on the focal length scale) read object size in feet on the altitude scale.

*For Example:*

A 2.3 inch image on a photograph made with a 24 inch lens at 40,000 feet corresponds to a 3850' ground object.

Clearly, the smallest *direct* measurement which can be so used, is one inch. For measurements less than one inch in length, multiply the measurement by 10 (or 100), set on the rule, and divide the answer by 10 (or 100).

*For Example:*

Consider an image .125" long on a photo made with a 12" lens at 27,500'. This cannot be set directly, so multiply the image size by 10. The image size of 1.25 inches is then found on the slide (after 12 inches is set opposite 27,500). Opposite the 1.25 inch setting, we read 2850 feet. Hence the object size for a .125 inch image is 285 feet.

**F. FUNCTIONAL VALUE OF THE RULE IN PLANNING AERIAL PHOTOGRAPHY PROBLEMS:**

Since it is possible to work problems in reverse when suitable data is available, plans for photographing desired areas to obtain specific data can be developed in advance.

(a) To find minimum altitude at which adequate 60% overlap on 9" film can be obtained, given the cycling time as 2 seconds and the flying speed as 200 M.P.H.

Set index mark cycling time on image size scale opposite 2 on the ground size scale. Under hairline at index mark 3.6, read 1.8 on image size scale. Move slide upward until 1.8 is opposite 200 on ground speed scale. Without moving the slide, set hairline over index mark M.P.H. on Focal Length Scale. Move slide down until Index Mark Scale 12 is under the hairline. The altitude for several settings may then be read as follows:

F.L. 24" 3,950'      F.L. 36" 5,990'      F.L. 40" 6,520'

(b) To find number of exposures on a proposed survey of a given area.

Find the number of exposures in covering an area 125x80 miles. Flight to be lengthwise of the area with 60% overlap and 50% sidelap, with 9" film at scale 1/10,000.

First determine individual photo cover by placing Index Mark scale 12 at 10,000 on altitude scale. Since 3.6" forward travel will give 60% overlap, set hairline over 3.6 on focal length scale and opposite on altitude scale read 3000' forward travel for each exposure. To obtain 50% sidelap, divide 9" by .50, giving a figure of 4.5". Opposite this number on the focal length scale, read 3850 on the altitude scale. This gives the flight line separation. To find the number of exposures per line, find the number of feet in 125 miles and divide by 3000' forward travel. 660000' gives 220 exposures per line. To find number of lines, find the number of feet in 80 miles which is 422400 and dividing by 3850, we get approximately 110 lines required for sidelap. Multiplying 220x110 gives 24,200 exposures required to cover an area 125 miles long and 80 miles wide.

- (c) At what minimum scale must a 38' long object be photographed to yield an image at least .1 cm. in size? What minimum altitude is required with a 24" lens?

With one setting of slide — .1 cm. against 24' — read scale of 11,500 and altitude (with 24" lens) of 25,000'.

## AERIAL PHOTO SLIDE RULE

### PART II

Instructions for solving slide rule problems using the scales on the back face of the rule.

#### THE SCALES

The scales on the rule consist of accurately graduated markings representing the logarithms of numbers. The left margin of each scale is called the left index, and the right margin is called the right index. Each scale is designated by a reference letter at both ends of the scale to simplify its location in relation to the other scales.

The basic scales on every standard slide rule are the C scale on the lower edge of the slide and the D scale on the upper edge of the lower bar. They are exactly alike and can be used together for multiplication of numbers, division of numbers, solving proportion problems, and for other basic computations.

The A scale has two sections, each being one-half the length of, and looking like, the D scale. It is used with the D scale to find squares and square roots.

The K scale has three sections, each being one-third the length of, and looking like, the D scale. It is used with the D scale to find cubes and cube roots.

The CF and DF scales are folded C and D scales with the index near the center. Each reading on the CF or DF scale is  $\pi$  times the reading on the C or D scale. They are very helpful in solving multiplication, division, and proportion problems.

The DI scale is an inverted D scale reading from right to left. Each reading on the DI scale is the reciprocal of the reading on the D scale and vice versa.

The CIF scale is an inverted CF scale reading from right to left. It has the same relationship to the CF scale that the DI scale has to the D scale.

The S (sine) and T (tangent) scales are the trigonometric scales on the rule. They are used with the C, D, DI, and A scales to solve triangles and other problems dealing with the trigonometric functions of angles.

The L scale is the only equal parts scale on the rule. It gives the decimal parts or mantissae of the logarithms of numbers read on the D scale and vice versa.

The lines marked with numbers on each scale represent the position of primary or secondary scale divisions or circular measures (in degrees.) They are laid down on the basis of the logarithm of the number or the function value of the circular measure.

*When two numbers are set on the rule for multiplication or division, the logarithms of the numbers are respectively added or subtracted.*

When the logarithms of two numbers are added, the result is the answer obtained when one of the numbers is multiplied by the other. If the logarithm of a number is subtracted from the logarithm of another number, the answer is the division of the second number by the first number.

#### Illustrations:

(1)  $2 \times 3 = 6$

Set the left index of C over 2 on D (Fig. 1). Below 3 on C read 6 on D.

That is:  $\log 2 = 0.3010$

$\log 3 = 0.4771$

$\log 6 = 0.7781$

(2)  $3 \div 4 = 12$

Set the left index of C over 3 on D. Below 4 on C read 12 on D.

That is:  $\log 3 = 0.4771$

$\log 4 = 0.6021$

$\log 12 = 1.0792$

(3)  $9 \div 3 = 3$

Set 3 on C over 9 on D. Read 3 on D under the left index of C.

That is:  $\log 9 = 0.9542$

$\log 3 = 0.4771$

$\log 3 = 0.4771$

(4)  $8 \div 2 = 4$

Set 2 on C over 8 on D. Read 4 on D under the left index of C.

