Pickett

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how to use...

log log SLIDE RULES

Pickett

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Price 50 Cents

PREFACE

A computer who must make many difficult calculations usually has a book of tables of the elementary mathematical functions, or a slide rule, close at hand. In many cases the slide rule is a very convenient substitute for a book of tables. It is, however, much more than that, because by means of a few simple adjustments the actual calculations can be carried through and the result obtained. One has only to learn to read the scales, how to move the slide and indicator. and how to set them accurately, in order to be able to perform long and otherwise difficult calculations.

When people have difficulty in learning to use a slide rule, usually it is not because the instrument is difficult to use. The reason is likely to be that they do not understand the mathematics on which the instrument is based, or the formulas they are trying to evaluate. Some slide rule manuals contain relatively extensive explanations of the theory underlying the operations. Some explain in detail how to solve many different types of problems – for example, various cases which arise in solving triangles by trigonometry. In this manual such theory has deliberately been kept to a minimum. It is assumed that the *theory* of exponents, of logarithms of trigonometry, and of the slide rule is known to the reader, or will be recalled or studied by reference to formal textbooks on these subjects. This is a brief manual on operational technique and is not intended to be a textbook or workbook. Relatively few exercises are included, and the answers of these (to slide rule accuracy) are given immediately so that learning may proceed rapidly, by self-correction if necessary. Any textbook on mathematics which contains problems suitable for slide rule calculation, and their answers, will provide additional practice.

Some of the special scales described in this manual may not be available on your slide rule. All of the illustrations and problems shown can be worked on the slide rule you purchased. However, the special scales simplify the calculations. Pickett & Eckel Slide Rules are available with all of the special scales shown in this manual.

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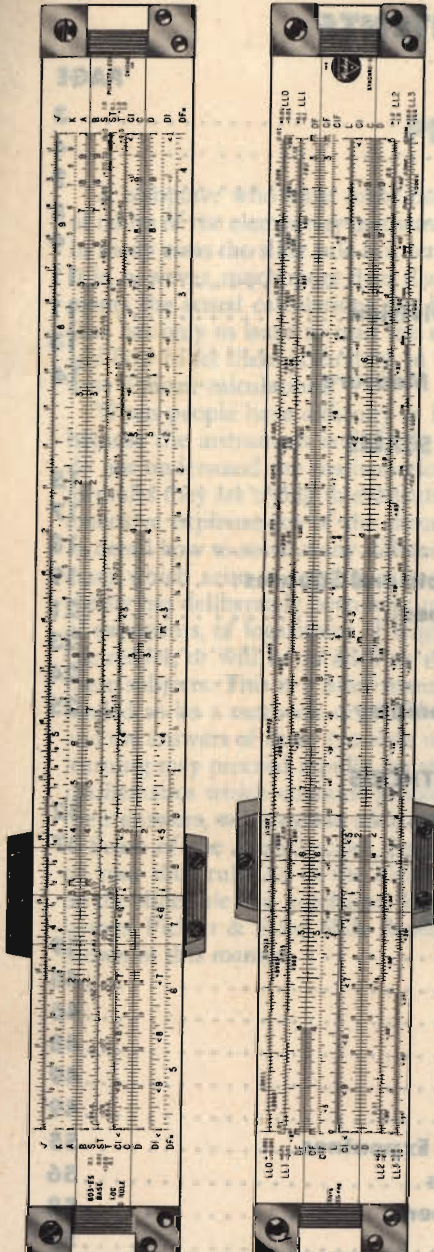
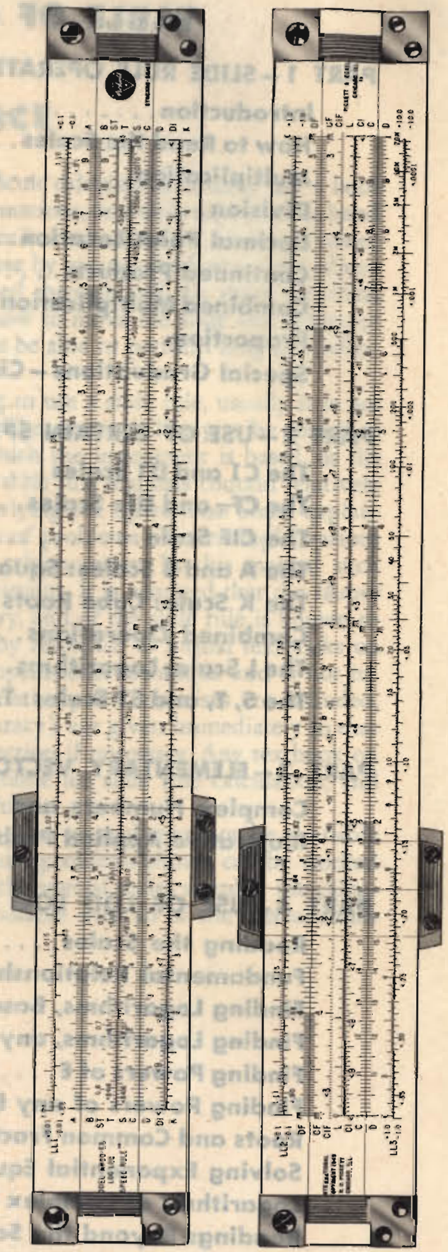


MODEL N803-ES



MODEL N800-ES

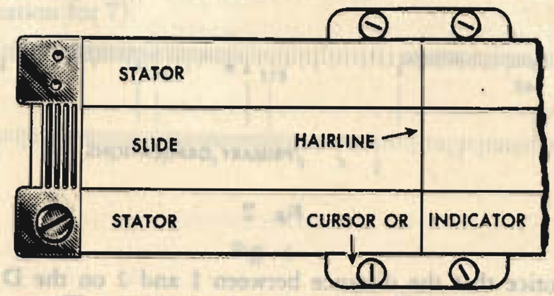
PART 1. SLIDE RULE OPERATION

INTRODUCTION

The slide rule is a fairly simple tool by means of which answers to involved mathematical problems can be easily obtained. To solve problems easily and with confidence it is necessary to have a clear understanding of the operation of your slide rule. Speed and accuracy will soon reward the user who makes a careful study of the scale arrangements and the manual.

The slide rule consists of three parts: (1) the stator (upper and lower bars); (2) the slide; (3) the cursor or indicator. The scales on the bars and slide are arranged to work together in solving problems. The hairline on the indicator is used to help the eyes in reading the scales and in adjusting the slide.

Fig. 1



Each scale is named by a letter (A, B, C, D, L, S, T) or other symbol at the left end.

The table below shows some of the mathematical operations which can be done easily and quickly with an ordinary slide rule.

OPERATIONS	INVERSE OPERATIONS
Multiplying two or more numbers	Dividing one number by another
Squaring a number	Finding the square root of a number
Cubing a number	Finding the cube root of a number
Finding the logarithm of a number	Finding a number whose logarithm is known
Finding the sine, cosine, or tangent of an angle	Finding an angle whose sine, cosine, or tangent is known

Various *combinations* of these operations (such as multiplying two numbers and then finding the square root of the result) are also easily done. Numbers can be added or subtracted with an ordinary slide rule, but it is usually easier to do these operations by arithmetic.*

In order to use a slide rule, a computer must know: (1) how to read the scales; (2) how to "set" the slide and indicator for each operation to be done; and (3) how to determine the decimal point in the result.

*By putting special scales on a slide rule, these and certain other operations much more difficult than those shown in the table above can be done easily.

HOW TO READ THE SCALES

The scale labeled C (on the slide) and the scale D (on the stator bar) are used most frequently. These two scales are exactly alike. The total length of these scales has been separated into many smaller parts by fine lines called "graduations."

Some of these lines on the D scale have large numerals (1, 2, 3, etc.) printed just below them. These lines are called *primary* graduations. On the C scale the numerals are printed above the corresponding graduations. A line labeled 1 at the left end is called the *left* index. A line labeled 1 at the right end is called the *right* index.

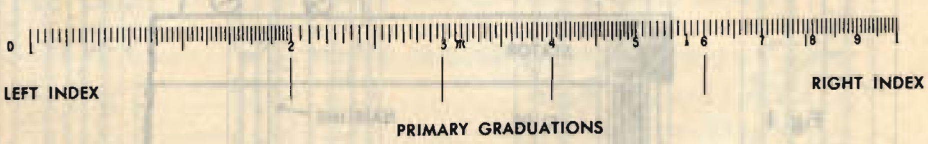


Fig. 2

Next notice that the distance between 1 and 2 on the D scale has been separated into 10 parts by shorter graduation lines. These are the *secondary* graduations. (On 10 inch slide rules these lines are labeled with smaller numerals 1, 2, 3, etc. On 6 inch rules these lines are not labeled.) Each of the spaces between the larger numerals 2 and 3, between 3 and 4, and between the other primary graduations is also sub-divided into 10 parts. Numerals are not printed beside these smaller secondary graduations because it would crowd the numerals too much.

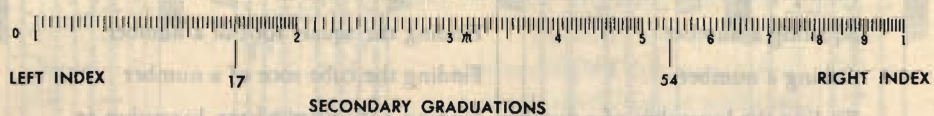


Fig. 3.

When a number is to be located on the D scale, the *first* digit is located by use of the *primary* graduations. The *second* digit is located by use of the *secondary* graduations. Thus when the number 17 is located, the 1 at the left index represents the 1 in 17. The 7th secondary graduation represents the 7. When 54 is to be located, look first for primary graduation 5, and then for secondary graduation 4 in the space immediately to the right.

There are further sub-divisions, or *tertiary graduations*, on all slide rules. The meaning of these graduations is slightly different at different parts of the scale. It is also different on a 6 inch slide rule than on a 10 inch rule. For this reason a separate explanation must be given for each.

Tertiary graduations on 10 inch rules.

The space between each secondary graduation at the left end of the rule (over to primary graduation **2**) is separated into ten parts, but these shortest graduation marks are not numbered. In the middle part of the rule, between the primary graduations **2** and **4**, the smaller spaces between the *secondary* graduations are separated into five parts. Finally, the still smaller spaces between the secondary graduations at the right of **4** are separated into only two parts.

To find 173 on the D scale, look for primary division **1** (the left index), then for secondary division 7 (numbered) then for smaller subdivision 3 (not numbered, but found as the 3rd very short graduation to the right of the longer graduation for 7).

Fig. 4

Similarly, 149 is found as the 9th small graduation mark to the right of the 4th secondary graduation mark to the right of primary graduation **1**.

To find 246, look for primary graduation **2**, then for the 4th secondary graduation after it (the 4th long line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are fifths. Since $\frac{3}{5} = \frac{6}{10}$, then the third graduation, marking *three fifths*, is at the same point as *six tenths* would be.

Tertiary graduations on 6 inch rules.

The space between each secondary graduation at the left end of the rule (over to primary graduation 2) is separated into five parts. In the middle of the rule, between the primary graduations 2 and 5, the smaller spaces between the secondary graduations are separated into two parts. Finally, the still smaller spaces between the secondary graduations at the right of 5 are not subdivided.

To find 170 on the D scale, look for the primary division 1 (the left index), then for the secondary division 7 which is the 7th secondary graduation.

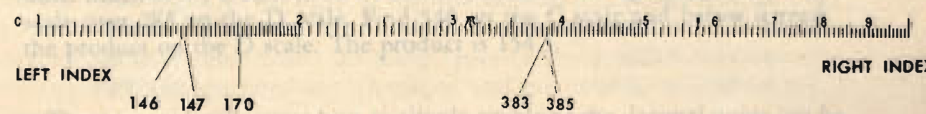


Fig. 5

To find 146, look for the primary graduation 1, then for the 4th secondary graduation after it (the 4th secondary line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are fifths. Since $3/5 = 6/10$, then the third graduation, marking *three fifths*, is at the same point as *six tenths* would be.

The number 147 would be half of a small space beyond 146. With the aid of the hairline on the runner the position of this number can be located approximately by the eye. The small space is mentally "split" in half.

The number 385 is found by locating primary graduation 3 and then secondary graduation 8 (the 8th long graduation after 3). Following this, one observes that between secondary graduations 8 and 9 there is one short mark. Think of this as the "5 tenths" mark, which represents 385. The location of 383 can be found approximately by mentally "splitting" the space between 380 and 385 into fifths, and estimating where the 3rd "fifths" mark would be placed. It would be just a little to the right of halfway between 380 and 385.

On the scale below are some sample readings.

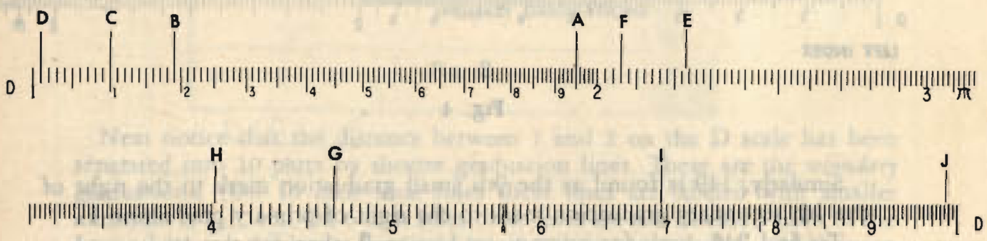


Fig. 6

A: 195	F: 206
B: 119	G: 465
C: 110	H: 402
D: 101	I: 694
E: 223	J: 987

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used in writing numbers are called *digits*. One way to describe a number is to tell how many digits are used in writing it. Thus 54 is a "two-digit number", and 1,348,256 is a "seven-digit number." In many computations only the first two or three digits of a number need to be used to get an approximate result which is accurate enough for practical purposes. Usually not more than the first three digits of a number can be "set" on a six inch slide rule scale. In many practical problems this degree of accuracy is sufficient. When greater accuracy is desired, a ten inch slide rule is generally used.

MULTIPLICATION

Numbers that are to be multiplied are called factors. The result is called the product. Thus, in the statement $6 \times 7 = 42$, the numbers 6 and 7 are factors, and 42 is the product.

EXAMPLE: Multiply 2×3

Setting the Scales: Set the left index of the C scale on 2 of the D scale. Find 3 on the C scale, and below it read the product, 6 on the D scale.

Think: The length for 2 plus the length for 3 will be the length for the product. This length, measured by the D scale, is 6.

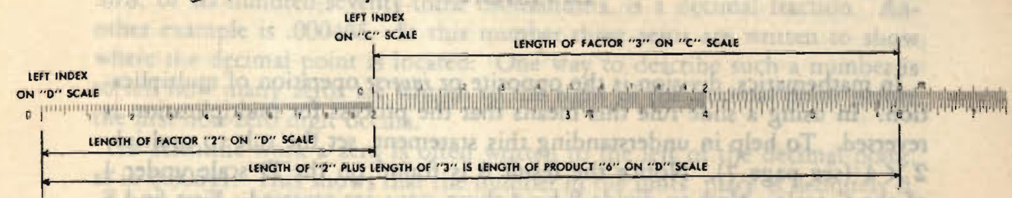


Fig. 7

EXAMPLE: Multiply 4×2

Setting the Scales: Set the left index of the C scale on 4 of the D scale. Find 2 on the C scale, and below it read the product, 8, on the D scale.

Think: The length for 4 plus the length for 2 will be the length for the product. This length, measured by the D scale, is 8.

***Rule for Multiplication:* Over one of the factors on the D scale, set the index of the C scale. Locate the other factor on the C scale, and directly below it read the product on the D scale.**

EXAMPLE. Multiply 2.34×36.8

Estimate the result: First note that the result will be roughly the same as 2×40, or 80; that is, there will be two digits to the left of the decimal point. Hence, we can ignore the decimal points for the present and multiply as though the problem was 234×368.

Set the Scales: Set the left index of the C scale on 234 of the D scale. Find 368 on the C scale and read product 861 on the D scale.

Think: The length for 234 plus the length for 368 will be the length for the product. This length is measured on the D scale. Since we already knew the result was somewhere near 80, the product must be 86.1, approximately.

EXAMPLE: Multiply 28.3×5.46

Note first that the result will be about the same as 30×5, or 150. Note also that if the left index of the C scale is set over 283 on the D scale, and 546 is then found on the C scale, the slide projects so far to the right of the rule that the D scale is no longer below the 546. When this happens, the other index of the C scale must be used. That is, set the *right* index on the C scale over 283 on the D scale. Find 546 on the C scale and below it read the product on the D scale. The product is 154.5.

These examples illustrate how in simple problems the decimal point can be placed by use of an estimate.

PROBLEMS	ANSWERS
1. 15×3.7	55.5
2. 280×0.34	95.2
3. 753×89.1	67,100
4. 9.54×16.7	159.3
5. 0.0215×3.79	0.0815

DIVISION

In mathematics, division is the opposite or *inverse* operation of multiplication. In using a slide rule this means that the process for multiplication is reversed. To help in understanding this statement, set the rule to multiply 2×4 (see page 7). Notice the result 8 is found on the D scale under 4 of the C scale. Now to divide 8 by 4 these steps are reversed. First find 8 on the D scale, set 4 on the C scale over it, and read the result 2 on the D scale under the index of the C scale.

Think: From the length for 8 (on the D scale) *subtract* the length for 4 (on the C scale). The length for the difference, read on the D scale, is the result, or quotient.

With this same setting you can read the quotient of $6 \div 3$, or $9 \div 4.5$, and in fact all divisions of one number by another in which the result is 2.

***Rule for Division:* Set the *divisor* (on the C scale) opposite the number to be divided (on the D scale). Read the result, or quotient, on the D scale under the index of the C scale.**

EXAMPLES:

(a) Find $63.4 \div 3.29$. The quotient must be near 20, since $60 \div 3 = 20$. Set indicator on 63.4 of the D scale. Move the slide until 3.29 of the C scale is under the hairline. Read the result 19.27 on the D scale at the C index.

(b) Find $26.4 \div 47.7$. Since 26.4 is near 25, and 47.7 is near 50, the quotient must be roughly $25/50 = \frac{1}{2} = 0.5$. Set 47.7 of C opposite 26.4 of D, using the indicator to aid the eyes. Read 0.553 on the D scale at the C index.

PROBLEMS	ANSWERS
1. $83 \div 7$	11.86
2. $75 \div 92$	0.815
3. $137 \div 513$	0.267
4. $17.3 \div 231$	0.0749
5. $8570 \div .0219$	391,000

DECIMAL POINT LOCATION

In the discussion which follows, it will occasionally be necessary to refer to the number of "digits" and number of "zeros" in some given numbers.

When numbers are greater than 1 the number of *digits* to the left of the decimal point will be counted. Thus 734.05 will be said to have 3 digits. Although as written the number indicates accuracy to *five* digits, only three of these are at the left of the decimal point.