That is:  $\log 8 = .9031$

$\log 2 = .3010$

$\log 4 = .6021$

Tables of logarithms are given at the end of the Manual and can be used to verify the above readings.

#### READING THE SCALES

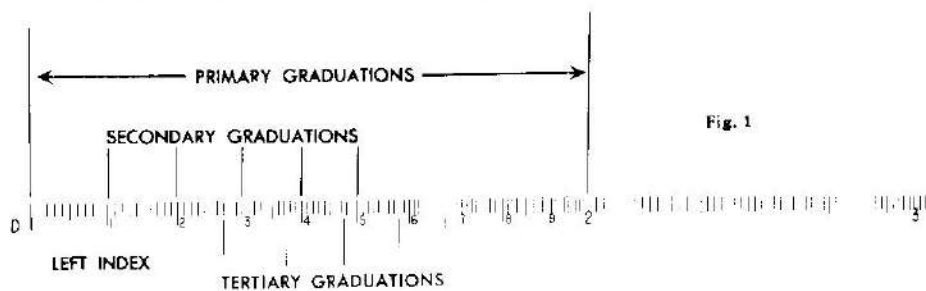
In starting your study of the rule, make a careful analysis of the scales and note that each scale has a series of major parts which become shorter as you read the part numbers from left to right. Observe that the A scale has two sections of equal length with 9 major parts in each section, and that the K scale has three such sections. The L (logarithm) scale is the only scale that has parts of equal length.

*A thorough study of the C and D scales will give you a better understanding of the layout of all the scales on the rule.* Starting at 1, at the left end of either scale (the left index), observe that there are 9 major or primary parts of unequal length. The long lines separating these parts are known as primary lines. Each major part is divided into 10 secondary parts indicated by secondary lines slightly shorter than the primary lines.

The secondary parts are also divided into smaller parts by still shorter lines called tertiary lines. Each tertiary part is considered to contain ten parts; but there is not enough space for all the lines needed except in the first major part. As a result, the operator of the rule must learn to estimate the location of some lines that are missing. As a result only the even number tertiary lines are shown and odd numbers must be estimated at about one-half the distance between the even tertiary lines. In the primary parts between 3 and 4 (the right index), the ten secondary lines (the digit 5). Therefore, in each of these spaces, the operator must estimate the location of four imaginary tertiary lines which if present, would be about one-fifth of the secondary space apart. The tertiary digit 1 would be about one-fifth the distance across the space.



Since the first primary part is so long, each secondary part is denoted by a small number on the 10 inch rule. (Fig. 1.) The first space between the primary line 1 and the secondary line 1, includes all numbers having 10 as the first two significant digits. In like manner, if the number has the first two significant digits reading 12, the number will be found between the secondary lines 2 and 3 on the 10 inch rule and implied on the 5 inch rule.

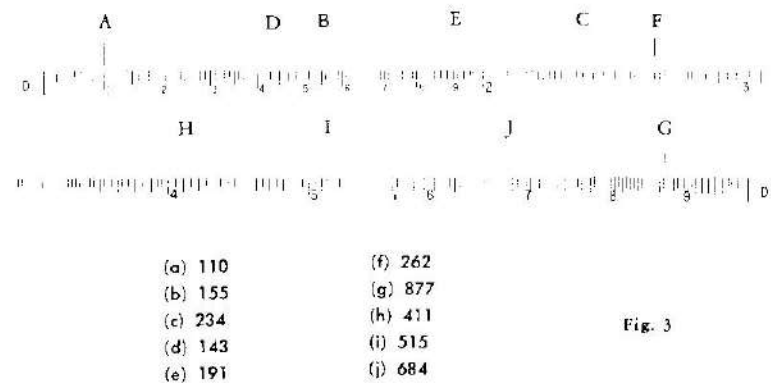


On a slide rule only the first three significant digits of a number are considered. The numbers 0.0101, 0.101, 1.01, 10.1, 101, 1010, and so on, are all located at the same place on a scale. That is, disregard the location of the decimal point in a number when making a setting. Thus to find 101 (read one-o-one) set the hairline at the first tertiary line beyond the secondary line marked 1, and so on with 121, 131, ... 191. In like manner, any number with the third digit of 6 would fall at the first mark past 5. To locate any number between 191 and 199, set the hairline on the proper tertiary line between the secondary line marked 9 and the primary line marked 2.

All numbers beginning with a first significant digit of 2 will be found between the primary lines 2 and 3. If the second significant digit is 1, 2, 3, or more, the hairline will fall on the first, second, third or more secondary line beyond 2. If the third significant digit of the number is an even digit, such as in 248, the hairline will fall in a tertiary line. But if it is odd, such as in 249, the hairline will fall about midway between the 248 and the 250 lines. (Fig. 2.)



In the primary parts of a 10 inch rule where the first significant digit is a 4 or more, or in the primary parts of a 5 inch rule where the first significant digit is 2 or 3, care must be exercised in setting the hairline since there is only 1 tertiary line in each secondary part. That is, if the third significant digit of a number is a 1 to 4 or a 6 to 9, its location must be estimated. Sample readings are shown in (Fig. 3).



### MULTIPLICATION AND DIVISION

Multiplication and division of numbers on a slide rule become simple operations once the operator has learned to read the scales. If the values assigned to the lines can be accurately read, the operator needs only to develop the ability of placing the hairline in its correct position even in spaces where the lines are missing.

*To multiply two numbers together, set one index of the C scale over one of the numbers on the D scale. Then find the other number on the C scale and below it on the D scale find the desired product.*



#### Illustration:

- (1)  $2 \times 3 = 6$   
Set the left index of C to 2 on D. Move hairline to 3 on C. Read 6 on D under hairline. (See Fig. 4).
- (2)  $4 \times 8 = 32$   
Set the right index of C to 8 on D. Move hairline to 4 on C. Read 32 on D under hairline.
- (3)  $27 \times 41 = 1107$   
Set the right index of C to 41 on D. Move hairline to 27 on C. Read 1107 on D under hairline.
- (4)  $31.5 \times 31.5 = 992$  (true answer is 992.25)  
Set the left index of C to 315 on D. Move hairline to 315 on C. Read 992 on D under hairline.

PROBLEMS	ANSWERS
(1) $13 \times 4.1$	53.3
(2) $11.7 \times 65$	737.1
(3) $631 \times 77$	48,590
(4) $72.1 \times 3.44$	248.02
(5) $0.031 \times 41.1$	1.2741

Division is the inverse process of multiplication. To divide one number by another, using the hairline, set the dividing number on the C scale over the number to be divided on the D scale and read the answer on D under the index of the C scale.

*Illustrations:*

(1)  $198 \div 11 = 18$

Using the hairline, place 11 on C over 198 on D. Read 18 on D under the left index of C.

(2)  $1107 \div 41 = 27$

Using the hairline, place 41 on C over 1107 on D. Read 27 on D under the right index of C.

(3)  $992 \div 31.5 = 31.5$

Using the hairline, place 315 on C over 992 on D. Read 31.5 on D under the left index of C.

PROBLEMS	ANSWERS
(1) $47 \div 8$	5.875
(2) $182 \div 21$	8.667
(3) $141 \div 521$	.2706
(4) $98 \div 133$	.737
(5) $4621 \div 037$	1248.92

**DECIMAL POINT LOCATION**

The position of the decimal point in an answer is usually determined by estimating the size of the answer since it cannot be done directly from the rule. Four illustrations should suffice to show how it can be done.

*Illustrations:*

(1)  $27.8 \times 425$ . The estimated product would be  $30 \times 400$  since 27.8 is closer to 30 than it is to 20, and 425 is closer to 400 than it is to 500. Since  $30 \times 400 = 12,000$ , the answer is 11,800, not 1180 or 118,000.

(2)  $0.0582 \times 0.382$ . The estimated product would be  $0.06 \times 0.4$ , since 0.0582 is closer to 0.06 than it is to 0.05, and 0.382 is closer to 0.4 than it is to 0.3. Since  $0.06 \times 0.4 = 0.024$ , the answer is 0.0222, not 0.222 or 0.00222.

(3)  $528 \times 0.0716$

The estimated product would be  $500 \times 0.07$ , since 528 is closer to 500 than it is to 600, and 0.0716 is closer to 0.07 than it is to 0.06. Since decimal points in division can be moved in the same direction without changing the answer, we have  $500 \times 0.07 = 5 \times 7 = 35$ . The answer is therefore 37.8, not 3.78 or 378.

(4)  $38.2 \div 0.0693$ . The estimated quotient would be  $40 \div 0.07$  since 38.2 is closer to 40 than it is to 30 and 0.0693 is closer to 0.07 than it is to 0.06. Since decimal points in division can be moved in the same direction without changing the answer, make the denominator a number less than 10. Thus  $40 \div 0.07 = 4000 \div 7 = 600$ . The answer is therefore 551.

Note that your estimated numbers will have only one significant digit in them but they will still be close in value.

A second method which may be used requires expressing numbers in standard form. To change numbers to their standard form, the exponent of 10 is plus, if one or more digits are to the left of the decimal point, and minus, if all digits are to the right of the decimal point. The following example will clarify the system.

**EXAMPLES:**

NUMBERS	IN	STANDARD FORM
4.78		$4.78 \times 10^0$
47.8		$4.78 \times 10^1$
478		$4.78 \times 10^2$
4780		$4.78 \times 10^3$
478000		$4.78 \times 10^5$
0.78		$7.8 \times 10^{-1}$
0.078		$7.8 \times 10^{-2}$
0.0078		$7.8 \times 10^{-3}$
0.00078		$7.8 \times 10^{-4}$
0.000078		$7.8 \times 10^{-5}$

(A) In multiplying  $47.8 \times 0.0078$ , if the numbers are changed to standard form, they become  $4.78 \times 10^1 \times 7.8 \times 10^{-3}$ . Combining the exponents by addition this becomes  $4.78 \times 7.8 \times 10^{-2}$ . 4.78 is near 5 and 7.8 is near 8 and the product is near  $40 \times 10^{-2}$  or 0.4. The product of 478 and 78 on the rule is 373 and the decimal point is located to the left of the first digit, making the answer 0.373.

(B) In multiplying  $0.078 \times 0.0078$ , if the numbers are changed to standard form they become  $7.8 \times 10^{-2} \times 7.8 \times 10^{-3}$ . Exponents are combined in the present problem by addition. For example,  $10^{-2} \times 10^{-3} = 10^{-5}$ .  $7.8 \times 7.8 \times 10^{-5}$ . 7.8 is near 8 so by multiplication we have  $64 \times 10^{-5}$  or  $6.4 \times 10^{-4}$ , or  $0.64 \times 10^{-3}$ , or  $0.064 \times 10^{-2}$ , or  $0.0064 \times 10^{-1}$ , or 0.00064. The product of  $78 \times 78$  on the rule is 608 and the decimal point is located three places to the left of the digit six, making the number 0.000608.

(C) If we take the problem  $25.3 \div 0.0064$  and change the numbers to standard form we have  $2.53 \times 10^1 \div 6.4 \times 10^{-3} = 0.395 \times 10^4 (+3) = 0.395 \times 10^4 = 3950$ .

**CONTINUED PRODUCTS**

When a series of numbers are to be multiplied together, such as  $7 \times 13 \times 23$ , set the index of the C scale over the first number on the D scale and then move the hairline and an index alternately until all numbers are included.

*Illustrations:*

(1)  $7 \times 13 \times 23 = 2093$

Set the left index of C over 13 on D. Move hairline to 7 on C. Move right index of C under hairline. Move hairline to 23 on C and read 2093 below on D.

(2)  $9.1 \times 16 \times 4.2 \times 41 = 25,070$

Set right index of C over 9.1 on D. Move hairline to 16 on C. Move left index of C under hairline. Move hairline to 42 on C. Move right index of C under hairline. Move hairline to 41 on C and read 25,070 below on D.