Numbers that are less than 1 may be written as *decimal fractions.** Thus .673, or six-hundred-seventy-three thousandths, is a decimal fraction. Another example is .000465. In this number three zeros are written to show where the decimal point is located. One way to describe such a number is to tell how many zeros are written to the right of the decimal point before the first non-zero digit occurs.

In scientific work a zero is often written to the left of the decimal point, as in 0.00541. This shows that the number in the units' place is definitely 0, and that no digits have been carelessly omitted in writing or printing. The zeros will *not* be counted unless they are (a) at the *right* of the decimal point, (b) before or at the *left* of the first non-zero digit, and (c) are not between other digits. The number 0.000408 will be said to have 3 zeros (that is, the number of zeros between the decimal point and the 4).

In many, perhaps a majority, of the problems met in genuine applications of mathematics to practical affairs, the position of the decimal point in the result can be determined by what is sometimes called "common sense." There is usually only one place for the decimal point in which the answer is "reasonable" for the problem. Thus, if the calculated speed in miles per hour of a powerful new airplane comes out to be 4833, the decimal point clearly belongs between the 3's, since 48 m.p.h. is too small, and 4833 m.p.h. is too large for such a plane. In some cases, however, the data are such that the position of the point in the final result is not easy to get by inspection.

Another commonly used method of locating the decimal point is by estimation or approximation. For example, when the slide rule is used to find 133.4×12.4, the scale reading for the result is 1655, and the decimal point is to be determined. By rounding off the factors to 133.0×10.0, one obtains 1330 by mental arithmetic. The result would be somewhat greater than this but certainly contains four digits on the left of the decimal point. The answer, therefore, must be 1655.

In scientific work numbers are often expressed in *standard form.* For example, 428 can be written 4.28×10^2, and 0.00395 can be written as 3.95×10^{-3}. When a number is written in standard form it always has two factors. The first factor has one digit (not a zero) on the left of the decimal point, and usually other digits on the right of the decimal point. The other factor is a power of 10 which places the decimal point in its true position if the indicated multiplication is carried out. In many types of problems this method of writing numbers simplifies the calculation and the location of the decimal point.

*Only positive real numbers are being considered in this discussion.

When a number is written in standard form, the exponent of 10 may be called "the characteristic." It is the characteristic of the logarithm of the number to base 10. The characteristic may be either a positive or a negative number. Although the rule below appears long, in actual practice it may be used with great ease.

Rule. To express a number in standard form:
(a) place a decimal point at the right of the first non-zero digit.*
(b) start at the right of the first non-zero digit in the original number and count the digits and zeros passed over in reaching the decimal point. The result of the count is the numerical value of the characteristic, or exponent of 10. If the original decimal point is toward the right, the characteristic is *positive* (+). If the original decimal point is toward the left, the characteristic is *negative* (−). Indicate that the result of (a) is to be multiplied by 10 with the exponent thus determined in (b).

EXAMPLES:

Number	Number in standard form.	Characteristic
(a) 5,790,000	5.79×10^6	6
(b) 0.000283	2.83×10^{-4}	−4
(c) 44	4.4×10^1	1
(d) 0.623	6.23×10^{-1}	−1
(e) 8.15	8.15×10^0	0
(f) 461,328	4.61328×10^5	5
(g) 0.0000005371	5.371×10^{-7}	−7
(h) 0.0306	3.06×10^{-2}	−2
(i) 80.07	8.007×10^1	1

If a number given in standard form is to be written in "ordinary" form, the digits should be copied, and then starting at the right of the first digit the number of places indicated by the exponent should be counted, supplying zeros as necessary, and the point put down. If the exponent is positive, the count is toward the right; if negative, the count is toward the left. This converse application of the rule may be verified by studying the examples given above.

Consider now the calculation of 5,790,000 × 0.000283. From examples (a) and (b) above, this can be written $5.79 \times 10^6 \times 2.83 \times 10^{-4}$, or by changing order and combining the exponents of 10, as $5.79 \times 2.83 \times 10^2$. Then since 5.79 is near 6, and 2.83 is near 3, the product of these two factors is known to be near 18. The multiplication by use of the C and D scales shows it to be about 16.39, or 1.639×10^1. Hence, $5.79 \times 2.83 \times 10^2 = 1.639 \times 10^1 \times 10^2 = 1.639 \times 10^3 = 1639$. If, however, one has

5,790,000 ÷ 0.000283, the use of standard form yields

$$\frac{5.79 \times 10^6}{2.83 \times 10^{-4}} = 2.04 \times 10^{6-(-4)} = 2.04 \times 10.^{10}$$

In scientific work the result would be left in this form, but for popular consumption it would be written as 20,400,000,000. The general rule is as follows.

*In using this rule, "first" is to be counted from the left; thus, in 3246, the digit 3 is "first."

Rule. To determine the decimal point, first express the numbers in standard form. Carry out the indicated operations of multiplication or division, using the laws of exponents* to combine the exponents until a single power of 10 is indicated. If desired, write out the resulting number, using the final exponent of 10 to determine how far, and in what direction, the decimal point in the coefficient should be moved.

CONTINUED PRODUCTS

Sometimes the product of three or more numbers must be found. These "continued" products are easy to get on the slide rule.

EXAMPLE: Multiply 38.2 × 1.65 × 8.9.

Estimate the result as follows: 40 × 1 × 10 = 400. The result should be, very roughly, 400.

Setting the Scales: Set left index of the C scale over 382 on the D scale. Find 165 on the C scale, and *set the hairline* on the indicator on it.** Move the index on the slide under the hairline. In this example if the *left* index is placed under the hairline, then 89 on the C scale falls outside the D scale. Therefore move the *right* index under the hairline. Move the hairline to 89 on the C scale and read the result (561) under it on the D scale.

Below is a *general rule* for continued products: $a \times b \times c \times d \times e \cdots$

Set hairline of indicator at a on D scale.
Move index of C scale under hairline.
Move hairline over b on the C scale.
Move index of C scale under hairline.
Move hairline over c on the C scale.
Move index of C scale under hairline.

Continue moving hairline and index alternately until all numbers have been set.

Read result under the hairline on the D scale.

PROBLEMS	ANSWERS
1. 2.9 × 3.4 × 7.5	73.9
2. 17.3 × 43 × 9.2	6,840
3. 343 × 91.5 × 0.00532	167
4. 19 × 407 × 0.0021	16.24
5. 13.5 × 709 × 0.567 × 0.97	5260

COMBINED MULTIPLICATION AND DIVISION

Many problems call for both multiplication and division.

EXAMPLE: $\frac{42 \times 37}{65}$.

*See any textbook on elementary algebra. The theory of exponents and the rules of operation with signed numbers are both involved in a complete treatment of this topic. In this manual it is assumed that the reader is familiar with this theory.

**The product of 382 × 165 could now be read under the hairline on the D scale, but this is not necessary.

First, set the division of 42 by 65; that is, set 65 on the C scale opposite 42 on the D scale.* Move the hairline on indicator to 37 on the C scale. Read the result 239 on the D scale under the hairline. Since the fraction $\frac{42}{65}$ is about equal to $\frac{2}{3}$, the result is about two-thirds of 37, or 23.9.

EXAMPLE: $\frac{273 \times 548}{692 \times 344}$

Set 692 on the C scale opposite 273 on the D scale. Move the hairline to 548 on the C scale. Move the slide to set 344 on the C scale under the hairline. Read the result .628 on the D scale under the C index.

In general, to do computations of the type $\frac{a \times c \times e \times g \cdots}{b \times d \times f \times h \cdots}$, set the rule to divide the first factor in the numerator *a* by the first factor in the denominator *b*, move the hairline to the next factor in the numerator *c*; move the slide to set next factor in denominator, *d*, under the hairline. Continue moving hairline and slide alternately for other factors (*e, f, g, h, etc.*). Read the result on the D scale. If there is one more factor in the numerator than in the denominator, the result is under the hairline. If the number of factors in numerator and denominator is the same, the result is under the C index. Sometimes the slide must be moved so that one index replaces the other.**

EXAMPLE: $\frac{2.2 \times 2.4}{8.4}$

If the rule is set to divide 2.2 by 8.4, the hairline cannot be set over 2.4 of the C scale and at the same time remain on the rule. Therefore the hairline is moved to the C index (opposite 262 on the D scale) and the slide is moved end for end to the right (so that the *left* index falls under the hairline and over 262 on the D scale). Then the hairline is moved to 2.4 on the C scale and the result .63 is read on the D scale.

If the number of factors in the numerator exceeds the number in the denominator by more than one, the numbers may be grouped, as shown below. After the value of the *group* is worked out, it may be multiplied by the other factors in the usual manner.

$$\left(\frac{a \times b \times c}{m \times n}\right) \times d \times e$$

PROBLEMS	ANSWERS
1. $\frac{27 \times 43}{19}$	61.1
2. $\frac{5.17 \times 1.25 \times 9.33}{4.3 \times 6.77}$	2.07
3. $\frac{842 \times 2.41 \times 173}{567 \times 11.52}$	53.7
4. $\frac{1590 \times 3.64 \times 0.763}{4.39 \times 930}$	1.081

*The quotient, .646, need not be read.

**This statement assumes that up to this point only the C and D scales are being used. Later sections will describe how this operation may be avoided by the use of other scales.

5. $\frac{0.0237 \times 3970 \times 32 \times 6.28}{0.00029 \times 186000}$	351
6. $\frac{231 \times 58.6 \times 4930}{182.5 \times 3770}$	97.0
7. $\frac{875 \times 1414 \times 2.01}{661 \times 35.9}$	104.8
8. $\frac{558 \times 1145 \times 633 \times 809}{417 \times 757 \times 354}$	2930
9. $\frac{0.691 \times 34.7 \times 0.0561}{91,500}$	0.0000147 or 1.47×10^{-5}
10. $\frac{19.45 \times 7.86 \times 361 \times 64.4}{32.6 \times 9.74}$	11,190

PROPORTION

Problems in proportion are very easy to solve. First notice that when the index of the C scale is opposite 2 on the D scale, the ratio 1 : 2 or $\frac{1}{2}$ is at the *same time* set for all other opposite graduations; that is, 2 : 4, or 3 : 6, or 2.5 : 5, or 3.2 : 6.4, etc. It is true in general that for any setting the numbers for *all pairs of opposite graduations have the same ratio.* Suppose one of the terms of a proportion is unknown. The proportion can be written as $\frac{a}{b} = \frac{c}{x}$, where *a*, *b*, and *c*, are known numbers and *x* is to be found.

***Rule:* Set *a* on the C scale opposite *b* on the D scale. Under *c* on the C scale read *x* on the D scale.**

EXAMPLE: Find *x* if $\frac{3}{4} = \frac{5}{x}$

Set 3 on C opposite 4 on D. Under 5 on C read 6.67 on D.

The proportion above could also be written $\frac{b}{a} = \frac{x}{c}$, or "inverted," and exactly the same rule could be used. Moreover, if C and D are interchanged in the above rule, it will still hold if "under" is replaced by "over." It then reads as follows:

Set *a* on the D scale opposite *b* on the C scale. Over *c* on the D scale read *x* on the C scale.

***Rule:* In solving proportions, keep in mind that the position of the numbers as set on the scales is the same as it is in the proportion written in the form $\frac{a}{b} = \frac{c}{d}$**

Proportions can also be solved *algebraically*. Then $\frac{a}{b} = \frac{c}{x}$ becomes $x = \frac{bc}{a}$, and this may be computed as combined multiplication and division.

PROBLEMS	ANSWERS
1. $\frac{x}{42.5} = \frac{13.2}{1.87}$	300.
2. $\frac{90.5}{x} = \frac{3.42}{1.54}$	40.7
3. $\frac{43.6}{89.2} = \frac{x}{2550}$	1247.
4. $\frac{0.063}{0.51} = \frac{34.1}{x}$	276.
5 $\frac{18}{91} = \frac{13}{x}$	65.7

SPECIAL GRADUATIONS—CIRCULAR MEASURE

A special graduation on the C scale at $\pi/4 = 0.7854$ is especially convenient in finding the area or the diameter of a circle, and in similar problems. Since $A = (\pi/4)d^2$, where d is the diameter, the following rule applies.

Rule: To find the area (A) of a circle whose diameter d is known, set the right index of the C scale over d on the D scale. Move the hairline over .7854 mark on righthand section of B scale. Read area under hairline on A scale.

Rule: To find the diameter of a circle whose area A is known, set hairline over area on A scale (See page 19 for rule regarding section of A scale to use). Move slide until .7854 mark on righthand section of B scale is under hairline. Read diameter on D scale below right index of C scale.

A special graduation on the C and D scales at 57.3 is indicated by a small R. Since one radian = 57.3° (approximately), this R graduation is useful in changing radian measure to degree measure, and conversely.

Rule: To convert radians to degrees, set an index of the C scale to the number of radians on D. Under R on C read the number of degrees on D. To convert degrees to radians, set R on C over the number of degrees on D. At the index of C read the number of radians on D.

EXAMPLES:

(a) Convert $\pi/2$ radians to degrees. Set 2 on C over π on D. At the left-index of C read $\pi/2$, or 1.57, on D.

(b) Convert 30° to radians. Set R on C over 30 on D. At the index of C read 0.524 radians on D.

Note that this method puts the result on the D scale in convenient form for continued computation. In similar fashion, by interchanging C and D and the words "under" and "over," in the above rules, the result can be put on the C scale.

PART 2. USE OF CERTAIN SPECIAL SCALES

THE CI AND DI SCALES

It should be understood that the use of the CI and DI scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

The CI scale on the slide is a C scale which *increases from right to left*. It may be used for finding reciprocals. When any number is set under the hairline on the C scale its reciprocal is found under the hairline on the CI scale, and conversely.

EXAMPLES:

(a) Find 1/2.4. Set 2.4 on C. Read .417 directly above on CI.

(b) Find 1/60.5. Set 60.5 on C. Read .0165 directly above on CI. Or, set 60.5 on CI, read .0165 directly below on C.

The CI scale is useful in replacing a division by a multiplication. Since $\frac{a}{b} = a \times 1/b$, any division may be done by multiplying the numerator (or dividend) by the reciprocal of the denominator (or divisor). This process may often be used to avoid settings in which the slide projects far outside the rule.

EXAMPLES:

(a) Find $13.6 \div 87.5$. Consider this as $13.6 \times 1/87.5$. Set left index of the C scale on 13.6 of the D scale. Move hairline to 87.5 on the CI scale. Read the result, .155, on the D scale.

(b) Find $72.4 \div 1.15$. Consider this as $72.4 \times 1/1.15$. Set right index of the C scale on 72.4 of the D scale. Move hairline to 1.15 on the CI scale. Read 63.0 under the hairline on the D scale.

An important use of the CI scale occurs in problems of the following type.

EXAMPLE: Find $\frac{13.6}{4.13 \times 2.79}$ This is the same as $\frac{13.6 \times (1/2.79)}{4.13}$.

Set 4.13 on the C scale opposite 13.6 on the D scale. Move hairline to 2.79 on the CI scale, and read the result, 1.180, on the D scale.

By use of the CI scale, factors may be transferred from the numerator to the denominator of a fraction (or vice-versa) in order to make the settings more readily. Also, it is sometimes easier to get $a \times b$ by setting the hairline on a, pulling b on the CI scale under the hairline, and reading the result on the D scale under the index.

The DI scale (inverted D scale) below the D scale corresponds to the CI scale on the slide. Thus the D and DI scales together represent reciprocals. The DI scale has several important uses, of which the following is representative.

Expressions of the type 1/X, where X is some complicated expression or formula, may be computed by first finding the value of X. If the result for X falls on D, then 1/X may be read under the hairline on DI.

EXAMPLE:

(a) Find $\frac{1}{0.265 \times 138}$. Multiply 0.265×138 using the C and D scales. Read the reciprocal .0273 under the hairline on the DI scale. Or set the hairline on 265 of the DI scale, pull 138 of the C scale under the hairline, and read the result on the D scale under the left index of the C scale. This is equivalent to writing the expression as $\frac{(1/.265)}{138}$.

PROBLEMS	ANSWERS
1. $\frac{1}{7}$	.143
2. $\frac{1}{35.2}$	.0284
3. $\frac{1}{.1795}$	5.57
4. $\frac{1}{6430}$	.0001555
5. $\frac{1}{\pi}$	.318
6. $\frac{1}{.00417}$	240

THE CF/π AND DF/π SCALES

It should be understood that the use of the CF and DF scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of certain settings. In this way the speed can be increased and errors minimized. When π on the C scale is opposite the right index of the D scale, about half the slide projects beyond the rule. If this part were cut off and used to fill in the opening at the left end, the result would be a "folded" C scale, or CF scale. Such a scale is printed at the top of the slide. It begins at π and the index is near the middle of the rule. The DF scale is similarly placed. Any setting of C on D is automatically set on CF and DF. Thus if 3 on C is opposite 2 on D, then 3 on CF is also opposite 2 on DF. The CF and DF scales can be used for multiplication and division in exactly the same way as the C and D scales.

The most important use of the CF and DF scales is to avoid resetting the slide. If a setting of the indicator cannot be made on the C or D scale, it can be made on the CF or DF scale.

EXAMPLES:

(a) Find 19.2×6.38. Set left index of C on 19.2 of D. Note that 6.38 on C falls outside the D scale. Hence, move the indicator to 6.38 on the CF scale, and read the result 122.5 on the DF scale. Or set the index of CF on 19.2 of DF. Move indicator to 6.38 on CF and read 122.5 on DF.

(b) Find $\frac{8.39 \times 9.65}{5.72}$. Set 5.72 on C opposite 8.39 on D. The indicator cannot be moved to 9.65 of C, but it can be moved to this setting on CF and the result, 14.15, read on DF. Or the entire calculation may be done on the CF and DF scales.

These scales are also helpful in calculations involving π and $1/\pi$. When the indicator is set on any number N on D, the reading on DF is $N\pi$. This can be symbolized as $(DF) = \pi(D)$. Then $(D) = \frac{(DF)}{\pi}$. This leads to the following simple rule.

***Rule:* If the diameter of a circle is set on D, the circumference may be read immediately on DF, and conversely.**

EXAMPLES:

(a) Find 5.6π. Set indicator over 5.6 on D. Read 17.6 under hairline on DF.

(b) Find $8/\pi$. Set indicator over 8 on DF. Read 2.55 under hairline on D.

(c) Find the circumference of a circle whose diameter is 7.2. Set indicator on 7.2 of D. Read 22.6 on DF.

(d) Find the diameter of a circle whose circumference is 121. Set indicator on 121 of DF. Read 38.5 on D.