PROBLEMS	ANSWERS
1. $7 \div 4 \times 12$	336
2. $5.4 \div 4.2 \div 8.7$	124.24
3. $51 \div 4.7 \times 0.78$	124.64
4. $2.6 \div 18 \div 0.05$	.1404
5. $12.12 \div 1.6 \div 28 \div 0.41$	222.25

### COMBINED MULTIPLICATION AND DIVISION

When the procedures of multiplication and division of numbers are well understood, the combination of the two is not difficult. *Start with the division of the first number above the line by the first number below the line. Then multiply and divide in rotation until the last number is included.* Note that you move the slide for division and the hairline for multiplication.

Move and set hairline  $a \times b \div c \times d$

Move and set slide  $e \times f \times g$

A is on D. All other numbers are read on C. Answer is on D.

*Illustrations:*

$$(1) \begin{array}{r} 24.8 \times 14.6 \\ \hline 12.4 \end{array} = 29.2 \quad \text{Estimated answer: } \frac{20 \times 10}{10} = 20$$

Set 124 on C over 248 on D. Move hairline from 124 to 146 on C. Read the answer 29.2 on D under hairline.

$$(2) \begin{array}{r} 26 \div 31 \div 56 \\ \hline 39 \div 47 \end{array} = 24.6 \quad \text{Estimated Answer: } \frac{30 \times 30 \times 60}{40 \times 50} = \frac{3 \times 3 \times 60}{4 \times 5} = 20 \div 27$$

Set 39 on C over 26 on D. Move hairline to 31 on C. Move 47 on C under hairline. Move hairline to 56 on C and read the answer, 24.6, on D under hairline.

PROBLEMS	ANSWERS
1. $\frac{18 \times 43}{16}$	48.4
2. $\frac{5.7 \times 8.1 \times 41.2}{23 \times 13.6}$	6.08
3. $\frac{41 \times 18 \div 37}{42 \times 91}$	7.14
4. $\frac{0.61 \div 43 \div 69}{556 \times 84.2}$	0.00387
5. $\frac{18 \div 61.1 \times .078 \div 3.9}{24 \div 32.7 \div 901}$	0.00474

### THE CF AND DF SCALES

The CF and DF scales, known as folded scales, are alike and have the same relationship to each other as is true for the C and D scales. They also may be used for multiplication and division of numbers.

*These scales are called folded scales because they are C and D scales divided at  $\pi$  and the indices joined together.* As a result, the left and right indices are at the same point. All settings on the C and D scales can be similarly located on the CF and DF scales.

One important use of these scales is when the slide projects beyond the end of the rule so that the indicator cannot be set over a number on the C scale. The setting can then be made on the CF scale and the answer read on the DF scale without making it necessary to shift the slide so that the other index of the C scale is used.

*Illustrations:*

$$(1) 21 \times 72 = 1512$$

Set the left index of C over 21 on D. Then 72 on C is beyond the D scale index. So move the hairline to 72 on CF and read the answer, 1512, on DF under the hairline.

$$(2) 3.7 \times 7.8 = 28.9$$

Set the left index of C over 37 on D. Move the indicator to 78 on CF and read the answer, 28.9 on DF.

The DF scale is also important in finding the circumference of a circle when the diameter is given and vice versa.

*Illustrations:*

(1) If the diameter of a circle is 6.2 feet, find the circumference. Set 6.2 on D and read the circumference, 19.5 on DF.

(2) If the circumference is 11 feet, find the diameter. Set 11 on DF and read 3.5 on D.

These scales can also be used in changing radians to degrees and vice versa. Set 180 on C opposite  $\pi$  on DF. Then move hairline to radians on DF and read degrees on C, and vice versa.

*Illustrations:*

$$(1) 3 \text{ radians} = 172^\circ$$

Set 180 on C under right end of DF scale, then 3 on DF is over 172 on C.

$$(2) 54^\circ = 0.942 \text{ radians.}$$

Set 180 on C under  $\pi$  on left end of DF scale, then 54 on C is below 0.942 on DF. Putting an index of C over the little R on D, or vice versa, these answers can be read between the C and D scales or between the CF and DF scales.

PROBLEMS:	ANSWERS:
1. $1.73 \times 8.1$	14.013
2. $2.11 \times 57.6$	121.536
3. $81 \times 7.48$	605.88
4. $3.7 \times \pi$	11.6239
5. $\frac{1955 \times 23.7}{50.7 \times \pi}$	.0291
6. $\frac{2.15 \times 16.35 \times 516 \times \pi}{655 \times 9620}$	9.04

### PROPORTION

The principle of proportion is the key to the solution of problems on the slide rule because the solution to a problem is developed from a simple arithmetic rule: *When three numbers are given, a fourth can be found that has the same ratio to the third as the second has to the first.* On a slide rule, when a number on the C scale is set next to a number on the D scale, a ratio is established, and the same ratio will be found to exist for all pairs of adjacent

graduations. Note that if 1 on the C scale is set over 2 on the D scale, 2 on C is over 4 on D, and so on. That is:  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$ . This indicates that 2 on the

D scale is the multiplier, and that any number selected on the D scale will have double the value of the numbers directly above on the C scale. Thus a proportion is formed whenever a problem is set for multiplication or division on the slide rule. For example, if we have  $25 \div 5$ , the proportion becomes  $\frac{5}{1} = \frac{25}{x}$  and in like manner  $5 \times 5$  becomes  $\frac{5}{5} = \frac{x}{25}$ .

If one of the numbers in any proportion is not known, it can be written in the form  $A:B = C:x$  or  $A \frac{C}{B} = x$ . Then A and C on the C scale are over B and X respectively on the D scale. Or A and C on the CF scale are under B and X on the DF scale.