Finally, these scales are useful in changing radians to degrees and conversely. Since π radians = 180 degrees, the relationship may be written as a proportion $\frac{r}{d} = \frac{\pi}{180}$, or $\frac{r}{\pi} = \frac{d}{180}$.

Rule: **Set 180 of C opposite π on D. To convert radians to degrees, move indicator to r (the number of radians) on DF, read d (the number of degrees) on CF; to convert degrees to radians, move indicator to d on CF, read r on DF.**

There are also other convenient settings as suggested by the proportion. Thus one can set the ratio $\pi/180$ on the CF and DF scales and find the result from the C and D scales.

EXAMPLES:

(a) The numbers 1, 2, and 7.64 are the measures of three angles in radians. Convert to degrees. Set 180 of C on π of D. Move indicator to 1 on DF, read 57.3° on CF. Move indicator over 2 of DF, read 114.6°. Move indicator to 7.64 of DF. Read 437° on CF.

(b) Convert 36° and 83.2° to radians. Use the same setting as in (a) above. Locate 36 on CF. Read 0.628 radians on DF. Locate 83.2 on CF. Read 1.45 radians on DF.

PROBLEMS	ANSWERS
1. 1.414×7.79	11.02
2. 2.14×57.6	123.3
3. $\frac{84.5 \times 7.59}{36.8}$	17.43
4. $2.65 \times \pi$	8.33
5. $\frac{.1955 \times 23.7}{50.7 \times \pi}$	.0291

THE CIF SCALE

Like the other special scales the CIF scale does not increase the power of the instrument to solve problems. It is used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

The CIF scale is a folded CI scale. Its relationship to the CF and DF scales is the same as the relation of the CI scale to the C and D scales.

EXAMPLES:

(a) Find 68.2×1.43. Set the indicator on 68.2 of the D scale. Observe that if the left index is moved to the hairline the slide will project far to the right. Hence merely move 14.3 on CI under the hairline and read the result 97.5 on D at the C index.

(b) Find $2.07 \times 8.4 \times 16.1$. Set indicator on 2.07 on C. Move slide until 8.4 on CI is under hairline. Move hairline to 16.1 on C. Read 280 on D under hairline. Or, set the index of CF on 8.4 of DF. Move indicator to 16.1 on CF, then move slide until 2.07 on CIF is under hairline. Read 280 on DF above the index of CF. Or set 16.1 on CI opposite 8.4 on D. Move indicator to 2.07 on C, and read 280 on D. Although several other methods are possible, the first method given is preferable.

THE A AND B SCALES: Square Roots and Squares

When a number is multiplied by itself the result is called the *square* of the number. Thus 25 or 5×5 is the square of 5. The factor 5 is called the *square root* of 25. Similarly, since $12.25 = 3.5 \times 3.5$, the number 12.25 is called the square of 3.5; also 3.5 is called the square root of 12.25. Squares and square roots are easily found on a slide rule.

Square Roots: To find square roots the A and D scales or the B and C scales are used.

Rule: **The square root of any number located on the A scale is found below it on the D scale.**

Also, the square root of any number located on the B scale (on the slide) is found on the C scale (on the reverse side of the slide).

EXAMPLES: Find the $\sqrt{4}$. Place the hairline of the indicator over 4 on the left end of the A scale. The square root, 2, is read below on the D scale. Similarly the square root of 9 (or $\sqrt{9}$) is 3, found on the D scale below the 9 on the left end of the A scale.

Reading the Scales: The A scale is a contraction of the D scale itself. The D scale has been shrunk to half its former length and printed twice on the same line. To find the square root of a number between 0 and 10 the left half of the A scale is used (as in the examples above). To find the square root of a number between 10 and 100 the right half of the A scale is used. For example, if the hairline is set over 16 on the right half of the A scale (near the middle of the rule), the square root of 16, or 4, is found below it on the D scale.

In general, to find the square root of any number with an odd number of digits or zeros (1, 3, 5, 7, . . .), the left half of the A scale is used. If the number has an even number of digits or zeros (2, 4, 6, 8, . . .), the right half of the A scale is used. In these statements it is assumed that the number is not written in standard form.

The table below shows the number of digits or zeros in the number N and its square root, and also whether right or left half of the A scale should be used.

	ZEROS				or	DIGITS				
	L R	L R	L R	L	R	L R	L R	L R	L R	
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.

This shows that numbers from 1 up to 100 have one digit in the square root; numbers from 100 up to 10,000 have two digits in the square root, etc. Numbers which are less than 1 and have, for example, either two or three zeros, have only one zero in the square root. Thus $\sqrt{0.004} = 0.0632$, and $\sqrt{0.0004} = 0.02$.

EXAMPLES:

(a) Find $\sqrt{248}$. This number has 3 (an *odd* number) digits. Set the hairline on 248 of the left A scale. Therefore the result on D has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000}$. The number has 6 (an *even* number) digits. Set the hairline on 563 of the right A scale. Read the figures of the square root on the D scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. The number of *zeros* is 4 (an *even* number.) Set the hairline on 1362 of the right half of the A scale. Read the figures 369 on the D Scale. The result has 2 zeros, and is .00369.

If the number is written in standard form, the following rule may be used. If the exponent of 10 is an even number, use the left half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the original. If the exponent of 10 is an odd number, move the decimal point one place to the right and decrease the exponent of 10 by one, then use the right half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the reduced exponent. This rule applies to either positive or negative exponents of 10.

EXAMPLES:

(1) Find the square root of 3.56×10^4. Place hairline of indicator on 3.56 on the left half of the A scale and read 1.887 on the D scale. Then the square root is $1.887 \times 10^2 = 188.7$.

(2) Find the square root of 7.43×10^{-5}. Express the number as 74.3×10^{-6} Now place the hairline of the indicator over 74.3 on the right half of the A scale and read 8.62 on the D scale. Then the desired square root is 8.62×10^{-3}.

All the above rules and discussion can be applied to the B and C scales if it is more convenient to have the square root on the slide rather than on the body of the rule.

***Squares: To find the square of a number, reverse the process* for finding the square root. Set the indicator over the number on the D scale and read the square of that number on the A scale; or set the indicator over the number on the C scale and read the square on the B scale.**

EXAMPLES:

(a) Find $(1.73)^2$ or 1.73×1.73. Locate 1.73 on the D scale. On the A scale find the approximate square 3.

(b) Find $(62800)^2$. Locate 628 on the D scale. Find 394 above it on the A scale. The number has 5 digits. Hence the square has either 9 or 10 digits. Since, however, 394 was located on the right half of the A scale, the square has the *even* number of digits, or 10. The result is 3,940,000,000.

(c) Find $(.000254)^2$. On the A scale read 645 above the 254 of the D scale. The number has 3 zeros. Since 645 was located on the side of the A scale for "odd zero" numbers, the result has 7 zeros, and is 0.0000000645.

PROBLEMS	ANSWERS
1. $\sqrt{7.3}$	2.7
2. $\sqrt{73}$	8.54
3. $\sqrt{841}$	29
4. $\sqrt{0.062}$	0.249
5. $\sqrt{0.00000094}$	0.00097
6. $(3.95)^2$	15.6
7. $(48.2)^2$	2320
8. $(0.087)^2$	0.00757
9. $(0.00284)^2$	0.00000807
10. $(635000)^2$	4.03×10^{11}

THE K SCALE: Cube Roots and Cubes

Just below the D scale on the back of the rule is a scale marked with the letter K; this scale may be used in finding the cube or cube root of any number.

***Rule:* The cube root of any number located on the K scale is found directly above on the D scale.**

EXAMPLE: Find the $\sqrt[3]{8}$. Place the hairline of the indicator over the 8 at the left end of the K scale. The cube root, 2, is read directly above on the D scale.

Reading the scales: The cube root scale is directly below the D scale and is a contraction of the D scale itself. The D scale has been shrunk to one third its former length and printed three times on the same line. To find the cube root of any number between 0 and 10 the left third of the K scale is used. To find the cube root of a number between 10 and 100 the middle third is used. To find the cube root of a number between 100 and 1000 the right third of the K scale is used to locate the number.

In general to decide which part of the K scale to use in locating a number, mark off the digits in groups of three starting from the decimal point. If the left group contains one digit, the left third of the K scale is used; if there are two digits in the left group, the middle third of the K scale is used; if there are three digits, the right third of the K scale is used. In other words, numbers containing 1, 4, 7, $\cdots$ digits are located on the left third; numbers containing 2, 5, 8, $\cdots$ digits are located on the middle third; and numbers containing 3, 6, 9, $\cdots$ digits are located on the right third of the K scale. The corresponding number of digits or zeros in the cube roots are shown in the table below and whether the left, center or right section of the K scale should be used.

	ZEROS				or	DIGITS			
	L C R	L C R	L C R	L C	R	L C R	L C R	L C R	L C R
N	11, 10, 9	8, 7, 6	5, 4, 3	2, 1	0	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12
$\sqrt[3]{N}$	3	2	1	0	0	1	2	3	4

EXAMPLES:

(a) Find $\sqrt[3]{6.4}$. Set hairline over 64 on the left most third of the K scale. Read 1.857 on the D scale.

(b) Find $\sqrt[3]{64}$. Set hairline over 64 on the middle third of the K scale. Read 4 on the D scale.

(c) Find $\sqrt[3]{640}$. Set hairline over 64 on the right most third of the K scale. Read 8.62 on the D scale.

(d) Find $\sqrt[3]{6,400}$. Set hairline over 64 on the left third of the K scale. Read 18.57 on the D scale.

(e) Find $\sqrt[3]{64,000}$. Set hairline over 64 on the middle third of the K scale. Read 40 on the D scale.

(f) Find $\sqrt[3]{0.0064}$. Use the left third of the K scale, since the first group of three, or 0.006, has only one non-zero digit. The D scale reading is then 0.1857.

(g) Find $\sqrt[3]{0.064}$. Use the middle third of the K scale, reading 0.4 on D.

If the number is expressed in standard form it can either be written in ordinary form or the cube root can be found by the following rule.

Rule: Make the exponent of 10 a multiple of three, and locate the number on the proper third of the K scale. Read the result on the D scale and multiply this result by 10 to an exponent which is ⅓ the former exponent of 10.

Examples: Find the cube root of 6.9×10^3. Place the hairline over 6.9 on the left third of the K scale and read 1.904 on the D scale. Thus the desired cube root is 1.904×10^1. Find the cube root of 4.85×10^7. Express the number as 48.5×10^6 and place the hairline of the indicator over 48.5 on the middle third of the K scale. Read 3.65 on the D scale. Thus the desired cube root is 3.65×10^2. or 365. Find the cube root of 1.33×10^{-4}. Express the number as 133×10^{-6} and place the hairline over 133 on the right third of the K scale. Read 5.10 on the D scale. The required cube root is 5.10×10^{-2}.

***Cubes:* To find the cube of a number, reverse the process for finding cube root. Locate the number on the D scale and read the cube of that number on the K scale.**

EXAMPLES:

(a) Find $(1.37)^3$. Set the indicator on 1.37 of the D scale. Read 2.57 on the K scale.

(b) Find $(13.7)^3$. The setting is the same as in example (a), but the K scale reading is 2570, or 1000 times the former reading.

(c) Find $(2.9)^3$ and $(29)^3$. When the indicator is on 2.9 of D, the K scale reading is 24.4. The result for 29^3 is therefore 24,400.

(d) Find $(6.3)^3$. When the indicator is on 6.3 of D, the K scale reading is 250.

PROBLEMS:	ANSWERS:
1. 2.45^3	14.7
2. 56.1^3	176,600
3. $.738^3$	.402
4. 164.5^3	4,451,000
5. $.0933^3$	.000812
6. $\sqrt[3]{5.3}$	1.744
7. $\sqrt[3]{71}$	4.14
8. $\sqrt[3]{815}$	9.34
9. $\sqrt[3]{.0315}$	.316
10. $\sqrt[3]{525,000}$	80.7
11. $\sqrt[3]{.156}$	.538

COMBINED OPERATIONS

Many problems involve expressions like $\sqrt{ab}$, or $(a \sin \theta)^2$, etc. With a little care, many such problems involving combined operations may be easily computed. The list of possibilities is extensive, and it is no real substitute for the thinking needed. to solve them. Consequently, only a few examples will be given.

The A and B scales may be used for multiplication or division in exactly the same way as the C and D scales. Since the scales are shorter, there is some loss in accuracy. Nevertheless, most computers employ the A and B scales (in conjunction with the C and D scales) to avoid extra steps which would also lead to loss of accuracy.

EXAMPLE:

(a) Find $\sqrt{3.25 \times 4.18}$. First find the product 3.25×4.18 using the left A and B scales. Set left index of B on 3.25 of A. Move indicator to 4.18 of B. Read the square root of this product under the hairline on D. The result is 3.69 approximately.

(b) Find 1.63×5.41^2. Set the left index on B under 1.63 of A. Move the indicator to 5.41 on C. Read the result 47.7 under the hairline on A.

(c) Find $5^{\frac{3}{2}}$ or $5^{1.5}$. This is the same as $(\sqrt{5})^3$. Hence set the indicator on 5 of the left A scale. Read 11.2 on the middle K scale under the hairline.

(d) Find $24^{\frac{2}{3}}$ or $(\sqrt[3]{24})^2$. Set the indicator on 24 of the middle K scale. Read the result after squaring as 8.3 on the A scale under the hairline.

The L Scale: LOGARITHMS

The L scale represents logarithms to the base 10, or common logarithms. The logarithm of a number is the exponent to which a given base must be raised to produce the number. For example, Log $10^2 = 2.00$; Log $10^3 = 3.00$, etc. A logarithm consists of two parts. The *characteristic* is the part on the left of the decimal point. The *mantissa* is the decimal fraction part on the right of the decimal point. The L scale is used for finding the mantissa of the logarithm (to the base 10) of any number. The mantissa of the logarithm is the same for any series of digits regardless of the location of the decimal point.

The position of the decimal point in the given number determines the characteristic of the logarithm, and conversely. The following rules apply in determining the characteristic.

1. For 1, and all numbers greater than 1, the characteristic is one less than the number of places to the left of the decimal point in the given number.
2. For numbers smaller than 1, that is for decimal fractions, the characteristic is negative. Its numerical value is one more than the number of zeros between the decimal point and the first significant figure in the given number.

The application of these rules is illustrated by the following chart:

Digits to Left of Decimal Point		Zeros to Right of Decimal Point*	
Digits in Number	1 2 3 4 5 6 7 8	Zeros in Number	0 1 2 3 4 5 6 7 8
Characteristic	0 1 2 3 4 5 6 7	Characteristic	-1-2-3-4-5-6-7-8-9

The Method described on page 10 is also easy to use.

*Note: Count only zeros between decimal point and first significant figure.

Rule: **Locate the number on the D scale (when L scale is on top or bottom stator), and read the mantissa of its logarithm (to the base 10) on the L scale. Determine the characteristic. If the L scale is on the slide, use the C scale instead of the D scale.**

EXAMPLE: Find the logarithm of 425.

Set the hairline over 425 on the D scale. Read the mantissa of the logarithm (.628) on the L scale. Since the number 425 has 3 digits, the characteristic is 2 and the logarithm is 2.628.

If the logarithm of a number is known, the number may be found by reversing the above process. The characteristic is ignored until the decimal point is to be placed in the number.

EXAMPLE: Find x, if $\log x = 3.248$.

Set the hairline over 248 on the L scale. Above it read the number 177 on the D scale. Since the characteristic is 3, there are 4 digits in the number, $x = 1770$.

EXAMPLE: Find the logarithm of .000627

Opposite 627 on the D scale find .797 on the L scale. Since the number has 3 zeros, the characteristic is —4 and the logarithm is —4 + .797 and is usually written 6.797—10.

Note that the mantissa of a logarithm is always positive but the characteristic may be either positive or negative. In computations, negative characteristics are troublesome and frequently are a source of error. It is customary to handle the difficulty by not actually combining the negative characteristic and positive mantissa. For example, if the characteristic is −4 and the mantissa is .797, the logarithm may be written 0.797 − 4. This same number may also be written 6.797 − 10, or 5.797 − 9. and in other ways as convenient. In each of these forms if the integral parts are combined, the result is −4. Thus 0 − 4 = −4; 6. − 10 = −4; 5. − 9 = −4. The form which shows that the number 10 is to be subtracted is the most common.

EXAMPLES:

Log 1 = 10.000 − 10
Log .4 = 9.602 − 10
Log .0004 = 6.602 − 10

PROBLEMS:	ANSWERS:
Log 3.26	.513
Log 735	2.866
Log .0194	8.288 − 10
Log 54800	4.739
Log X = 2.052	X = 112.7
Log X = 9.831 − 10	X = .678
Log X = .357	X = 2.28
Log X = 1.598	X = 39.6
Log X = 7.154 − 10	X = .001426

The S, T, and ST Scales: TRIGONOMETRY

The branch of mathematics called *trigonometry* arose historically in connection with the measurement of triangles. However, it now has many other uses in various scientific fields.

Some important formulas from trigonometry are listed here for ready reference.

The trigonometric ratios may be defined in terms of a right triangle as follows:

Sine of angle A $= \dfrac{\text{side opposite}}{\text{hypotenuse}}$ (written Sin A $= \dfrac{a}{h}$);

Cosine of angle A $= \dfrac{\text{side adjacent}}{\text{hypotenuse}}$ (written Cos A $= \dfrac{b}{h}$);

Tangent of angle A $= \dfrac{\text{side opposite}}{\text{side adjacent}}$ (written Tan A $= \dfrac{a}{b}$);

Cotangent of angle A $= \dfrac{\text{side adjacent}}{\text{side opposite}}$ (written Cot A $= \dfrac{b}{a}$);

Secant of angle A $= \dfrac{\text{hypotenuse}}{\text{side adjacent}}$ (written Sec A $= \dfrac{h}{b}$);

Cosecant of angle A $= \dfrac{\text{hypotenuse}}{\text{side opposite}}$ (written Cosec A $= \dfrac{h}{a}$).