#### Illustrations:

(1)  $7:21 = 4:x$ .  
Set 7 on C over 21 on D. Under 4 on C, read  $x=12$  on D.

(2)  $7:12 = 16:x$ .  
Set 7 on CF under 12 on DF, read  $x = 27.4$  on DE.

#### PROBLEMS:

1.  $\frac{x}{25} = \frac{12.3}{7}$

2.  $\frac{48.7}{x} = \frac{17.1}{24}$

3.  $\frac{42.1}{17} = \frac{x}{15.1}$

4.  $\frac{0.42}{0.31} = \frac{x}{293}$

5.  $\frac{0.021}{0.61} = \frac{7}{x}$

#### ANSWERS:

43.93

68.35

37.4

397

203.3

### POWERS AND ROOTS

To find squares and square roots, use the A and D scales. To find cubes and cube roots, use the K and D scales. The *square* of any number on the D scale is found on the A scale, and the *cube* of any number on the D scale is found on the K scale. The *square root* of any number on the A scale and the *cube root* of any number on the K scale are found on the D scale.

Roots are more difficult to find since there are two sections of the A scale and three sections of the K scale exactly alike. Thus to find a *square root*, mark off in pairs from the decimal point, such as 258 becomes 2'58 and 2580 becomes 25'80. If there is one digit in the left division, use the left half of the scale, and if there are two digits in the left division, use the right half of the scale. To find a *cube root*, mark off in three's from the decimal point, such as 2580 becomes 2'580, 25800 becomes 25'800, and 258000 becomes 258'000. If there is one digit in the left division, use the left third of the scale; if there are two digits, use the middle third; and if there are three digits, use the right third of the scale. Try this with familiar values.

#### EXAMPLES FOR SQUARES:

Find the square of 2.

Set hairline over 2 on D and read 4 on A.

Find the square of 5.

Set hairline over 5 on D and read 25 on A.

Find the square of 16.

Set hairline over 16 on D and read 256 on A.

Find the square of 35.

Set hairline over 35 on D and read 1225 on A.

#### EXAMPLES FOR SQUARE ROOTS:

Find the square root of 9.

Set the hairline over 9 on A (left hand) and read 3 on D.

Find the square root of 36.

Set the hairline over 36 on A (right hand) and read 6 on D.

Find the square root of 121.

Set the hairline over 121 on A (left hand) and read 11 on D.

Find the square root of 1296.

Set the hairline over 1296 on A (right hand) and read 36 on D.

#### EXAMPLES FOR CUBES:

(a) Find the cube of 4.

Set the hairline over 4 on D and read 64 on K (center section).

(b) Find the cube of 8.

Set the hairline over 8 on D and read 512 on K (right section).

(c) Find the cube of 18.

Set the hairline over 18 on D and read 5830 on K (left section).

#### EXAMPLES FOR CUBE ROOTS:

(a) Find the cube root of 8.

Set the hairline over 8 on K (left section) and read 2 on D.

(b) Find the cube root of 27.

Set the hairline over 27 on K (center section) and read 3 on D.

(c) Find the cube root of 216.

Set the hairline over 216 on K (right section) and read 6 on D.

(d) Find the cube root of 5,830.

Set the hairline over 5,830 on K (left section) and read 18 on D.

### THE RECIPROCAL SCALES DI AND CIF

The DI scale is an inverted D scale and the CIF scale is an inverted CF scale. Both sets of scale values increase from right to left.

Their first and simplest use is in finding reciprocals such as 2 on D is next to 0.5 on DI or vice versa. Also note that 2 on CF is next to 0.5 on CIF or vice versa.

Since  $4 \times 2 = 8$  can be written in the form  $4 \div \frac{1}{2} = 8$ , put 2 on DIF below

4 on DE and read the answer above the index of CIF on DE. That is, after a little practice, multiply numbers together using the CIF and DE scales by putting one above the other just as you would do for division of numbers when the C and D or CF and DF scales are used. Try this for several simple problems.



Problems such as  $1/(2.78)^2$  and  $1/(2.78)^2$  can be solved directly by putting 2.78 on DI and reading the first answer on A and the second answer on K.

These simple illustrations show how valuable these reciprocal scales can be to you. Get familiar with them and use them. In the next section other uses for these scales will be shown.

**EXAMPLES:**

(a) Find  $1/2.4$ .

Set hairline over 2.4 on D and read .417 below on DI.

(b) Find  $1/60.5$ .

Set hairline over 60.5 on D and read .01652 below on DI. Or set 60.5 on DI and read .0165 above on D.

(c) Find  $1/\pi$ .

Set hairline over  $\pi$  on D and read .3183 below on DI.

(d) Find  $1/(3.12)^2$  and  $1/(3.12)^2$ .

Set hairline over 312 on DI and read 0.103 on A and 0.0329 on K.

**CONVERTING IMAGE AREA TO GROUND AREA**

The conversion of image area in square inches (as might be determined by use of an Arcameter) to ground area in square feet may be accomplished as follows:

Set 12 on the C Scale under the image area (in square inches) on the A scale. Place the hairline over the Photo Scale Reciprocal on the C scale and read the answer on the A scale under the hairline.

To determine the number of digits in the answer double the number of digits in the Photo Scale Reciprocal and subtract the appropriate factor given below. Table to be used if the answer is found on the left half of the A scale.

If image area is	If C scale	
	Slides Left	Slides Right
Over 1.44 sq. in.	1	3
Under 1.44 sq. in.	3	5
Table to be used if the answer is found on the right half of the A scale.		
If image area is	If C scale	
	Slides Left	Slides Right
Over 1.44 sq. in.	0	2
Under 1.44 sq. in.	2	4

**EXAMPLE:**

After measuring on Aerial Photography an image area is found to be 3 square inches. The scale of the photography is 1:15,000. Place the hairline over 3 on the A scale and slide the C scale to a position where 12 is under the hairline. Next place the hairline over 15 on the C scale and read the answer under the hairline on the A scale—468. The answer is found on the left side of the A scale so the first table above should be used to determine the number of digits in the answer. The image area is greater than 1.44 square inches, and the slide is right. Therefore, the factor is 3. Double the number of digits in the scale reciprocal (5) minus the factor (3) = (2 × 5) - 3 or 7. There will be 7 digits in the answer—4,680,000 square feet.

**THE TRIGONOMETRIC FUNCTIONS AND THEIR USE**

The trigonometric function scales are the S and T scales. The S scale gives the logarithmic location of the natural function values of the sine of angles from 5.74° to 90°; and the T scale gives the logarithmic location of the natural function values of the tangent of angles from 5.72° to 45° when reading from left to right, and from 45° to 84.28° when reading from right to left. Table II, in the back of the manual, gives these natural function values as well as those for the other four function values of angles. These function values are defined as the ratios of the sides of a right triangle.

They are:

$b/c = \cos A = \sin B$ ;  $c/b = \sec A = \csc B$

$a/c = \sin A = \cos B$ ;  $c/a = \csc A = \sec B$

$a/b = \tan A = \cot B$ ;  $b/a = \cot A = \tan B$

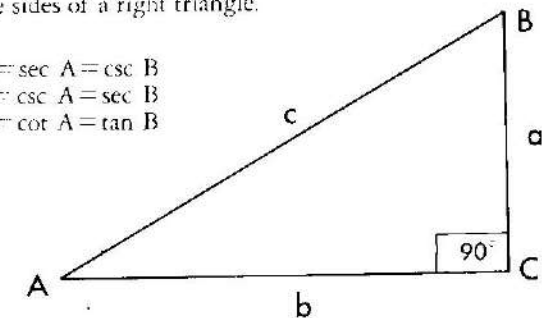


Fig. 1a

Also note that  $\cos A = \sin (90^\circ - A)$ ;  $\csc A = 1/\sin A$ ;  $\sec A = 1/\cos A = 1/\sin (90^\circ - A)$ ; and  $\cot A = 1/\tan A$  or  $\tan (90^\circ - A)$ .

The S and T scales can be used directly with other scales to solve problems, such as  $27.8 \times \sin 34^\circ$ ,  $34.6 \div \tan 27.6^\circ$ ,  $\sqrt{34.8} \times \tan 18.6^\circ$ ,  $\sin 12.5^\circ \times 24\pi$ , and so on. By using these relationships shown above, such problems as  $45.2 \times \cos 27^\circ$ ,  $32.8 \times \csc 18^\circ$ , and  $42.8 \div \csc 56.4^\circ$  can also be solved on your rule. In these problems, 27.8, 34.6, 45.2 and 32.8 would be on the D scale,  $\sqrt{34.8}$  would be on the A scale, and  $24\pi$  would be on the DF scale.  $45.2 \times \cos 27^\circ$  would be changed to read  $45.2 \times \sin (90^\circ - 27^\circ) = 45.2 \times \sin 63^\circ$ ;  $32.8 \times \csc 18^\circ$  would become  $32.8 \div \sin 18^\circ$ ; and  $42.8 \div \csc 56.4^\circ$  would become  $42.8 \times \sin 56.4^\circ$ . A problem like  $\sin 42^\circ = 2.8$  can be solved using the S and DI scales. Put the right index of S over 2.8 on DI and read the answer, 0.239 on D under 42 on S.

The greatest use of these trigonometric scales is in solving triangle problems. If two angles and a side are given, use the Law of Sines which is nothing more than a proportion problem between S and D. That is:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}, \text{ where } A + B + C = 180^\circ$$

where A, B, C, are the angles of the triangles and a, b, c, are the sides opposite these angles respectively. C would be a right angle (90°) and c would be the hypotenuse if the triangle is a right triangle.

**Illustrations:**

(1) Given A = 50°, B = 55°, and a = 26.5 feet. Find b and c.

$$\frac{\sin 50^\circ}{26.5} = \frac{\sin 55^\circ}{b} = \frac{\sin 75^\circ}{c} \quad 50^\circ + 55^\circ + 75^\circ = 180^\circ$$

Set 50° on S over 26.5 on D. Then 55° on S is over b, 28.3, on D, and 75° on S is over c, 33.4, on D. Thus b = 28.3 feet, c = 33.4 feet.

(2) Given A = 35°, B = 40°, and a = 32.6 feet. Find b and c.

Since C is greater than 90°, namely 105°, and since it can be shown that  $\sin 105^\circ = \sin (180^\circ - 105^\circ) = \sin 75^\circ$ , we have  $\frac{\sin 35^\circ}{32.6} = \frac{\sin 40^\circ}{b} = \frac{\sin 75^\circ}{c}$

Make this setting and show that b = 36.6 feet and c = 54.8 feet.

The Law of Sines can be used to check any triangle solution problem. Use the values given and determined, to verify the answers.

When given two sides and the included angle, the solution is more difficult no matter how you solve the problem. Note the diagram at your right where the triangle has been made into two triangles by drawing an altitude. Note the letters assigned and then note that

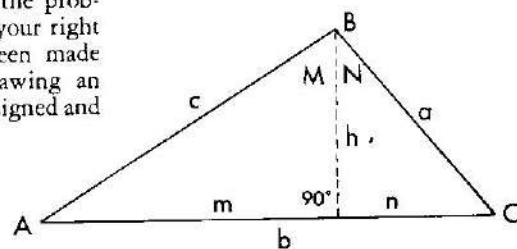


Fig. 2a

$$\frac{\sin A}{h} = \frac{\sin M}{m} = \frac{\sin 90^\circ}{c} \quad h = \tan C \text{ since } n = b - m; \text{ and } \frac{\sin C}{h} = \frac{\sin 90^\circ}{a}$$

Thus if A, b and c are given, we can find the values of B, C and a.