These ratios are functions of the angle. The definitions may be extended to cover cases in which the angle A is not an interior angle of a right triangle, and hence may be greater than 90 degrees. Note that the sine and cosecant are reciprocals, as are the cosine and secant, and the tangent and cotangent. Therefore,

$$\text{Sin A} = \frac{1}{\text{Cosec A}}, \text{ and Cosec A} = \frac{1}{\text{Sin A}},$$

$$\text{Tan A} = \frac{1}{\text{Cot A}}, \text{ and Cot A} = \frac{1}{\text{Tan A}};$$

$$\text{Cos A} = \frac{1}{\text{Sec A}}, \text{ and Sec A} = \frac{1}{\text{Cos A}}.$$

When the sum of two angles equals 90°, the angles are *complementary.*

$$\begin{aligned} \text{Sin A} &= \cos(90° - \text{A}) \\ \text{Cos A} &= \sin(90° - \text{A}) \\ \text{Tan A} &= \cot(90° - \text{A}) \\ \text{Cot A} &= \tan(90° - \text{A}) \end{aligned}$$

When the sum of two angles equals 180°, the angles are *supplementary.*

$$\begin{aligned} \sin(180° - \text{A}) &= \text{Sin A} \\ \cos(180° - \text{A}) &= -\text{Cos A} \\ \tan(180° - \text{A}) &= -\text{Tan A} \end{aligned}$$

The following laws are applicable to any triangle.

$$\text{A} + \text{B} + \text{C} = 180°$$

$$\text{Law of sines: } \frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$$

$$\text{Law of cosines: } a^2 = b^2 + c^2 - 2bc \cos \text{A}$$

THE S SCALE: Sines and Cosines

The scale marked S is used in finding the approximate sine or cosine of any angle between 5.7 degrees and 90 degrees, Since sin x = cos $(90-x)$, the same graduations serve for both sines and cosines. Thus sin 6° = cos (90°−6°) = cos 84°. The numbers printed at the right of the longer graduations are read when sines are to be found. Those printed at the left are used when cosines are to be found. On the slide rule, angles are divided decimally instead of into minutes and seconds. Thus sin 12.7° is represented by the 7th small graduation to the right of the graduation marked 78|12.

Sines (or cosines) of all angles on the S scale have no digits or zeros—the decimal point is at the left of figures read from the C (or D) scale.

Rule: **To find the sine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read sines from left to right and the numbers to the right of the graduation are for sines). Read the sine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the sine can also be read on the D scale, and the mantissa of the logarithm of the sine (log sin) may be read on the L scale.**

Example:

(1) Find sin 15° 30′ and log sin 15°30′. Set left index of C scale over left index of D scale. Set hairline on 15.5° (i.e., 15°30′) on S scale. Read sin 15.5° = .267 on C or D scale. Read mantissa of log sin 15.5° = .427 on L scale. According to the rule for characteristics of logarithms, this would be 9.427−10.

Rule: **To find the cosine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read cosines from right to left and the numbers to the left of the graduation are for cosines). Read the cosine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the cosine can also be read on the D scale, and the mantissa of the cosine (log cos) may be read on the L scale.**

Example:

(1) Find cos 42° 15′ and log cos 42° 15′. Set left index of C scale over left index of D scale. Set hairline on 42.25° (i.e., 42° 15′) on S scale. Read cos 42.25° = .740 on C or D scale. Read mantissa of log cos 42.25° = .869 on L scale. According to rule for characteristics of logarithms, this would be 9.869−10.

Finding the Angle

If the value of trigonometric ratio is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value of the ratio is set on the C scale, and the angle itself read on the S scale.

Examples:

(a) Given sin x = .465, find x. Set indicator on 465 of C scale, read x = 27.7° on the S scale.

(b) Given cos x = .289, find x. Set indicator on 289 on C scale. Read x = 73.2° on the S scale.

PROBLEMS:	ANSWERS:
1. Sin 9.6°	.167
2. Sin 37.2°	.605
3. Sin 79.0°	.982
4. Cos 12.2°	.977
5. Cos 28.6°	.878
6. Cos 37.2°	.794
7. Cosec 15.8°	3.68
Note: Cosec Θ = $\frac{1}{\sin\Theta}$	
8. Sec. 19.3°	1.060
Note: Sec Θ = $\frac{1}{\cos\Theta}$	
9. Sin Θ = .1737	Θ = 10°
10. Sin Θ = .98	Θ = 78°
11. Sin Θ = .472	Θ = 28.2°
12. Cos Θ = .982	Θ = 10.8°
13. Cos Θ = .317	Θ = 71.5°
14. Cos Θ = .242	Θ = 76°

15. $\sec \Theta = 1.054$ ($\sec \Theta = \frac{1}{\cos \Theta}$)	$\Theta = 18.7°$
16. $\operatorname{cosec} \Theta = 1.765$ ($\operatorname{cosec} \Theta = \frac{1}{\sin \Theta}$)	$\Theta = 34.5°$
17. log sin 10.4°	9.256−10
18. log sin 24.2°	9.613−10
19. log cos 14.3°	9.986−10
20. log cos 39.7°	9.886−10
21. log sin Θ = 9.773−10	$\Theta = 36.4°$
22. log sin Θ = 9.985−10	$\Theta = 75.°$
23. log cos Θ = 9.321−10	$\Theta = 77.9°$
24. log cos Θ = 9.643−10	$\Theta = 63.9°$

THE T SCALE: Tangents and Cotangents

The T scale, together with the C or CI scales, is used to find the value of the tangent or cotangent of angles between 5.7° and 84.3°. Since $\tan x = \cot (90-x)$, the same graduations serve for both tangents and cotangents. For example, if the indicator is set on the graduation marked 60|30, the corresponding reading on the C scale is .577, the value of tan 30°. This is also the value of cot 60°, since $\tan 30° = \cot (90° - 30°) = \cot 60°$. Moreover, $\tan x = 1/\cot x$; in other words, the tangent and cotangent of the same angle are reciprocals. Thus for the same setting, the reciprocal of cot 60°, or 1/.577, may be read on the CI scale as 1.732. This is the value of tan 60. These relations lead to the following rule.

Rule: Set the angle value on the T scale and read
(i) tangents of angles from 5.7° to 45° on C,
(ii) tangents of angles from 45° to 84.3° on CI,
(iii) cotangents of angles from 45° to 84.3° on C
(iv) cotangents of angles from 5.7° to 45° on CI.

If the slide is set so that the C and D scales coincide, these values may also be read on the D scale. Care must be taken to note that the T scale readings for angles between 45° and 84.3° *increase from right to left*.

In case (i) above, the tangent ratios are all between 0.1 and 1.0; that is, the decimal point is at the left of the number as read from the C scale.

In case (ii), the tangents are greater than 1.0, and the decimal point is placed to the right of the first digit as read from the CI scale. For the cotangent ratios in cases (iii) and (iv) the situation is *reversed*. Cotangents for angles between 45° and 84.3° have the decimal point at the left of the number read from the C scale. For angles between 5.7° and 45° the cotangent is greater than 1 and the decimal point is to the right of the first digit read on the CI scale. These facts may be summarized as follows.

***Rule:* If the tangent or cotangent ratio is read from the C scale, the decimal point is at the *left* of the first digit read. If the value is read from the CI scale, it is at the *right* of the first digit read.**

EXAMPLES:

(a) Find tan x and cot x when $x = 9°50'$. First note that $50' = \frac{50}{60}$ of 1 degree = .83°, approximately. Hence 9° 50′ = 9.83°. Locate $x = 9.83°$ on the T scale. Read tan x = .173 on the C scale, and read cot x = 5.77 on the CI scale.

(b) Find tan x and cot x when $x = 68.6°$. Locate $x = 68.6°$ on the T scale reading from right to left. Read 255 on the CI scale. Since all angles greater than 45° have tangents greater than 1 (that is, have one digit as defined above), tan x = 2.55. Read cot 68.6° = .392 on the C scale.

Finding the Angle

If the value of the trigonometric ratio is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value of the ratio is set on the C or CI scale, and the angle itself read on the T scale.

EXAMPLES:

(a) Given tan x = .324, find x. Set 324 on the C scale, read 17.9° on the T scale.

(b) Given tan x = 2.66, find x. Set 266 on the CI scale, read x = 69.4° on the T scale.

(c) Given cot x = 630, find x. Set .630 on the C scale, read x = 57.8° on the T scale.

(d) Given cot x = 1.865, find x. Set 1865 on the CI scale, read 28.2° on the T scale.

PROBLEMS:	ANSWERS:
1. tan 18.6°	.337
2. tan 66.4°	2.29
3. cot 31.7°	1.619
4. cot 83.85°	.1078
5. tan Θ = 1.173	$\Theta = 49.55°$
6. cot Θ = .387	$\Theta = 68.84°$

THE ST SCALE: Small Angles

The sine and the tangent of angles of less than about 5.7° are so nearly equal that a single scale, marked ST, may be used for both. The graduation for 1° is marked with the degree symbol (°). To the left of it the primary graduations represent tenths of a degree. The graduation for 2° is just about in the center of the slide. The graduations for 1.5° and 2.5° are also numbered.

***Rule:* For small angles, set the indicator over the graduation for the angle on the ST scale, then read the value of the sine or tangent on the C scale. Sines or tangents of angles on the ST scale have one zero.**

EXAMPLES:

(a) Find sin 2° and tan 2°. Set the indicator on the graduation for 2° on the ST scale. Read sin 2° = .0349 on the C scale. This is also the value of tan 2° correct to three digits.

(b) Find sin 0.94° and tan 0.94°. Set the indicator on 0.94 of ST. Read sin 0.94° = tan 0.94° = .0164 on the C scale.

Since cot $x = 1/\tan x$, the cotangents of small angles may be read on the CI scale. Moreover, tangents of angles between 84.3° and 89.42° can be found by use of the relation $\tan x = \cot (90 - x)$. Thus cot 2° = 1/tan 2° = 28.6, and tan 88° = cot 2° = 28.6. Finally, it may be noted that $\csc x = 1/\sin x$, and $\sec x = 1/\cos x$. Hence the value of these ratios may be readily found if they are needed. Functions of angles greater than 90° may be converted to equivalent (except for sign) functions in the first quadrant.

EXAMPLES:

(a) Find cot 1.41° and tan 88.59°. Set indicator at 1.41° on ST. Read cot 1.41° = tan 88.59° = 40.7 on CI.

(b) Find csc 21.8° and sec 21.8°. Set indicator on 21.8° of the S scale. Read csc 21.8° = 1/sin 21.8° = 2.69 on CI. Set indicator on 68.2° of the S scale (or 21.8 reading from right to left), and read sec 21.8° = 1.077 on the CI scale.

When the angle is less than 0.57° the approximate value of the sine or tangent can be obtained directly from the C scale by the following procedure.

Read the ST scale as though the decimal point were at the left of the numbers printed, and read the C scale (or D, CI, etc.) with the decimal point one place to the left of where it would normally be. Thus sin 0.2° = 0.00349; tan 0.16° = 0.00279, read on the C scale.

Two seldom used special graduations are also placed on the ST scale. One is indicated by a longer graduation found just to the left of the graduation for 2° at about 1.97°. When this graduation is set opposite any number of minutes on the D scale, the sine (or the tangent) of an angle of that many minutes may be read on the D scale under the C index.

Sin 0° = 0, and sin 1′ = .00029, and for small angles the sine increases by .00029 for each increase of 1′ in the angle. Thus sin 2′ = .00058; sin 3.44′ = .00100, and the sines of all angles between 3.44′ and 34.4′ have two zeros. Sines of angles between 34.4′ and 344′ (or 5.73°) have one zero. The tangents of these small angles are very nearly equal to the sines.

EXAMPLE: Find sin 6′. With the hairline set the "minute graduation" opposite 6 located on the D scale. Read 175 on the D scale under the C index. Then sin 6′ = .00175.

The second special graduation is also indicated by a longer graduation located at about 1.18°. It is used in exactly the same way as the graduation for minutes. Sin 1″ = .0000048, approximately, and the sine increases by this amount for each increase of 1″ in the angle, reaching .00029 for sin 60″ or sin 1′ = .00029.

Trigonometric Computations

Many formulas involve both trigonometric ratios and other factors. By using several different scales such computations are easily done.

EXAMPLES:

(a) Find the length of the legs of a right triangle in which the hypotenuse is 48.3 ft. and one acute angle is 25° 20′.

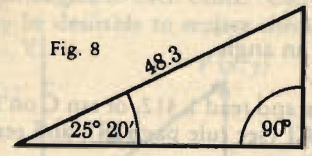


Fig. 8

The side opposite the given acute angle is equal to 48.3 sin 25° 20′. Hence we compute 48.3 × sin 25.3°. Set the index (right-hand index in this example) of the C scale on 48.3 of the D scale. Move the hairline over 25.3° on the S scale. Read 20.7 under the hairline on the D scale. Another method is to set the left index of the C scale and D scale opposite each other. Set the hairline over 25.3° on the S scale. Move the slide so that (right) index of the C scale is under the hairline. Read 20.7 on the D scale under 48.3 of the C scale. The length of the other leg is equal to 48.3 cos 25.3° or 48.3 sin 64.7° = 43.7.

(b) One angle of a right triangle is 68.3°, and the adjacent side is 18.6 ft. long. Find the other side and the hypotenuse.

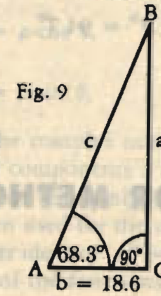


Fig. 9

a = 18.6 tan 68.3° or 18.6/cot 68.3
c = 18.6/cos 68.3°.

To find *a*, set the indicator on 18.6 of the D scale, pull the slide until 68.3 of the T scale (read from right to left) is under the hairline, and read *a* = 46.7 on the D scale under the right index of the C scale. To find c, pull the slide until 68.3° of the S scale (read from right to left) is under the hairline (which remains over 18.6), and read the result 50.3 on the D scale at the right index.

This problem may also be solved by the law of sines, namely,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}, \quad \text{or} \quad \frac{\sin 68.3}{a} = \frac{\sin 21.7}{18.6} = \frac{1}{c}$$

Set 21.7° on S opposite 18.6 on D. Read c = 50.3 on D under 1 of C. Move indicator to 68.3° on S, read 46.7 under the hairline on D.

(c) Find one side and two angles of an obtuse triangle when two sides and an included angle are known.

Given: c = 428; b = 537; A = 32.6°

Find: a, B, and C.

Construct: Line h from the vertex of B perpendicular to AC. This divides △ ABC into 2 right triangles.

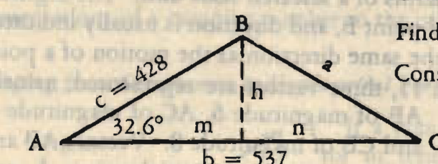


Fig. 10

Then h = c Sin A, from the formula for sine of an angle. Set right index of C over 428 on D and read h = 231 on D scale opposite 32.6° on S scale. Also,

$$m = \frac{h}{\text{Tan } A}, \text{ from formula for tangent of an angle.}$$

Opposite h (231) on D scale set 32.6° on T scale, and read m = 361 on D scale opposite right index on C scale. Since m is now known

$n = b - m = 176.$

$\text{Tan } C = \frac{h}{n}$, from formula for tangent of an angle.

Opposite 231 on D scale set 176 on C scale and read 1.312, or tan C on D opposite left index of C scale. Set 1.312 on CI (see rule page 28), and read C = 52.7° on T. Finally,

$a = \frac{h}{\text{Sin } C}$, from formula for sine of an angle.

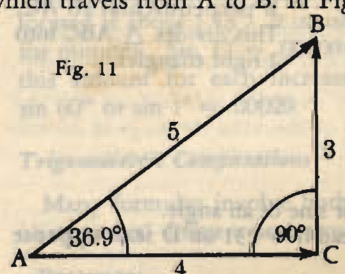
Set index of C over index of D and move hairline to 52.7° on S. Move right index of C under hairline. Move hairline to 231 on CI. Read a = 290 on DI.

Since A + B + C = 180°,
then B = 180° − (A+C) = 180° − 32.6° − 52.7° = 94.7°.

PART 3—ELEMENTARY VECTOR METHODS

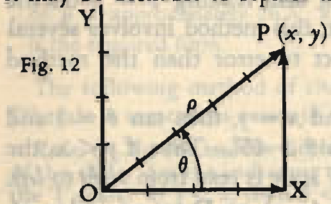
COMPLEX NUMBERS AND VECTORS

A *vector* quantity is one which has both *magnitude* and *direction.* For example, force and velocity are vector quantities. A quantity which has magnitude only is called a *scalar.* For example, mass is a scalar. Vector quantities are often represented by directed straight line segments. The length of the segment represents the magnitude in terms of a selected scale unit. The segment has an initial point A and a terminal point B, and direction is usually indicated by an arrowhead at B pointing in the same direction as the motion of a point which travels from A to B. In Fig. 11, three vectors are represented; namely AB of magnitude 5, AC of magnitude 4, and CB of magnitude 3. Vectors AB and AC have the same initial point, A, and form an angle, CAB, of 36.9°. The initial point of vector CB is at the terminal point of AC. Vectors CB and AB have the same terminal point.



Operations with vectors (for example, addition and multiplication) are performed according to special rules. Thus in Fig. 11, AB may be regarded as the *vector sum* of AC and CB. AB is called the *resultant* of AC and CB; the latter

are *components* of AB, and in this case are at right angles to each other. It is frequently desirable to express a given vector in terms of two such components at right angles to each other. Conversely, when the components are given, it may be desirable to replace them with the single resultant vector.



In algebra, the complex number $x + iy$, where $i = \sqrt{-1}$, is represented by a point P (x, y) in the complex plane, using a coordinate system in which an axis of "pure imaginary" numbers, OY, is at right angles to an axis of "real" numbers, OX.

The same point can be expressed in terms of polar coordinates (ρ, θ) in which the radius vector OP from the origin of coordinates has length ρ and makes an angle θ with the X-axis. The two systems of representation are related to each other by the following formulas:

(1) $x = \rho \cos \theta$, (3) $\tan \theta = \frac{y}{x}$ or $\theta = \arctan \frac{y}{x}$

(2) $y = \rho \sin \theta$, (4) $\rho = \sqrt{x^2 + y^2}$

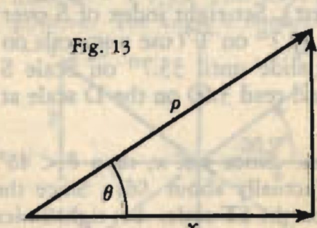
Finally, the complex number $x + iy$ may be regarded as a vector given in terms of its components x and y and the complex operator $i = \sqrt{-1}$. In practical work the symbol j is preferred to i, to avoid confusion with the symbol often used for the *current* in electricity.

The "Euler identity" $e^{j\theta} = \cos \theta + j \sin \theta$ can be proved by use of the series expansions of the functions involved. Then $\rho e^{j\theta}$ is an *exponential* representation of the complex number $x + jy$, since $\rho e^{j\theta} = \rho \cos \theta + j\rho \sin \theta = x + jy$. The notation is often simplified by writing $\rho \underline{/\theta}$ in place of $\rho e^{j\theta}$.

If two or more complex numbers are to be added or subtracted, it is convenient to have them expressed in the form $x + jy$, since if $N_1 = x_1 + jy_1$, and $N_2 = x_2 + jy_2$, then $N_1 + N_2 = (x_1 + x_2) + j(y_1 + y_2)$. If, however, two or more complex numbers are to be multiplied, it is convenient to have them expressed in the exponential form. Then if $N_1 = \rho_1 e^{j\theta_1}$ and $N_2 = \rho_2 e^{j\theta_2}$, then $N_1 N_2 = \rho_1 \rho_2 e^{j(\theta_1 + \theta_2)}$, or $(\rho_1 \underline{/\theta_1})(\rho_2 \underline{/\theta_2}) = \rho_1 \rho_2 \underline{/\theta_1 + \theta_2}$.

It is therefore necessary to be able to change readily from either of these representations of a complex number to the other.

Changing from Components to Exponential Form



If a complex number $x + jy$ (or vector in terms of perpendicular components) is given, the problem of changing to the form $\rho \underline{/\theta}$ is equivalent to finding the hypotenuse and one acute angle of a right triangle. The formulas $\tan \theta = \frac{y}{x}$ and $\rho = y/\sin \theta$, or $\rho = x/\cos \theta$, are the basis of the solution. Thus if N = 4 + j3, when 4 of C is set opposite 3 of D, the value of the ratio $\frac{y}{x}$, or $\frac{3}{4}$ = .75 is read on D under the C index.

If the indicator is set at the index, and the slide moved so that .75 is under the hairline, the value of $\theta = 36.9°$ may be read on the T scale. Then $\rho = 3/\sin 36.9$ may be computed by moving the indicator to 3 on the D scale, pulling 36.9 on the S scale under the hairline, and reading $\rho = 5$ on the D scale opposite the right index of C. However, this method involves several unnecessary settings and is thus more subject to error than the method given in the general rule below.

Observe that if x and y are both positive and $x = y$, then $\tan\theta = 1$ and $\theta = 45°$. If $y < x$, then $\theta < 45°$; if $y > x$ then $\theta > 45°$. Thus if $y < x$, the T scale is read from *left* to *right*. If $y > x$, the T scale is read from *right* to *left*.

Rule: **(i) To the *larger* of the two numbers (x, y) on D set an index of the slide. Set the indicator over the smaller value on D and read θ on the T scale. If $y < x$, then $\theta < 45$. If $y > x$, then $\theta > 45$, and is read from right to left (or on the left of the graduation mark).**

(ii) Move the slide until θ on scale S is under the indicator, reading S on the same side of the graduation as in (i). Read ρ on D at the index of the C-scale.

Observe that the reading both begins and ends at an index of the slide. By this method the value of the ratio y/x occurs on the C (or CI) scale of the *slide* over the smaller of the two numbers, and the angle may be read immediately on the T scale without moving the slide. In using any method or rule, it is wise to keep a mental picture of the right triangle in mind in order to know whether to read θ on the T or on the ST scale. Thus if the ratio y/x is a small number, the angle θ is a small angle, and must be read on the ST scale. To be precise, if $y/x < 0.1$, the ST scale must be used. Similarly, if the ratio $y/x > 10$, the angle θ will be larger than 84.3° and cannot be read on the T scale. The complementary angle $\varphi = (90 - \theta)$ will, however, then be on the ST scale, and then θ may be found by subtracting the reading on the ST scale from 90°, since $\theta = 90 - \varphi$.

EXAMPLES:

(a) Change $2 + j3.46$ to exponential or "vector" form. Note $\theta > 45$, since $y > x$ (or $3.46 > 2$). Set right index of S opposite 3.46 on D. Move indicator to 2 on D. Read $\theta = 60°$ on T at the *left* of the hairline. Move slide until 60° on scale S is under the hairline (numerals on the *left*), and read $\rho = 4$ on the D scale at the C-index. Then $2 + j3.46 = 4\rho e^{j60} = 4\,\underline{/60°}$.

(b) Change $3 + j2$ to exponential or vector form. Note that $\theta < 45°$ since $y < x$ (second component less than first). Set right index of S over 3 on D. Move indicator to 2 on D, read $\theta = 33.7°$ on T (use numerals on the right-hand side of graduations). Move slide until 33.7° on Scale S is under the hairline (numerals on right), and read 3.60 on the D scale at the C index. Hence $3 + j2 = 3.60\ \underline{/33.7}$.

(c) Change $2.34 + j.14$ to exponential form. Since $y < x$, then $\theta < 45°$. Moreover, the ratio y/x is a small number (actually about .06). Since the tangent has one zero, the angle may be read on the ST scale. Set right index of S opposite 2.34 of D. Move indicator to .14 on D. Read $\theta = 3.43°$ on ST. The slide need not be moved. The value of ρ is approximately 2.34. In other words, the angle is so small that the hypotenuse is approximately equal to the longer side. Then $2.34 + j.14 = 2.34\ \underline{/3.43}$.

(d) Change $1.08 + j26.5$ to exponential form. Here $y > x$, so that $\theta > 45°$. But $\frac{y}{x} = \frac{26.5}{1.08} > 10$. Set right index of S on 26.5 of D. Move indicator to 1.08 of D. Read $\varphi = 2.34°$ on ST. The slide need not be moved. The value of ρ is approximately 26.5; $\theta = 90 - 2.34° = 87.66°$. Hence $26.5\ \underline{/87.66°}$ is the required form.

The following method of changing $x + jy$ to the form $\rho\ \underline{/\theta}$ using the DI scale is sometimes easier to use than methods based on the D scale.

Rule: **(i) To the *smaller* of the two numbers (x, y) on DI set an index of the slide. Set the indicator over the larger value on DI and read θ on the T scale. If $y < x$, then $\theta < 45°$. If $y > x$, then $\theta > 45°$ and is read from right to left (or on the left of the graduation mark).**

(ii) Move the indicator over θ on scale S (or ST), reading S on the same side of the graduation as in (i). Read ρ on DI under the hairline.

EXAMPLES:

(a) Change $2 + j\,3.46$ to exponential form. Note that $y > x$ since $3.46 > 2$, and hence $\theta > 45°$. Set right index of C over 2 on DI. Move indicator to 3.46 on DI. Read $\theta = 60°$ on T. Move indicator to 60° on S. Read $\rho = 4$ on DI. Hence $2 + j\,3.46 = 4\underline{/60°}$.

(b) Change $114 + j\,20$ to exponential form. Here $y < x$, so $\theta < 45°$. Set left index of C over 20 on DI. Move indicator to 1.14 on DI. Read $\theta = 9.95°$ on T. Move hairline to 9.95° on S. Read $\rho = 116$ on DI. Hence $114 + j\,20 = 116\ \underline{/9.95°}$.

It will be observed that this rule is, in general, easy to use. In step (i) the value of $\tan\theta$ for $\theta < 45°$ may be observed under the hairline on the C scale, and the value of $\tan\theta$ for $\theta > 45°$ under the hairline on CI.

It may be noted that the rule given first (using the D scale) obtains the result in example (b) above without having the slide project far to the right. Thus, it appears that the relative advantages of the two methods depend in part upon the problem.

If x and y are both positive, $\theta < 90°$. If x and y are *not* both positive, the resultant vector does not lie in the first quadrant, and θ is not an acute angle. In using the slide rule, however, x and y must be treated as both positive. It is therefore necessary to correct θ as is done in trigonometry when an angle is not in the first quadrant.

EXAMPLES:

(a) Find the angle between the X-axis and the radius vector for the complex number $-4 + j3$. First solve the problem as though both components were positive. The angle θ obtained is 36.9°. In this case the required angle is $180° - \theta = 180° - 36.9° = 143.1°$. Hence $-4 + j3 = 5\ \underline{/143.1°}$. Similarly for $-4 - j3$, the required angle is $180 + \theta = 180 + 36.9° = 216.9°$, so $-4 - j3 = 5\ \underline{/216.9°}$. For $4 - j3$ the required angle is $360° - \theta = 323.1°$, so $4 - j3 = 5\ \underline{/323.1°}$, which may also be expressed in terms of a negative angle as $5\ \underline{/-36.9°}$.

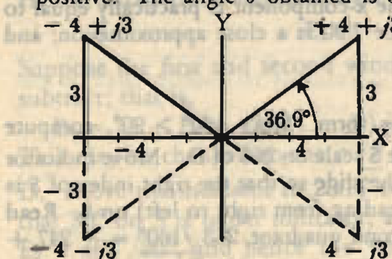


Fig. 14

(b) Change $17.2 - j6.54$ to exponential form. Here the ratio y/x is negative so θ can be expressed as a negative angle. In numerical value $y < x$, so the numerical or absolute value of $\theta < 45°$. Set left index of S opposite 17.2 on D. Move indicator over 6.54 of D, read $\theta = 20.8°$ on T. Pull 20.8 of S under hairline, read 18.4 on D at left index. Hence $17.2 - j6.54 = 18.4 \underline{/-20.8°}$, or $18.4 \underline{/339.2°}$.

Changing from Exponential Form to Components

The process of changing a complex number or vector from the form $\rho e^{j\theta} = \rho \underline{/\theta}$ to the form $x + jy$ depends upon the formulas $x = \rho \cos \theta$, $y = \rho \sin \theta$. These are simple multiplications using the C, D, and S (or ST) scales.

Rule: **Set an index of the S scale opposite ρ on the D scale. Move indicator to θ on the S (or ST) scale, reading from left to right (sines). Read y on the D scale. Moving indicator to θ on the S (or ST) scale, reading from right to left (cosines), read x on the D scale.**

If $\theta > 90°$ or $\theta < 0$, it should first be converted to the first quadrant, and the proper negative signs must later be associated with x or y.

EXAMPLES:

(a) Change $4 \underline{/60°}$ to component form. Set right index of S on 4 of D. Move indicator to 60° on S (reading scale from left to right). Read 3.46 on D under hairline. Move indicator to 60° on S, reading scale from right to left (cosines). Read 2 on D under hairline. Hence $4 \underline{/60°} = 2 + j3.46$.

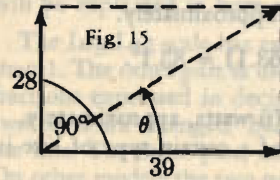
(b) Change $16.3 \underline{/15.4°}$ to the $x + jy$ form. Set left index of S on 16.3 of D. Move indicator to 15.4 of S, read 4.33 on D. Since 15.4° reading from right to left is off the D scale, exchange indices so the right index of C is opposite 16.3 of D. Move indicator to 15.4 of S, and read 15.7 on D. Hence $16.3 \underline{/15.4°} = 15.7 + j4.33$.

(c) Change $7.91 \underline{/3.25°}$ to component form. Set right index of S on 7.91 of D. Move indicator to 3.25 on ST. Read 0.448 on D. To determine the decimal point, observe that the angle is small, and hence the y component will also be small. Obviously, when the hypotenuse is near 8, 4.48 would be too large, and 0.0448 too small, to produce an angle of 3.25°. The cosine cannot be set on ST, but the angle is so small that the x-component is practically equal to the radius vector or hypotenuse. Hence 7.90 is a close approximation; and $7.91 \underline{/3.25°} = 7.90 + j0.448$.

(d) Convert $263 \underline{/160°}$ to the $x + jy$ form. Since $160° > 90°$, compute $180° - 160° = 20°$. Set left index of the S scale on 263 of D. Move indicator to 20° on S. Read 90.0 on D. Move the slide so that the right index of S is on 263 of D. Move indicator to 20 (reading from right to left) on S. Read 247 on D. Since the angle is in the second quadrant, $263 \underline{/160°} = -247 + j90$.

ILLUSTRATIVE APPLIED PROBLEMS

1. Two forces of magnitude 28 units and 39 units act on the same body but at right angles to each other. Find the magnitude and angle of the resultant force.



In complex number notation, the resultant is $39 + j28$. Change this to exponential form. Since $28 < 39$, then $\theta < 45°$. Set the right index of S on 39 of D. Move indicator to 28 of D. Read $\theta = 35.6°$ on T. Move slide so 35.6° on S is under the hairline. Read $\rho = 48.0$ on D under the S-index. Hence the resultant has magnitude 48 units, and acts in a direction 35.6° from the larger force and $90 - 35.6°$ or 54.4° from the smaller force. This angle can be read on the S scale under the hairline.

2. A certain alternating generator has three windings on its armature. In each winding the induced voltage is 266.4 volts effective. The windings are connected in such a way that the voltages in each are given by the following vector expressions.

$$E_1 = 266.4 (\cos 0° - j \sin 0°)$$

$$E_2 = 266.4 (\cos 120° - j \sin 120°)$$
$$= 266.4 \cos 120° - j\, 266.4 \sin 120°$$

$$E_3 = 266.4 (\cos 240° - j \sin 240°)$$
$$= 266.4 \cos 240° - j\, 266.4 \sin 240°$$

Express these numerically.

$$E_1 = 266.4 (1 - j0) = 266.4 - j0$$

To find E_2, reduce the angles to first quadrant by taking $180° - 120° = 60°$. Set the right index of S on 266.4 of D. Move the indicator to 60° of S (reading right to left). Read 133.2 on D. Move indicator to 60° on S, read 230.7 on D. Then

$$E_2 = -133.2 - j230.7$$

To find E_3, reduce 240° to the first quadrant by noting $240° = 180° + 60°$. Hence, except for a negative sign, E_3 is the same as E_2, and

$$E_3 = -133.2 + j230.7$$

Suppose the first and second windings are so connected that their voltages subtract; that is,

$$E_0 = E_1 - E_2 = (266.4 - j0) - (-133.2 - j230.7) = 399.6 + j230.7$$

This may be changed to the $\rho \underline{/\theta}$ form. Set the right index of S on 399.6 of D. Move the indicator to 230.7 of D. Read $\theta = 30°$ on T. Move slide so that 30° on S is under indicator, and read 461 on D at the S-index. Then $E_0 = 461 \underline{/30°}$, and hence the voltage is 461 volts and leads the voltage E_1 by 30°.

3. An alternating voltage of $104 + j60$ is impressed on a circuit such that the resulting current is $24 - j32$. Find the power and power factor. First convert each vector to exponential form.

$$E = 104 + j60 = 120\ \underline{/30^\circ} \text{ volts, approximately}$$

$$I = 24 - j32 = 40\ \underline{/-53.1} \text{ amperes, approximately.}$$

Hence the voltage leads the current by $30^\circ - (-53.1) = 83.1^\circ$.
The power factor $\cos 83.1^\circ = 0.120$.
The power $P = EI \cos \theta = (120)(40)(0.120) = 576$ watts, approximately.

4. The "characteristic impedance" of a section of a certain type of line is given by the formula $Z_0 = \sqrt{Z_1Z_2 + \frac{Z_1^2}{4}}$, where in each case, the symbol Z represents a vector quantity. Compute Z_0 when

$$Z_1 = 40 + j120,\ Z_2 = 220 - j110.$$

First convert to exponential form.

$$Z_1 = 40 + j120 = 126\ \underline{/71.6^\circ}$$

$$Z_2 = 220 - j110 = 246\ \underline{/-26.6^\circ}$$

Hence

$$Z_1Z_2 = (126)(246)\ \underline{/71.6 - 26.6} = 31{,}000\ \underline{/45.0^\circ}$$

$$\frac{Z_1^2}{4} = \frac{126^2}{4}\ \underline{/2(71.6)} = \frac{15{,}900}{4}\ \underline{/143.2} = 3{,}975\ \underline{/143.2}$$

$$Z = \sqrt{31{,}000\ \underline{/45.0^\circ} + 3{,}975\ \underline{/143.2}}$$

Since vectors are to be added before the square root is found, it is now convenient to convert them to component form.

$$31{,}000\ \underline{/45.0^\circ} = 21{,}900 + j21{,}900$$

$$3{,}975\ \underline{/143^\circ} = -3{,}180 + j2{,}390$$

To compute the latter, take $180^\circ - 143^\circ = 37^\circ$, compute the components using 37°, and observe that the x or real component must be negative since 143° is an angle in the second quadrant. Then

$$Z = \sqrt{(21{,}900 - 3180) + j(21{,}900 + 2390)}$$

$$= \sqrt{18{,}720 + j24{,}290}$$

In order to find the square root, it is convenient to change back to exponential form.

$$Z = \sqrt{18{,}720 + j24{,}290} = \sqrt{30{,}600\ \underline{/52.4^\circ}}$$

$$= 175\ \underline{/26.2^\circ} \text{ ohms.}$$

The final result is obtained by setting 30,600 on A and reading 175 on D; the angle 52.4° is merely divided by 2. This problem shows the value of being able to change readily from one form of vector representation to the other.

PART 4. USE OF LOG LOG SCALES

To find the value of 1.3^7, $5.6^{3.21}$, $\sqrt[5]{38}$, $\sqrt[3.5]{84}$, and many other types of expressions, Log Log scales are used. The method of computing such expressions will be explained in later sections. First the Log Log scales will be described.

The Log Log scale has two main parts. One part is used for numbers greater than 1. The other part is used for numbers between 0 and 1; that is, for proper fractions expressed in decimal form. On some models of the slide rule these two parts are arranged "back to back." One part, indicated by LL1+, is above the line. The other part, indicated by LL1−, is under the line. (See Fig. 16). On other models the two parts are separated or have different designations.

Fig. 16

Slide rules differ as to the range of the Log Log scales provided, and in the arrangement or position of the scales on the body of the rule. The discussion that follows will therefore have separate sections that apply to different slide rule models. There are two types of scale arrangements to be considered:

Standard Type — found on Models 501, 600 and 800.

Traditional Type — found on Models 300 and 500.

The owner of any particular model may omit the discussions that do not apply to his rule.

READING THE SCALES

Numbers greater than 1:

On an ordinary logarithmic scale, such as the D scale, any particular graduation represents many different numbers. Thus the graduation labeled 2 represents not only 2, but also 20, 200, .2, .02, etc. In contrast, any graduation on a Log Log scale represents only one number. The principal graduations are labeled with a number in which the decimal point is shown.

Standard Type: (Models 501, 600, 800)

The scales marked LL1+, LL2+, and LL3+ are the first, second, and third parts of one continuous scale. It begins at the left end of the scale marked LL1+. The index graduation is marked 1.01. Set the indicator on this mark and move the hairline slowly to the right, noting the graduations marked 1.02, 1.03, etc. to 1.10. The scale now continues on LL2+ through 1.11, 1.15, etc., and ends with e=2.718, the base of the Napierian system of logarithms. The LL3+ scale begins at e, continues through 3, 4, 5, etc., up to about 22,026.

Traditional Type: (Models 300 and 500)

For numbers greater than 1, the scales of Traditional Type rules are identical in arrangement with those of Standard Type. Hence no specific discussion of this type is needed until numbers less than 1 are to be considered. The discussion for Standard Type should be read.

All Types:

There is no difficulty in reading the principal graduations since they are labeled and the decimal point is shown. Between the principal graduations the intervals are subdivided in several different ways. Thus the graduations between the numbers shown do not have the same meaning on all sections of the scale. Also, the graduations are not the same on 6 inch slide rules as they are on 10 inch slide rules. To the beginner, this variation in the meaning of the scale divisions is often confusing. However, as one gains familiarity with the instrument, the proper reading usually may be obtained at a glance. The basic scheme is the same as that used in sub-dividing ordinary logarithmic scales, such as C and D.