**Illustrations:** A = 35°, b = 42.8, c = 27.4. Find B, C, and a.

(1)  $\frac{\sin 35^\circ}{h} = \frac{\sin 55^\circ}{m} = \frac{\sin 90^\circ}{27.4}$

Set 90° (right index) over 27.4 on D. The 35° on S is over h, 15.75, on D and 55° on S is over m, 22.5, on D.

(2) Since  $42.8 - 22.5 = 20.3$ , we have  $\frac{15.75}{22.5} = \frac{\tan C}{1}$

Set 15.75 on C over 22.5 on D. Read angle C on T, namely 35°, over the right index of the D scale.

(3)  $\frac{\sin 35^\circ}{15.75} = \frac{\sin 90^\circ}{a}$

Set 35° on S over 15.75 on D. Read a, 27.4, on D under 90° on S. Since C turned out to be less than 45°, and if in any other two sides and the included angle problem it is obvious that C will be less than 45°, the following procedure can be used with the same letters used above.

$$\frac{\sin A}{c} = \frac{\tan A}{m} = \frac{\tan C}{n} = \frac{\sin C}{a} \quad \text{where } n = b - m.$$

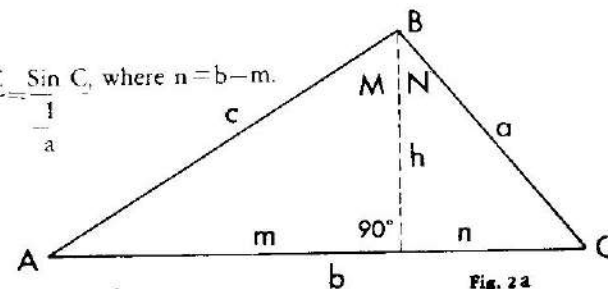


Fig. 2a

**Illustrations:**

A = 30°, b = 45.8, c = 28.2. Find a.

$$\frac{\sin 30^\circ}{28.2} = \frac{\tan 30^\circ}{m} = \frac{\tan C}{n} = \frac{\sin C}{a} \quad n = 45.8 - m.$$

Set 30° on S over 28.2 on DI. Under 30° on T, read m, 24.41, on DI.  $n = 45.8 - 24.4 = 21.4$ . Without changing the original setting of your rule, over 21.4 on DI, read C, 33.4°. Finally under 33.4° on S, read a, 25.6, on DI.

**SOLVING SMALL ANGLES WITH RADIAN MEASURE**

A special graduation mark R has been set on the C and D scales at about 57.4 to be used in changing degrees to radians.

By setting R on C over the index of D and moving the indicator to the number of degrees on C, the angle can be read in radians on D. Angles between .57 and 5.7 have one zero between the decimal and the first digit read on D.

Since the sine and tangent of angles of less than 5.7° are nearly equal, if the angle is given in radian measure, the sine and tangent are relatively equal to the angle itself. Therefore, the value of the angle may be set on C, or D, DI or A as needed in a computation.

When the angle is below 0.57, a near value for the sine or tangent can be found by this method, but may have two or more zeros.

**EXAMPLES:**

(a) Find  $\sin 2^\circ$  and  $\tan 2^\circ$ .

Place R on C over the right index of D. Set the hairline on the graduation for 2° on the C scale. Read  $\sin 2^\circ = .0349$  on the D scale. This is also the value of  $\tan 2^\circ$  correct to three digits.

(b) Find  $\sin 0.94^\circ$ .

Place R on C over the left index of D. Set the hairline on 94 of C and read 0.0164 on the C scale which is also  $\tan 0.94^\circ$ .

(c) Find  $\cot 1.41^\circ$  and  $\tan 88.59^\circ$ .

Place right index of C over R on D. Set hairline at 1.41° on DF and read  $\cot 1.41^\circ = \tan 88.59^\circ = 40.7$  on CIE.

**THE L SCALE**

The L (logarithm) scale is used for finding the logarithm of any number to the base 10.

To find the logarithm, set the number on the D scale and read the mantissa of its logarithm on the L scale. Or if the number of a known logarithm is desired, reverse the procedure.

**EXAMPLES:**

(a) To find the logarithm of 526, set the hairline over 526 and read the mantissa 721 on the L scale. The characteristic is 2, so the logarithm is 2.721.

## FOUR-PLACE LOGARITHMS

(b) To find the logarithm of .526, set the rule as before but since there are no digits before the decimal point, the characteristic will be -1, or 9-10 and the logarithm will be 9.721-10.

(c) To find x when the log x = 5.46, set the hairline over 5.46 on the L scale and read 3.51 on D.

(d) To find x when log x = 2.546, set the rule as before but note that the characteristic is 2, so the number is 351.

The logarithm of a number (Briggs) has in general two parts, a whole number part called the characteristic and the decimal part called the mantissa. The mantissas are the numbers given in a logarithm table. They are greater than 1 if the characteristic is positive and decimal if the characteristic is negative. In a number greater than 10, the characteristic is equal to one less than the number of figures. In a decimal number, the characteristic is equal to the number of zeros that appear before the significant figures.

### TYPE FORM

$$a^2 + b^2 = c^2, \text{ where } b > a$$

If you desire to find only the hypotenuse of a right triangle given the two sides of the right angle, a single procedure to do this is shown in the following illustration. Try it.

*Illustrations:*

$$3^2 + 4^2 = 5^2$$

Put 3 on C under the left index of A. Put the hairline over 4 on C and read 178 on A. Add 100 to 178 giving 278. (You will always add 100 to the A scale reading if the value of b is not more than 3.1 times a. If b lies between 3.1 times a and 10 times a, add only 10.) Move hairline to 278 on A and read 5 on C under hairline.

Type Form:  $\frac{1}{a} + \frac{1}{b} = \frac{1}{c} \quad a > b$

Using a procedure similar to that followed for  $a^2 - b^2 = c^2$ , this type form can be readily solved.