(1) To locate a number, look first for the nearest smaller number that appears on the scale.

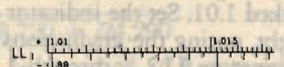
(2) Second, observe the major subdivisions between the nearest smaller number and the one following it. Sometimes there are 10, at other times 5, and at still other times only 2 or 3 major parts of the interval.

The general idea used in reading the scales may be stated informally as follows: Starting with the smaller printed number, decide how you must "count" the major graduation marks to come out correctly at the larger printed number.

(3) Third, in most cases there are still other or minor subdivisions between the major ones. These minor subdivisions divide the major intervals into 10 sub-parts, 5 sub-parts, or 2 sub-parts. Use the slide rule and check the location of the numbers in the table below.

NUMBER		SCALE Standard and Traditional Type
1.01278	is between 1.01 and 1.015 on	LL1+
1.173	is between 1.15 and 1.2 on	LL2+
4.78	is between 4 and 5 on	LL3+
1.054	is between 1.05 and 1.06 on	LL1+
1.862	is between 1.8 and 1.9 on	LL2+
25.6	is between 20 and 30 on	LL3+

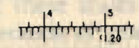
Examples:
(a) *Count by "ones" (thousandths)*
Then by "twos" (tenths of thousandths)



6 inch rule
5 major sub-parts
5 minor sub-parts

In this example there are only 5 minor sub-parts. Each represents one-fifth of the interval. The graduations are read in order as follows: 1.01, 1.0102, 1.0104, 1.0106, 1.0108, 1.0110, 1.0112, etc.

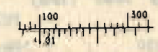
(b) *Count by "twos" (tenths)*



6 inch rule
5 major sub-parts
No minor sub-parts

In this example there are no minor subdivisions. However, one more digit may be read by estimation. Thus the number 4.65 may be set by placing the hairline about one-fourth of the distance from 4.6 to 4.8.

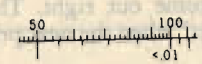
(c) *Count by "hundreds" (hundred)*
Then by "twos" (twenty)



6 inch rule
2 major sub-parts
5 minor sub-parts

In this example the minor subdivisions represent fifths. The numbers represented are 100, 120, 140, 160, 180, 200, 220, etc.

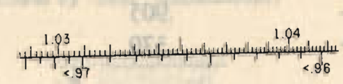
(d) *Count by "tens" (ten)*
Then by "twos" (two)



10 inch rule
5 major sub-parts
5 minor sub-parts

In this example each of the five minor subdivisions represents a *fifth* of the major interval. Hence, starting at 50 the graduations represent 52, 54, 56, 58, 60, 62, etc. The odd numbers may be set by estimation.

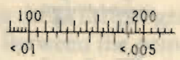
(e) *Count by "ones" (thousandths)*
Then by "twos" (tenths of thousandths)



10 inch rule
10 major sub-parts
5 minor sub-parts

In this example the minor subdivisions represent fifths of the major interval. Hence, the numbers represented are 1.0302, 1.0304, 1.0306, 1.0308, 1.0310, 1.0312, etc.

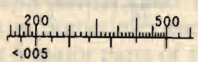
(f) *Count by "tens" (ten)*
Then by "fives" (five)



10 inch rule
10 major sub-parts
2 minor sub-parts

In this example the minor subdivisions represent halves of the major subinterval. The readings in order are 100, 105, 110, 115, etc.

(g) *Count by "hundreds" (hundred)*
Then by "tens" (ten)



10 inch rule
3 major sub-parts
10 minor sub-parts

In this example the minor subdivisions represent tenths of the major subintervals. The graduations read in order represent 200, 210, 220, 230, etc.

The general idea is the same in all cases. It is necessary to decide how the marks must be "counted" to come out right. That is, if the "counting" is properly done, it "comes out right" when the next principal graduation (labelled with a number) is reached.

Numbers from 0 to 1

Standard Type: (Models 501, 600, and 800.)

The scales marked LL1–, LL2–, and LL3– are the first, second, and third sections of one continuous scale. Note that these scales *decrease* from left to right and hence *increase from right to left.* The ranges of these scales are approximately as follows:

SCALE	LEFT INDEX	RIGHT INDEX
LL1–	.990	.905
LL2–	.905	.370
LL3–	.370	.00005

Traditional Type: (Model 500)

The scales labeled LL0 and LL00 are parts of a continuous Log Log scale. The ranges are approximately as follows:

SCALE	LEFT INDEX	RIGHT INDEX
LL0	.999	.905
LL00	.905	.00005

All Types:

The methods of subdividing these scales are the same as those used for numbers greater than 1. The methods of reading the scales are also the same. Use the slide rule to check the location of the numbers in the table below. As before, look first for the nearest smaller number at a principal graduation mark.

NUMBER	SCALE Standard Type	SCALE Traditional Type
.984 between .98 and .99 on		LL0
.984 between .98 and .985 on	LL1–	
.813 between .80 and .85 on		LL00
.813 between .80 and .82 on	LL2–	
.231 between .20 and .25 on	LL3–	LL00
.026 between .01 and .05 on	LL3–	LL00

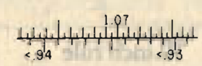
The examples below show how the scales may be read.

(h) *Count by "ones" (thousandths)*
Then by "fives" (tenths of thousandths)

10 inch rule—standard type
5 major sub-parts
2 minor sub-parts

In this example there are only 2 minor sub-parts. Each represents one-half of the interval. The graduations read in order from right to left represent .9800, .9805, .9810, .9815, .9820, .9825, etc.

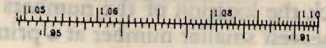
(i) *Count by "ones" (thousandths)*



10 inch rule
10 major sub-parts
No minor sub-parts

In this example there are no minor sub-parts. The graduations read in order from right to left represent .931, .932, .933, .934, .935, .936, etc.

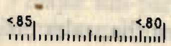
(j) *Count by "ones" (hundredths)*
Then by "ones" again (thousandths)



6 inch rule
4 major sub-parts
10 minor sub-parts

In this example there are 10 minor sub-parts. Each represents one-tenth of the interval. The graduations read in order from right to left represent .910, .911, .912, .913, .914, .915, .916, etc.

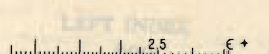
(k) *Count by "ones" (hundredths)*
Then by "twos" (thousandths)



10 inch rule—traditional type
5 major sub-parts
5 minor sub-parts

In this example there are 5 minor sub-parts. Each represents one-fifth of the interval. The graduations read in order from right to left represent .800, .802, .806, .808, .810, .812, etc.

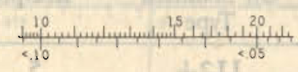
(l) *Count by "ones" (hundredths)*
Then by "fives" (thousandths)



10 inch rule
5 major sub-parts
2 minor sub-parts

In this example there are 2 minor sub-parts. Each represents one-half of the interval. The graduations read in order from right to left represent .400, .405, .410, .415, .420, .425, etc.

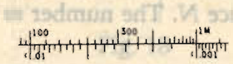
(m) *Count by "ones" (hundredths)*
Then by "fives" (thousandths)



6 and 10 inch rules
5 major sub-parts
2 minor sub-parts

In this example there are 2 minor sub-parts. Each represents one-half of the interval. The graduations read in order from right to left represent .050, .055, .060, .065, .070, .075, etc.

(n) *Count by "ones" (thousandths)*
Then by "fives" (tenths of thousandths)



6 inch rule
10 major sub-parts
2 minor sub-parts

In this example there are 2 minor sub-parts. Each represents one-half of the interval. The graduations read in order from right to left represent .0010, .0015, .0020, .0025, .0030, etc.

FUNDAMENTAL RELATIONSHIPS

On slide rules which have scale arrangements of Standard Type, there is a reciprocal relationship between numbers set on the Log Log scales. This relationship plays an important role in the use of these models of the slide rule.

Fig. 17

Check the readings in the table below on the slide rule. Observe that the symbols for the scales have been chosen to emphasize this relationship.

Ex.	Number	Scale On Standard Type	Reciprocal	Scale On Standard Type
(a)	2	LL2+	.5	LL2–
(b)	5	LL3+	.20	LL3–
(c)	1.25	LL2+	.80	LL2–
(d)	1.0131	LL1+	.9871	LL1–
(e)	52	LL3+	.0192	LL3–

This relationship does not exist for Log Log scales on Traditional Type slide rules.

It should be recalled that, in general, a number *N* may be represented by the form b^m. In this form the number *b* is called the *base* and the number *m* is called the *exponent*. The number *N* is called the *power*. In this discussion the number *b* will always be greater than 0 and not equal to 1, (i.e., $b>o$ and $b\neq 1$).

By definition, the logarithm of a number *N* to the base *b* is the exponent that must be given to *b* to produce *N*. The number *m* in the expression above is also the logarithm.

Exponential Form	*Logarithmic Form*
$N=b^m$	$\log_b N=m$

Although the Log Log scales of a slide rule have important uses in connection with the exponential form, it will be convenient to consider first their use in finding logarithms.

In general, if N represents a number under the indicator hairline on a Log Log scale, the logarithm of N will be under the hairline of an ordinary logarithmic scale, such as the D scale. The choice of the base *b* and the appropriate ordinary logarithmic scale is, however, affected by the type of scale arrangement available on the slide rule.

In scientific work the most convenient base is often the number *e* (approximately 2.718). Slide rule scales are therefore arranged to favor this base. When the base is *e* the logarithms are called natural, hyperbolic, or Napierian. Common logarithms have the number 10 as the base.

FINDING LOGARITHMS, BASE *e*.

Standard Type: (Models 501, 600, 800.) When logarithms to base *e* are to be found the following rule applies.

Rule (a): Position. When the indicator is set over any number N on a Log Log scale, the numerical value of the natural logarithm may be read under the hairline on the D scale, and conversely.

(b) *Sign:* If the number is greater than 1 (set on LL1+, LL2+, or LL3+) the logarithm is positive.

If the number is less than 1 (set on LL1–, LL2–, LL3–) the logarithm is negative.

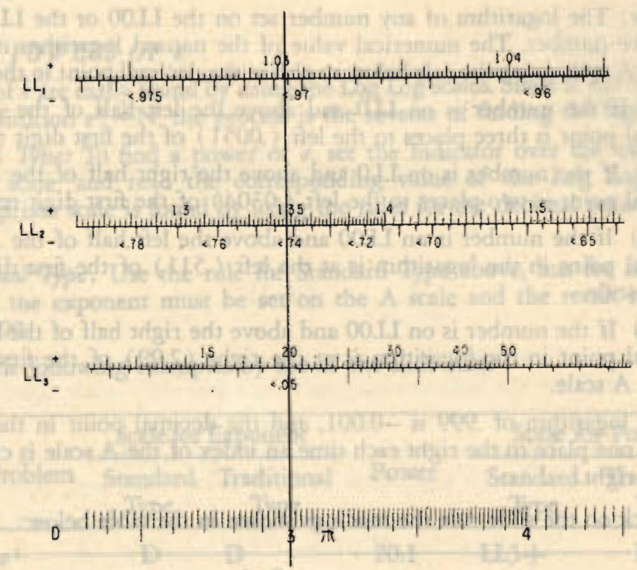


Fig. 18

(c) *Decimal Point.* The following cases must be carefully noted.

(i) If the number is on LL3±, the decimal point of the logarithm is at the right of the first digit read on D.

(ii) If the number is on LL2±, the decimal point of the logarithm is moved one place to the left (i.e., is at the left) of the first digit read on D.

(iii) If the number is on LL1±, the decimal point of the logarithm is moved two places to the left of the first digit read on D; or in other words, the logarithm has one zero to the left of the first digit. In each case the logarithm of the right index figure is governed by the rule for the scale on which it also appears as the left index. For example, *e* appears as the left index of LL3+ and also as the right index of LL2+. It is governed by the rule for LL3+.

For small values of X, the relation $\log_e (1+X) = X$ is approximately true. Thus $\log_e 1.01 = .01$, approximately. If this fact is remembered the rule for the LL1+ scale may be recalled. As one proceeds to LL2+ the decimal point in the logarithm moves one place to the right, and similarly as one proceeds from LL2+ to LL3+ the decimal point in the logarithm moves one place to the right.

Traditional Type: (Models 300 and 500) The scale arrangement for this type provides scales LL1, LL2, and LL3 like the scales LL1+, LL2+, and LL3+, of Standard Type. For numbers greater than 1, the rule for Standard Type applies. (See Above.)

Numbers less than 1 are set on the scales LL0 and LL00 and *the numerical value of the natural logarithm is read on the A scale.* To place the decimal point in the logarithm observe the following:

Rule: The logarithm of any number set on the LL00 or the LL0 scale is a negative number. The numerical value of the natural logarithm may be read on the A scale using the rule below to obtain the decimal point in the logarithm.

(i) If the number is on LL0 and above the left half of the A scale, the decimal point is three places to the left (.0051) of the first digit read.

(ii) If the number is on LL0 and above the right half of the A scale, the decimal point is two places to the left (.0304) of the first digit read.

(iii) If the number is on LL00 and above the left half of the A scale, the decimal point in the logarithm is at the left (.511) of the first digit read on the A scale.

(iv) If the number is on LL00 and above the right half of the A scale, the decimal point in the logarithm is at the right (2.99) of the first digit read on the A scale.

The logarithm of .999 is –0.001, and the decimal point in the logarithm moves one place to the right each time an index of the A scale is crossed from left to right.

Check on the slide rule the readings shown in the table below:

Ex.	Number	Scale: Standard	Scale: Traditional	$\log_e N$
(a)	4	LL3+	LL3	1.386
(b)	1.15	LL2+	LL2	0.1398
(c)	1.02	LL1+	LL1	0.0198
(d)	30	LL3+	LL3	3.4
(e)	1.405	LL2+	LL2	0.34
(f)	1.0346	LL1+	LL1	0.034
(g)	0.05	LL3–	LL00	–3.00
(h)	0.63	LL2–	LL00	–0.462
(i)	0.946	LL1–	LL0	–0.0555
(j)	0.9964	Use .9964-1	LL0	–0.0036

FINDING LOGARITHMS, ANY BASE

Logarithms to any base a may be found by the formula $\log_a N = (\log_e N) \div (\log_e a)$. For common logarithms $a = 10$, and $\log_e 10 = 2.303$. Since $1/2.303 = .4343$, the formula becomes

$$\log_{10} N = .4343 \log_e N.$$

Standard Types: On rules with the Standard Type scale arrangement, $\log_e N$ may be located on the D scale and multiplied by .4343 by the use of the C scale.

Traditional Type: On slide rules of this type logarithms to any base are found by the method described above for rules of the Standard Type except that for numbers less than 1 the A and B scales must be used instead of the C and D scales.

FINDING POWERS OF e

Powers of e are easily found by using the Log Log scales. Since if $m = \log_e N$, then by definition $e^m = N$, the process is the reverse of finding the logarithm.

Standard Type: To find a power of e, set the indicator over the exponent on the D scale, and read the corresponding value of the Log Log scale. The appropriate scale is found by using the rules for the decimal point in the logarithm.

Traditional Type: Use the rule for Standard Type above, but for negative exponents, the exponent must be set on the A scale and the result read on LL0 or LL00.

Verify the following examples by use of the slide rule.

Ex.	Problem	Scale for Exponent: Standard Type	Scale for Exponent: Traditional Type	Power	Scale for Power: Standard Type	Scale for Power: Traditional Type
(a)	e^{3}	D	D	20.1	LL3+	LL3
(b)	$e^{0.25}$	D	D	1.284	LL2+	LL2
(c)	$e^{0.081}$	D	D	1.0844	LL1+	LL1
(d)	$e^{-2.0}$	D	A (Right)	0.135	LL3–	LL00
(e)	$e^{-0.20}$	D	A (Left)	0.819	LL2–	LL00
(f)	$e^{-0.020}$	D	A (Right)	0.9802	LL1–	LL0
(g)	$e^{-0.002}$		A (Left)	0.998	1. ÷ 1.002	LL0
(h)	e^{5}	D	D	148.	LL3+	LL3
(i)	$e^{8.35}$	D	D	4230.	LL3+	LL3
(j)	$e^{-3.4}$	D	A (Right)	0.0334	LL3–	LL00

FINDING POWERS OF ANY BASE

The Log Log scales may be used to find any power of any base. Since roots may be expressed by exponents that are fractions in decimal form, the Log Log scales may also be used to find any root of a positive number. These statements are, of course, subject to certain restrictions which are of minor importance in practical work.

The problem is to compute $N = b^m$ when b and m are known numbers. The general method is given by the following rule.

Rule: To find b^m, when $m > 0$ set the index of an ordinary logarithmic scale on the slide (C, CF,) opposite b on a Log Log scale. Move the indicator to m of the ordinary logarithmic scale, and read b^m under the hairline on the Log Log scale. (On slide rules of Traditional Type, the B scale must be used with numbers set on LL0 and LL00.)

Rule: The logarithm of any number set on the LL00 or the LL0 scale is a negative number. The numerical value of the natural logarithm may be read on the A scale using the rule below to obtain the decimal point in the logarithm.

(i) If the number is on LL0 and above the left half of the A scale, the decimal point is three places to the left (.0051) of the first digit read.

(ii) If the number is on LL0 and above the right half of the A scale, the decimal point is two places to the left (.0304) of the first digit read.

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The problem is to compute $N = b^m$ when b and m are known numbers. The general method is given by the following rule.

Rule: To find b^m, when $m > 0$ set the index of an ordinary logarithmic scale on the slide (C, CF,) opposite b on a Log Log scale. Move the indicator to m of the ordinary logarithmic scale, and read b^m under the hairline on the Log Log scale. (On slide rules of Traditional Type, the B scale must be used with numbers set on LL0 and LL00.)

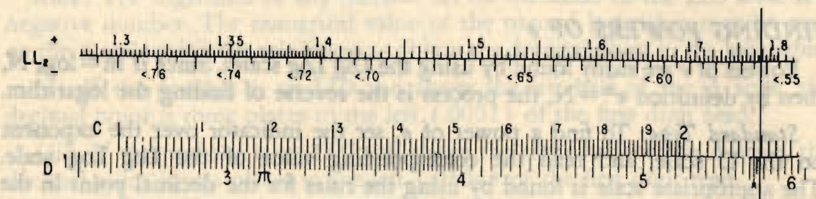


Fig. 19

Example: To find $N=1.3^{2.2}$, set the index of the C scale over 1.3 of the Log Log scale. Move the indicator hairline over 2.2 of the C scale. Read 1.78 on the Log Log scale.

The rule is based on mathematical theory which may be illustrated as follows: Given $N=1.3^{2.2}$, take logarithms of both sides. Then $\log_a N=2.2 \log_a 1.3$.