*Illustrations:*

$$\frac{1}{2} + \frac{1}{3} = \frac{1}{1.2}$$

Put left index of C over 3 on DI. Move hairline to 2 on DI and read 150 on C. Add 100 to 150, giving 250. Move hairline to 250 on C and read 1.2 on DI.

### EQUIVALENT RATIOS

A table of Equivalent Ratios may be found in the window of the case. The table is often useful for conversion of different units, (i.e.: inches to centimeters, etc.) Each pair of equivalents is followed by a suggested setting on the C and D scales to give a quick, accurate proportion from which either equivalent may be found with the other known.

**EXAMPLE:**

On the Equivalent Ratio table we see that 1 inch = 2.54 centimeters and the suggested setting on the C and D scales is 100 opposite 254 or 100 on the C scale opposite 254 on the D scale. Now by simply moving the hairline, we find that; 2 inches = 5.08 cm; 3.5 inches = 8.89 cm. Conversely the number of inches may be found when the centimeters are known: 9 cm. = 3.54 inches; 5 cm. = 1.97 inches, etc.

N.	0	1	2	3	4	5	6	7	8	9	Ave. cd
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	42
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	38
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	35
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	32
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	30
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	28
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	26
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	25
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	24
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	19
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	18
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	17
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	11
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8
N.	0	1	2	3	4	5	6	7	8	9	Ave. cd

**ERRATA:** NOTE:  
 Equivalent Correction Setting for 1 cu. ft. = .02832m<sup>3</sup> should be 600 opp. 17.  
 Table in Settings Setting for 1 cu. yd. = .7646m<sup>3</sup> should be 85 opp. 65.

FOUR-PLACE LOGARITHMS (Continued)

N.	0	1	2	3	4	5	6	7	8	9	Ave. cd
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	8
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	6
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	6
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	5
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	5
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	5
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	5
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	4
N.	0	1	2	3	4	5	6	7	8	9	Ave. cd

TABLE OF NATURAL TRIGONOMETRIC FUNCTIONS

Angle	sin	cos	tan	cot	sec	csc	Angle
0°	.0000	1.0000	.0000		1.0000		90°
1°	.0175	.9998	.0175	57.2900	1.0002	57.2987	89°
2°	.0349	.9994	.0349	28.6363	1.0006	28.6537	88°
3°	.0523	.9986	.0524	19.0811	1.0014	19.1073	87°
4°	.0698	.9976	.0699	14.3007	1.0024	14.3356	86°
5°	.0872	.9962	.0875	11.4301	1.0038	11.4737	85°
6°	.1045	.9945	.1051	9.5144	1.0055	9.5668	84°
7°	.1219	.9925	.1228	8.1443	1.0075	8.2055	83°
8°	.1392	.9903	.1405	7.1154	1.0098	7.1853	82°
9°	.1564	.9877	.1584	6.3138	1.0125	6.3925	81°
10°	.1736	.9848	.1763	5.6713	1.0154	5.7588	80°
11°	.1908	.9816	.1944	5.1446	1.0187	5.2408	79°
12°	.2079	.9781	.2126	4.7046	1.0223	4.8097	78°
13°	.2250	.9744	.2309	4.3315	1.0263	4.4454	77°
14°	.2419	.9703	.2493	4.0108	1.0306	4.1336	76°
15°	.2588	.9659	.2679	3.7321	1.0353	3.8637	75°
16°	.2756	.9613	.2867	3.4874	1.0403	3.6280	74°
17°	.2924	.9563	.3057	3.2709	1.0457	3.4203	73°
18°	.3090	.9511	.3249	3.0777	1.0515	3.2361	72°
19°	.3256	.9455	.3443	2.9042	1.0576	3.0716	71°
20°	.3420	.9397	.3640	2.7475	1.0642	2.9238	70°
21°	.3584	.9336	.3839	2.6051	1.0711	2.7904	69°
22°	.3746	.9272	.4040	2.4751	1.0785	2.6695	68°
23°	.3907	.9205	.4245	2.3559	1.0864	2.5593	67°
24°	.4067	.9135	.4452	2.2460	1.0946	2.4586	66°
25°	.4226	.9063	.4663	2.1445	1.1034	2.3662	65°
26°	.4384	.8988	.4877	2.0503	1.1126	2.2812	64°
27°	.4540	.8910	.5095	1.9626	1.1223	2.2027	63°
28°	.4695	.8829	.5317	1.8807	1.1326	2.1301	62°
29°	.4848	.8746	.5543	1.8040	1.1434	2.0627	61°
30°	.5000	.8660	.5774	1.7321	1.1547	2.0000	60°
31°	.5150	.8572	.6009	1.6643	1.1666	1.9416	59°
32°	.5299	.8480	.6249	1.6003	1.1792	1.8871	58°
33°	.5446	.8387	.6494	1.5399	1.1924	1.8361	57°
34°	.5592	.8290	.6745	1.4826	1.2062	1.7883	56°
35°	.5736	.8192	.7002	1.4281	1.2208	1.7434	55°
36°	.5878	.8090	.7265	1.3764	1.2361	1.7013	54°
37°	.6018	.7986	.7536	1.3270	1.2521	1.6616	53°
38°	.6157	.7880	.7813	1.2799	1.2690	1.6243	52°
39°	.6293	.7771	.8098	1.2349	1.2868	1.5890	51°
40°	.6428	.7660	.8391	1.1918	1.3054	1.5557	50°
41°	.6561	.7547	.8693	1.1504	1.3250	1.5243	49°
42°	.6691	.7431	.9004	1.1106	1.3456	1.4945	48°
43°	.6820	.7314	.9325	1.0724	1.3673	1.4663	47°
44°	.6947	.7193	.9657	1.0355	1.3902	1.4396	46°
45°	.7071	.7071	1.0000	1.0000	1.4142	1.4142	45°
Angle	cos	sin	cot	tan	csc	sec	Angle

By interpolation, answers in degrees and tenths of a degree, or vice versa, are possible except for the first five or six values under **cot** and **csc**.