To compute the right member, take logarithms of both sides again. Then

$$\log_{10} \log_a N=\log_{10} 2.2+\log_{10} \log_a 1.3.$$

The graduation at 1.3 of a Log Log scale of the slide rule represents a length from the left index that is proportional to $\log_{10} \log_a 1.3$. The graduation at 2.2 of the C scale represents a length that is proportional to $\log_{10} 2.2$. The lengths are added on the slide rule. The graduation on the Log Log scale at the sum represents N.

Since the *method* is simple, the main difficulty is to decide on which Log Log scale to read the result. The following principles will be helpful:

1) Remember that the Log Log scales for numbers greater than 1 are really sections of a single continuous scale. They could be arranged end-to-end on a long slide rule, with repeated lengths of D scale opposite them.

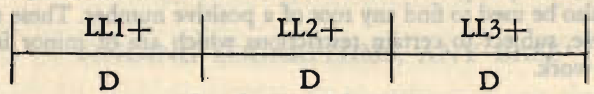


Fig. 20

Similarly, the Log Log scales for numbers less than 1 are really sections of a single scale. They also could be arranged end-to-end on a long slide rule. On rules of the Traditional Type, repeated lengths of B scale could be arranged opposite them.

2) Think of the scales arranged end-to-end as above. If the exponent m is greater than 1, then $N=b^m$ would be to the right of b. If the exponent m is positive but less than 1, then $N=b^m$ would be to the left of b.

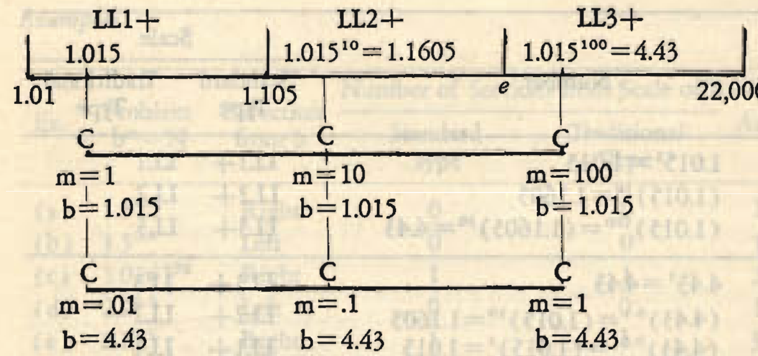


Fig. 21

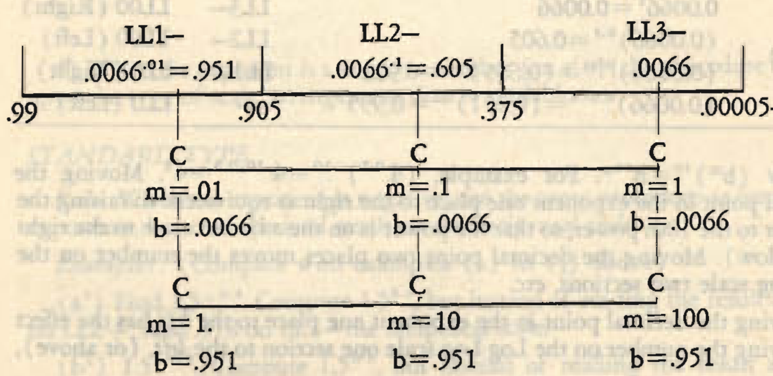


Fig. 22

3) Think of the scales arranged end-to-end as above. For any setting of the indicator hairline, the reading on any section of the Log Log scale is the 10th power of the reading on the adjacent section at its left. That is, *moving one scale section-length to the right has the effect of raising the number to the 10th power.* Conversely, moving one section-length to the left has the effect of taking the one-tenth power of the number.

Ex.	Number	Scale: Standard Type	Scale: Traditional Type
	$1.015^1 = 1.015$	LL1+	LL1
(a)	$(1.015)^{10} = 1.1605$	LL2+	LL2
	$(1.015)^{100} = (1.1605)^{10} = 4.43$	LL3+	LL3
	$4.43^1 = 4.43$	LL3+	LL3
(a')	$(4.43)^{0.1} = (1.015)^{10} = 1.1605$	LL2+	LL2
	$(4.43)^{0.01} = (1.015)^1 = 1.015$	LL1+	LL1
	$0.995^1 = 0.995$	—	LL0 (Left)
(b)	$(0.995)^{10} = 0.951$	LL1–	LL0 (Right)
	$(0.995)^{100} = (0.951)^{10} = 0.605$	LL2–	LL00 (Left)
	$(0.995)^{1000} = (0.951)^{100} = 0.0066$	LL3–	LL00 (Right)
	$0.0066^1 = 0.0066$	LL3–	LL00 (Right)
(b')	$(0.0066)^{0.1} = 0.605$	LL2–	LL00 (Left)
	$(0.0066)^{0.01} = (0.605)^{0.1} = 0.951$	LL1–	LL0 (Right)
	$(0.0066)^{0.001} = (0.951)^{0.1} = 0.995$	—	LL0 (Left)

Now $(b^m)^{10} = b^{10m}$. For example, $(4^{0.5})^{10} = 4^{10 \times 0.5} = 4^5$. Moving the decimal point in the exponent one place to the right is equivalent to raising the number to the 10th power, so that the power is on the adjacent scale to the right (or below). Moving the decimal point two places moves the number on the Log Log scale two sections, etc.

Moving the decimal point in the exponent one place to the *left* has the effect of moving the number on the Log Log scale one section to the *left*, (or above), etc.

Rule: Imagine the decimal point is at the right of the first significant digit of the exponent, and decide on which scale b^m would then be found. Then look to the "right" or "left" as many sections as are needed to adjust the decimal point to its true position.

Examples: (a) Find $(1.03)^{200}$. Set the left index of the C scale opposite (1.03) of the Log Log scale. Move the indicator over 2 on the C scale. Observe that $(1.03)^2 = 1.0609$ is on the same section, and that $(1.03)^{20} = 1.806$ is on the adjacent or next section of the Log Log scale. Finally, $(1.03)^{200} = 370$ is on the second section beyond $(1.03)^2$.

(b) Find $4^{0.05}$. Set the index of the C scale opposite 4 of the Log Log scale. Move the indicator over 5 on the C scale. Observe that 4^5 would be to the right of 4 at 1024, and $4^{0.5}$ is to the left of 4. In fact, $4^{0.5} = 4^{\frac{1}{2}} = \sqrt{4} = 2$. Then $4^{0.05}$ is one scale length farther to the left, and is 1.0718 on the next scale.

Examples:

Ex.	Problem $b^m = N$	Direction from b	Number of Sections from Scale of b: Standard Type	Number of Sections from Scale of b: Traditional Type	Answer N
(a)	$1.5^{2.4}$	Right	0	0	2.646
(b)	$1.5^{0.5}$	Left	0	0	1.225
(c)	1.0267^{32}	Right	1	1	2.32
(d)	$2.2^{0.3}$	Left	0	0	1.267
(e)	$2.2^{1.5}$	Right	1	1	3.26
(f)	$0.5^{1.2}$	Right	0	0	0.435
(g)	0.4^2	Right	1	0	0.16
(h)	$0.88^{0.25}$	Left	1	1	0.9685
(i)	$0.5^{0.05}$	Left	1	1	0.9659
(j)	$0.2^{0.008}$	Left	2	1	0.9872

4) When the exponent is a negative number $(m < o)$, the procedure depends upon the type of scale arrangement that is available.

STANDARD TYPE

Rule. When the exponent is negative, use the same procedure as for positive exponents, but read the final result on the *reciprocal scale.*

Examples: (Compare with examples (a) to (j) above)

(a¹) Find $1.5^{-2.4}$. Compute $1.5^{2.4}$, but instead of reading the result as 2.646 on LL2+, read .378 on LL2– the reciprocal scale.

(b¹) $1.5^{-0.5}$. Compute $1.5^{0.5}$, but instead of reading the result as 1.225 on LL2+, read 0.8165 on LL2– the reciprocal scale. Fig. 23

(f¹) Find $0.5^{-1.2}$. Compute $0.5^{1.2}$ as in example (f) above, but instead of reading 0.435 on LL2–, read the result 2.3 on LL2+ the reciprocal scale.

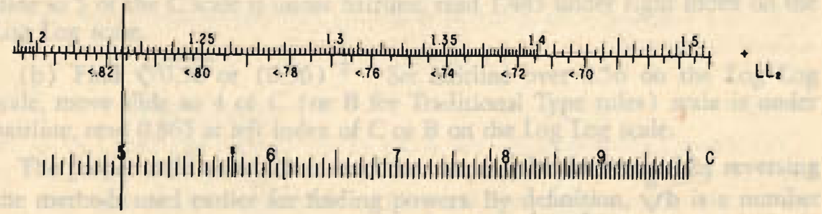


Fig. 23

TRADITIONAL TYPE

Rule. When the exponent is negative, as indicated by b^{-m}, either of two procedures may be used to convert the problem to one with a positive exponent. Since $b^{-m} = (b^{-1})^m, = (1/b)^m$, the reciprocal of b may be found by using the C and CI scales, and the *m*th power of the number may then be found. Of course 1/b may be divided by use of the C and D scales if this method is preferred.

Since $b^{-m} = (b^m)^{-1} = 1/b^m$, the value of b^m may be found first, and then the reciprocal of this number computed by either method suggested above.

The choice between these two procedures depends upon the value of b. If $b < 1$, then $(1/b) > 1$, and the first method will employ the scales LL1, LL2, or LL3, which provide greater accuracy but less range than the LL0 and LL00 scales. If $b > 1$, then $(1/b) < 1$. In this case the second method is preferable. In short, by proper choice of method the use of the LL0 and LL00 scales with negative exponents may be avoided.

PROBLEMS	ANSWERS
1. $4.3^{5.21}$	2000
2. $16.3^{0.107}$	1.348
3. $2.23^{0.073}$	1.060
4. $.325^{5.6}$	0.0018
5. $.734^{0.058}$	0.9822
6. $1.075^{-4.5}$	0.7223
7. $8.5^{-0.107}$	0.795

ROOTS, AND COMMON FRACTIONAL EXPONENTS

There are two methods of finding roots

Rule: To find $\sqrt[m]{b}$ or $b^{\frac{1}{m}}$ set the index of C scale on b on the Log Log scale, move indicator to m on CI, read result on Log Log scale under hairline.

This method uses the theory of exponents to express a root by using a fractional exponent (e.g. = 1/m). This fraction can be divided out and the result used as an exponent as described under finding powers. Thus $\sqrt[4]{3} = (3)^{1/4} = 3^{.25}$. However, the CI scale does the division automatically, since it gives reciprocals of numbers on the C scale.

Moreover, in some applied problems the formulas being used express the exponent as a common fraction, and it is then more convenient to use the CI scale.

On rules of the Traditional Type, if the base b is less than 1, the CI scale cannot be used. The exponent may, however, be found by ordinary division and then set on the B scale.

Examples:

(a) Find $\sqrt[4.2]{8.5}$ or $(8.5)^{\frac{1}{4.2}}$. Set left index of C scale over 8.5 on the Log Log scale. Move hairline to 4.2 on CI scale, read 1.664 on the Log Log scale under hairline.

(b) Find $\sqrt[.03]{0.964}$ or $(0.964)^{\frac{1}{.03}} = (0.964)^{33.3}$. On slide rules of the Standard Type, set the index of the C scale opposite 0.964 of the Log Log scale. Move the indicator over 3 on CI. Since 1/.03 = 33, approximately, the result is one scale section to the right of the one on which 0.964 is set. Read the result as 0.295. On rules of the Traditional Type, set the right index of B scale on 0.964 on LL0. Move hairline to 33.3 on B. The result in this problem is under the hairline on LL00, and is 0.295, approximately.

A second procedure treats roots as the inverse of powers.

Rule: To find $\sqrt[m]{b}$, or $b^{\frac{1}{m}}$ set hairline over *b* on a Log Log scale, pull *m* on the C scale under the hairline, read the result on the Log Log scale at the index. On rules of the Traditional Type, the B scale must be used when $b < 1$

Examples:

(a) Find $\sqrt[5]{6.3}$ or $(6.3)^{\frac{1}{5}}$. Set hairline over 6.3 on the Log Log scale, move slide so 5 of the C scale is under hairline, read 1.445 under right index on the Log Log scale.

(b) Find $\sqrt[4]{0.56}$ or $(0.56)^{\frac{1}{4}}$ Set hairline over 0.56 on the Log Log scale, move slide so 4 of C (or B for Traditional Type rules) scale is under hairline, read 0.865 at left index of C or B on the Log Log scale.

The proper scale on which to read the root may be determined by reversing the methods used earlier for finding powers. By definition, $\sqrt[m]{b}$ is a number which raised to the *m* power produces *b*, that is $(\sqrt[m]{b})^m = b$. Suppose $b > 1$, and $m > 1$; then $\sqrt[m]{b} < b$ and $\sqrt[m]{b}$ would be to the left of *b*, if the Log Log scales were on one continuous line. In example (a) above, $\sqrt[5]{6.3}$ or 1.445 is less than 6.3, and $1.445^5 = 6.3$. Although the reading on the Log Log scale, or 1.0375, is also less than 6.3, it is the 50th root of 6.3, or $\sqrt[5c]{6.3}$. On the other hand, the value of $\sqrt[0.5]{6.3}$ is to the right of 6.3 at about 40; observe that $\sqrt[0.5]{6.3}$ or $6.3^{1/.5} = 6.3^2$ is about 40.

Examples:

(a) Find $y = \sqrt[400]{100}$. Set 4 of the C scale on 100 of the Log Log scale and move indicator to the C index. Note that the 4th root would be about 3, the 40th root about 1.12 and the 400th root must be 1.0116.

(b) Find $\sqrt[50]{0.05}$ or $0.05^{\frac{1}{50}}$. On slide rules of the Standard Types, pull 5 on the C scale over 0.05 on the Log Log scale. At the index of the C scale read 0.9418 two scale sections to the left (above). On rules of the Traditional Type, set 5 on the B scale opposite 0.05 on LL00. Above the right index of B read 0.9418, which would be two sections (or cycles) of the B scale to the left if the LL0 and LL00 scales were end-to-end.

PROBLEMS	ANSWERS
1. $6.5^{\frac{1}{1.51}}$	3.45
2. $3400^{\frac{1}{75}}$	1.114
3. $1.606^{\frac{1}{21.5}}$	1.0223
4. $7.4^{\frac{53.6}{27.9}}$	47
5. $1.357^{\frac{4.21}{7.36}}$	1.191
6. $1.0411^{\frac{74.5}{9.8}}$	1.381

SOLVING EXPONENTIAL EQUATIONS:

The method of solving equations of the type $b^m = N$, where b and N are known and m is unknown, is very similar to the process of finding $b^m = N$ when m is known and N unknown. (See Finding Powers, above.)

The rule below holds for the Standard Type, and also for the Traditional Type when $b > 1$.

Rule: Set the index of the C scale (or CF scale) on b. Move the hairline to N on a Log Log scale. Read m under the hairline on the C scale (or CF scale, if it was used).

Examples:

(a) Solve $1.37^m = 8.43$. Set the index of the C scale opposite 1.37 of the Log Log scale. Move the hairline to 8.43 on the Log Log scale. Read 6.77 on the C scale under the hairline. It should be observed that 8.43 is greater than 1.37 and is to the right of 1.37 on the Log Log scale. Hence the exponent m must be larger than 1. The exponent 67.7 would obviously be too large, and it follows that the decimal point must be to the right of the 6 as in 6.77

(b) Solve $(0.75)^x = 0.872$. On Standard Type, set the index of the C scale opposite 0.75 of the Log Log scale. Move the hairline to .872 on the Log Log scale. Read .476 on the C scale under the hairline. Observe that 0.872 is to the left of 0.75, so the exponent is less than 1. On standard type rules, 0.872 is on the same scale as 0.75, but the right index of C must be used. Hence the decimal point must be at the left of the 4.

On Traditional Type slide rules, the LL00 and B scales are used. Set the middle index of B opposite 0.75 on LL00. At 0.872 on LL00 read 0.476 on B. Here the middle index of B is used as a right index, so the decimal point must be to the left of the 4.

(c) Solve for y if $(0.94)^y = 2.37$.

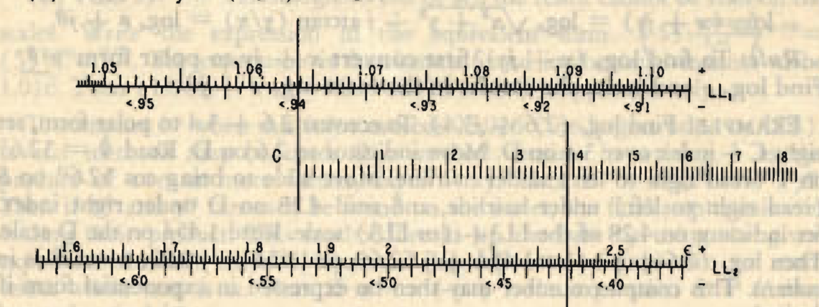


Fig. 24

Standard Types: Set the left index of the C scale opposite 0.94 on the Log Log scale. Move the indicator to 2.37 on the Log Log scale. Read 13.9 on the C scale. (The use of a folded C scale, such as CF, is convenient in this example). Observe that in this example 0.94 is less than 1 and 2.37 is greater than 1. Hence the exponent y must be a negative number. The reciprocal of 0.94 is about 1.064, and 2.37 is one scale section to the right. Hence $y = -13.9$.

Traditional Type: Use the C and CI scales to find $1/0.94 = 1.064$. Solve $(1.064)^y = 2.37$ as described above.

(d) Solve for p if $(5.27)^p = 0.818$.

Standard Types: Set the index of the C scale at 5.27 of the Log Log scale. Move the indicator to 0.818 on the Log Log scale. Read 1209 on the C scale. Note that $5.27 > 1$ and $0.818 < 1$, so the exponent is a negative number. The reciprocal of 0.818 is about 1.222, and since this is less than 5.27 the numerical value of the exponent must be less than 1. Hence $p = -0.1209$.

Traditional Type: Use C and CI to change 0.818 to 1.222 and proceed as above.

PROBLEMS	ANSWERS
1. $4^x = 3.75$	$x = 0.953$
2. $.963^x = 0.823$	$x = 5.17$
3. $5.25^x = 1.0141$	$x = 0.00844$
4. $2.11^x = 11{,}000$	$x = 12.46$
5. $3.04^x = 0.85$	$x = -.146$
6 $1.475^x = 0.015$	$x = -10.8$

The logarithm of a complex number $z = x + jy$ is a complex number. Let $\log_e (x + jy) = u + jv$. Then
$x + jy = e^{u+jv} = e^u . e^{jv} = e^u (\cos v + j \sin v) = e^u \cos v + j\, e^u \sin v$.
Equating the real and then the imaginary parts gives two equations

$$x = e^u \cos v$$
$$y = e^u \sin v$$

which may be solved for u and v. By division, $\tan v = y/x$, and hence $v = \arctan (y/x)$. Squaring and adding, $x^2 + y^2 = e^{2u}$, and hence $u = \log_e \sqrt{x^2 + y^2}$. Then

$$\log_e (x + jy) = \log_e \sqrt{x^2 + y^2} + j \arctan (y/x) = \log_e \rho + j\theta.$$

Rule: **To find $\log_e (x + jy)$, first convert $x + jy$ to polar form $\rho/\underline{\theta}$. Find $\log_e \rho$ and write the results in the form $\log_e \rho + j\theta$.**

EXAMPLE: Find $\log_e (2.6 + j3.4)$. To convert 2.6 + 3.4 to polar form, set right C – index over 3.4 on D. Move indicator to 2.6 on D. Read $\theta = 52.6°$ on T (read right to left) under hairline. Move slide to bring cos 52.6° on S (read right to left) under hairline, and read 4.28 on D under right index. Set indicator on 4.28 of the LL3+ (or LL3) scale. Read 1.454 on the D scale. Then $\log_e (2.6 + j\,3.4) = 1.454 + j\,52.6°$, or $1.454 + j\,0.92$, when θ is in radians. This complex number may then be expressed in exponential form if desired.

Occasionally there is need to compute an expression which involves values not on the scales. To compute b^m for b less than 1.01, note that by the binomial expansion $(1+xy)^{\frac{m}{y}} = 1+mx+ \ldots$, and if xy is sufficiently small, these first two terms will give a good approximation.

Examples:

(a) Find $(1.0004)^{2.7}$. Since 1.0004 cannot be set on the scales, compute $1+(2.7)(.0004) = 1.00108$, approximately.

(b) Find $53.^{0.00008}$. Although 53 can be set, the result cannot be read on the scales. Write the expression in the equivalent form $\left(53^{\frac{0.00008}{0.02}}\right)^{0.02} = (53^{0.004})^{0.02}$. The expression in brackets is found in the usual manner to be 1.016. Then $(1.016)^{0.02} = 1+0.02\times0.016 = 1.0003$, approximately.

To compute b^m when this value exceeds 22,026 (the largest value on LL3), several methods may be used. One method consists in writing b as the product of two or more factors, say $b = xy$. Then $b^m = (xy)^m = x^m.\ y^m$. Then if x^m and y^m can each be found on the scales, the final step consists in finding their product. This method breaks up the base by using the given exponent. Another method separates the exponent m into two or more terms whose sum is m. Thus, if $m = r + s + t$, the expression $b^r\ b^s\ b^t$ may be used to find b^m

Examples:

(a) Find $x = 19.3^6$. Let $5\ y = 19.3$. Then $y = 3.86$. Hence $x = 5^6 \times 3.86^6 = 19.3^6$. Then $5^6 = 15{,}600$, approximately, or 1.56×10^4, and $3.86^6 = 3300$ or 3.3×10^3. Then $x = 1.56 \times 3.3 \times 10^7$ or 5.15×10^7 or 51,500,000. It is, however, easier to get this result by logarithms directly.

(b) Find $x = 19.3^6$. Consider this as $(19.3)^3\ (19.3)^3$. Then $19.3^3 = 7200$, approximately, and 7200×7200 is about 5.18×10^7.

ILLUSTRATIVE APPLIED PROBLEMS

1. A volume of 1.2 cu. ft. of air at 60° F (or 520° absolute) and atmospheric pressure (14.7 lbs./sq. in.), is compressed adiabatically to a pressure of 70 lbs./sq. in. What is the final volume and final temperature?

(a) Compute: $V = 1.2\left(\frac{14.7}{70}\right)^{\frac{1}{1.4}}$ Ans. 0.394 cu. ft.

Standard Type: Set 70 of C over 14.7 on D. At the right index of C read 0.21 on D. Set right index of C over .21 on LL3— using hairline. Under 1.4 on CI read 0.328 on LL3—. Set 0.328 on D and multiply by 1.2 using the CI or C scale. Read final answer 0.394 on D.

Traditional Type: Set 70 of C over 14.7 on D. At the right index of C read 0.21 on D. Set middle index of B under 0.21 on LL00 using hairline. Over 0.714 (reciprocal of $\frac{1}{1.4}$) on B read 0.328 on LL00. Set 0.328 on D and multiply by 1.2 using the CI or C scale. Read final answer 0.394 on D.

(b) Compute: $T = 520\left(\frac{70}{14.7}\right)^{\frac{0.4}{1.4}}$ Ans. 812° Absolute or 352°F.

Standard and Traditional Type: Set right index of CI opposite 70 on D. Move hairline to 14.7 on CI and read 70 ÷ 14.7 = 4.76 under hairline on D. Set result 4.76 under hairline on LL3+. Move left index of C under hairline and move hairline to 0.4 on C. Set right index of C under hairline and read 1.562 on LL2+ directly opposite 1.4 on the CI scale. Multiply 1.562 by 520 on the C-D scales and obtain final temperature of 812 degrees absolute or 812° − 460 = 352° F.

2. Find the compound amount on an investment of $1200 at 3½% compounded annually for 20 years. The formula is $A = P\ (1+i)^n$, or, in this example, $A = 1200\ (1.035)^{20}$. Set index of the C scale on 1.035 on the LL1 scale. Move hairline over 20 on the C scale, read 1.99 on the LL2 scale. Multiply this by 1200, obtaining $2390, approximately.

3. In how many years does money double itself at 4.2% compounded annually? This problem requires finding n in the expression $(1.042)^n = 2$. Set the index of the C scale over 1.042 on LL1 scale, move the hairline over 2 on the LL2 scale, read 17 years, approximately, on the C scale under the hairline.

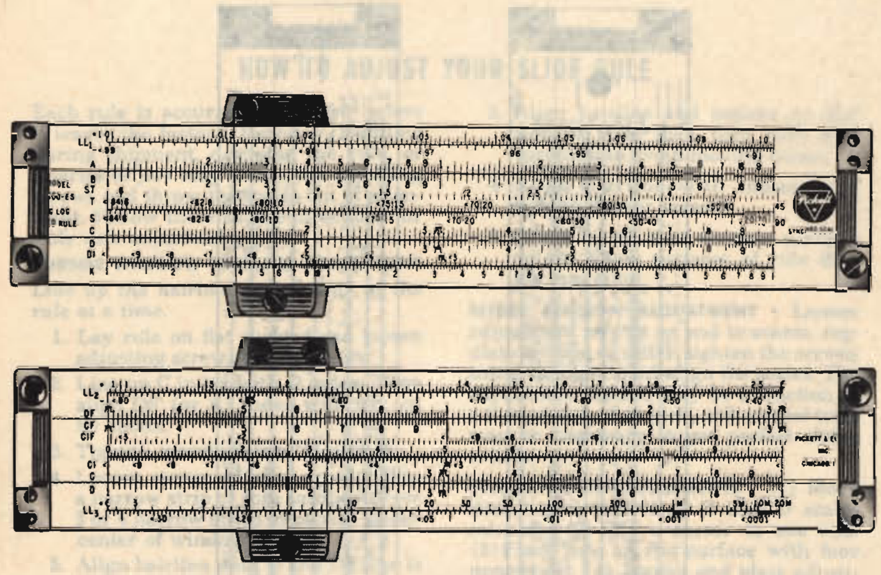
EXERCISES FOR PRACTICE

(Answers by slide rule on page 62).

1. 143×0.387	2. 29.8×4.87	3. $.0642 \times 80.6$
4. $18.9 \times 132. \times .0481$	5. $79.1 \times 3.62 \times 5.55$	6. $0.0427 \times 91.4 \times 169$
7. $832 \div 6.41$	8. $16.35 \div 8.02$	9. $40.7 \div 13.3$
10. $\frac{643 \times 8.12}{5.91}$	11. $\frac{11.95 \times 9.12}{3.40 \times .0202}$	12. $\frac{7.75 \times .0414}{1.91 \times 6.83}$
13. $\frac{7}{9} = \frac{3.2}{x}$	14. $\frac{2.81}{6.02} = \frac{x}{8.11}$	15. $\frac{132}{x} = \frac{65}{18}$
16. $1 \div 3.43$	17. $1/9.07$	18. $1/.055$
19. $\sqrt{8.34}$	20. $\sqrt{.0344}$	21. $\sqrt{54800}$
22. $(1.81)^2$	23. $(60.7)^2$	24. $(5.9)^2 \times 13.$
25. $\sqrt[3]{2.48}$	26. $\sqrt[3]{24.8}$	27. $\sqrt[3]{248}$
28. $(1.63)^3$	29. $(3.825)^3$	30. $(58.3)^3$
31. $\sin 22.5°$	32. $1.57 \times \sin 3.48°$	33. $96.2 \sin 72°$
34. $\cos 66°$	35. $\tan 19.2°$	36. $\tan 69.7°$
37. $\sqrt{18.3 \sin 26.2°}$	38. $\sqrt{\sin 36.1°/37.6}$	39. $\sqrt[3]{\tan 16.1°}$
40. $\frac{\sin 13.5°}{18} = \frac{\sin 32.4°}{x}$	41. $\frac{\sin 21.4°}{98.0} = \frac{\sin 6°}{x}$	42. $\frac{24.7 \sin 28.2°}{\sin 42.7°}$
43. $(1.352)^4$	44. $(2.81)^{3.81}$	45. $(1.031)^{.2}$
46. $(.693)^{2.4}$	47. $(.237)^{.03}$	48. $(.9943)^{324}$
49. $e^{1.2}$	50. $e^{0.3}$	51. $e^{.003}$
52. $2.2^{-3.5}$	53. $(1.054)^{-.7}$	54. $(.523)^{-3.76}$
55. $e^{-3.6}$	56. $e^{-.416}$	57. $e^{-.082}$
58. $(1.084)^{\frac{2.7}{1.6}}$	59. $(42.2)^{\frac{3.14}{7.08}}$	60. $(1.009)^{\frac{8.26}{3.47}}$
61. $\log 1.00291$	62. $\log 1.0448$	63. $\log 1.414$
64. $\log .99652$	65. $\log .863$	66. $\log .0042$
67. $\log_e 1.00291$	68. $\log_e 1.06$	69. $\log_e 2.5$

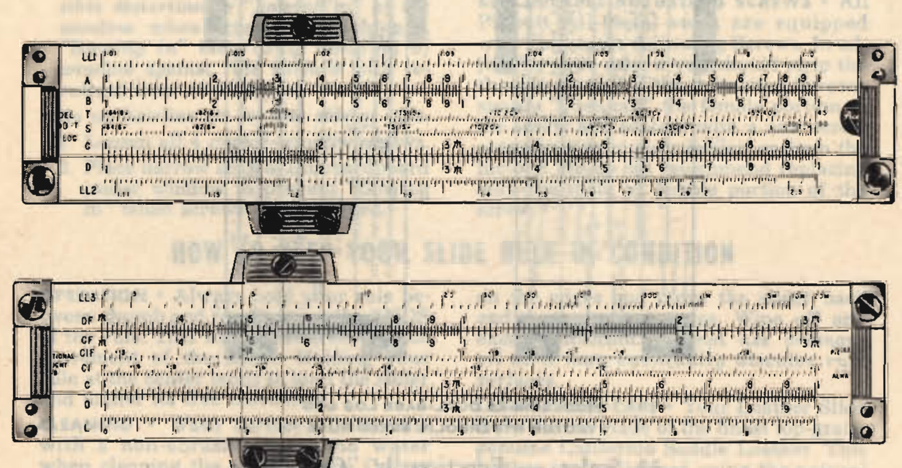
ANSWERS

1. 55.3	2. 145.0	3. 5.17	4. 120.
5. 1589.	6. 659.	7. 129.8	8. 2.038
9. 3.05	10. 883.	11. 1585.	12. 0.0246
13. 4.12	14. 3.79	15. 36.55	16. 0.0292
17. .1102	18. 18.	19. 2.888	20. .1856
21. 234.	22. 3.28	23. 3690.	24. 453.
25. 1.354	26. 2.920	27. 6.280	28. 4.325
29. 56.0	30. 198,000.	31. .384	32. 0.0954
33. 91.4	34. .407	35. .348	36. 2.70+
37. 2.844	38. .1252	39. .664	40. 41.3
41. 28.0	42. 17.2	43. 3.34	44. 50.8
45. 1.00612	46. 0.416	47. .9576	48. .158
49. 3.31	50. 1.35	51. 1.003	52. .063
53. .9637	54. 11.4	55. 0.0275	56. 0.659
57. 0.921	58. 1.146	59. 5.26	60. 1.0216
61. 0.00126	62. 0.0191	63. .1505	64. 9.99849–10.
65. 9.9361–10	66. 7.62–10	67. .0029	68. 0.0584
69. .916			



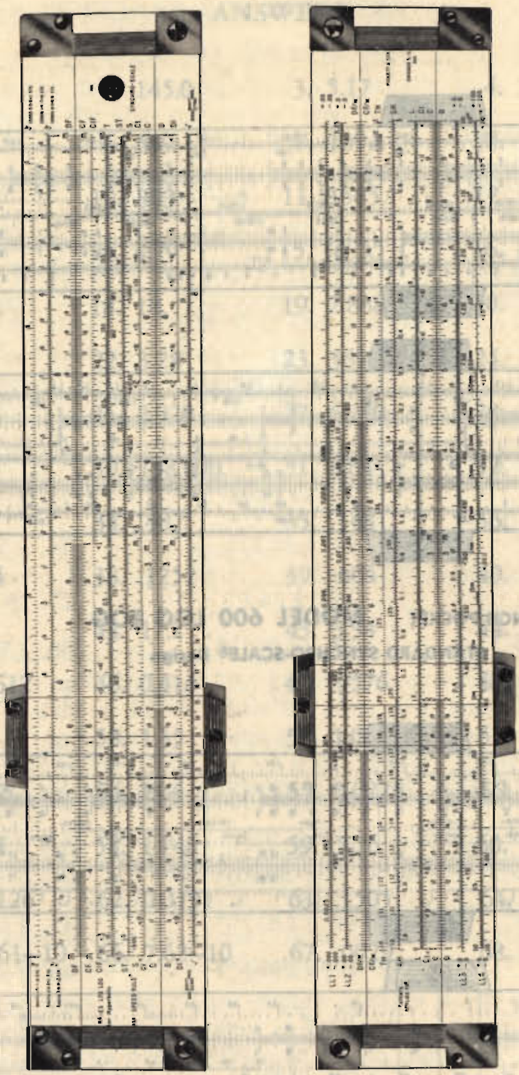
6-INCH POCKET **MODEL 600 LOG LOG**

STANDARD SYNCHRO-SCALE® Design



6-INCH POCKET **MODEL 300 LOG LOG**

TRADITIONAL Log Log Scale Design



MODEL N4-ES DUAL-BASE LOG LOG
VECTOR HYPERBOLIC SPEED RULE

33 Scales — Functionally "Grouped"

Logarithms to Base and Base 10, same setting

Range: 10^{-10} to .9977 and 1.0023 to 10^{10}

3-place Extended Roots Scales:
30-inch Cube Root Scales
20-inch Square Root Scales

HOW TO ADJUST YOUR SLIDE RULE

Each rule is accurately adjusted before it leaves the factory. However, handling during shipment, dropping the rule, or a series of jars may loosen the adjusting screws and throw the scales out of alignment. Follow these simple directions for slide rule adjustment.

CURSOR WINDOW HAIRLINE ADJUSTMENT

Line up the hairline on one side of the rule at a time.

1. Lay rule on flat surface and loosen adjusting screws in end plates.
2. Line up C index with D index. Then align DF (or A) index with CF (or B) index.
3. Tighten screws in end plates.
4. Loosen cursor window screws. Slip a narrow strip of thin cardboard (or 3 or 4 narrow strips of paper) under center of window.
5. Align hairline with D and DF (or D and A) indices, and tighten cursor window screws. Check to see that *window surfaces do not touch or rub against rule surfaces.*

Note: The narrow strip of cardboard under the window will prevent possible distortion or "bowing in" of the window when screws are tightened. "Bowing in" may cause rubbing of window against rule surface with resultant wear or scratches.

Line up hairline on reverse side of rule.

1. Loosen all 4 cursor window screws.
2. Place narrow strip of thin cardboard under window to prevent "Bowing in" when screws are tightened.
3. Align hairline and indices on first side of rule, then turn rule over carefully to avoid moving cursor.
4. Align hairline with indices and tighten cursor screws.
5. Check to see that window surfaces do not touch surfaces of rule during operation.

SLIDER TENSION ADJUSTMENT • Loosen adjustment screws on end brackets; regulate tension of slider, tighten the screws using care not to misalign the scales. The adjustment needed may be a fraction of a thousandth of an inch, and several tries may be necessary to get perfect slider action.

SCALE LINE-UP ADJUSTMENTS • (1) Move slider until indices of C and D scales coincide. (2) Move cursor to one end. (3) Place rule on flat surface with face uppermost. (4) Loosen end plate adjusting screw slightly. (5) Adjust upper portion of rule until graduations on DF scale coincide with corresponding graduations on CF scale. (6) Tighten screws in end plates.

REPLACEABLE ADJUSTING SCREWS • All Pickett All-Metal rules are equipped with Telescopic Adjusting Screws. In adjusting your rule, if you should strip the threads on one of the Adjusting Screws, simply "push out" the female portion of the screw and replace with a new screw obtainable from your dealer, or from the factory. We do not recommend replacing only the male or female portion of the screw.

HOW TO KEEP YOUR SLIDE RULE IN CONDITION

OPERATION • Always hold your rule between thumb and forefinger at the ENDS of the rule. This will insure free, smooth movement of the slider. Holding your rule at the center tends to bind the slider and hinder its free movement.

CLEANING • Wash surface of the rule with a non-abrasive soap and water when cleaning the scales. If the Cursor Window becomes dulled clean and brighten the surfaces with a small rag and tooth powder.

LUBRICATION • The metal edges of your slide rule will require lubrication from time to time. To lubricate, put a *little* white petroleum jelly (White Vaseline) on the edges and move the slider back and forth several times. Wipe off any excess lubricant. *Do not use ordinary oil as it may eventually discolor rule surfaces.*

LEATHER CASE CARE • Your Leather Slide Rule Case is made of the finest top-grain, genuine California Saddle Leather. This leather is slow-tanned using the natural tanbark from the rare Lithocarpus Oak which grows only in California. It polishes more and more with use and age.

To clean your case and to keep the leather pliable and in perfect condition, rub in a good harness soap such as Propert's Harness Soap.

Pickett & Eckel, Inc.
CHICAGO 5 • ALHAMBRA, CALIF.