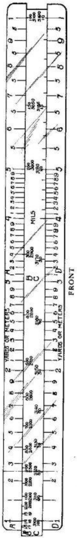
Military Slide Rule.

Used for the Solution of Triangle in Computing Range Target

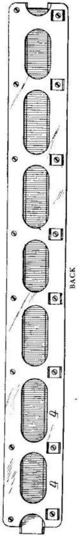
Size 2 in. x 18 in. x ¼ in.

Scales engraved on white celluloid and mounted on Mexican mahogany. The scale members are secured to a $\frac{1}{16}$ in. brass metal back, with seven elliptic holes to reduce weight. The A scale however is provided with eight adjustable clamps (see lower cut of back) to compensate for changes in atmospheric condition and proper alignment of the scales.



FRONT

The slide is graduated in mils on front and back. All Scales have subdivisions not shown in cuts on this page (see figure 2, page 96). This rule adopted by U. S. Field Artillery. Model 1917.



BACK

This rule is constructed very rigid to meet the hard service of which it will be called upon to perform. Price with complete directions $20.00

DIRECTIONS

for using the

Military Slide Rule

By J. J. Clark, M. E. (Lehigh)

Purpose of the Military Slide Rule.—The purpose of the military slide rule is to solve quickly, almost instantly, the problem illustrated in Fig. 1. Here G represents the position of a gun, T is the target, and O is the position of the observer. The distance between G and O is known, and the angles C and A (or C′ and A′) are also known, being readily measured; it is required to find the distance between the gun and target. In other words, given the triangle GTO, in which the side GO and the angles C and A are known, it is required to find the side GT. This problem would be solved by trigonometry in the following manner: represents the side GT opposite the angle A by *a*, the side TO opposite the angle C by *c*, and the side GO opposite the angle B by *b*. Angle B = 180° − (C + A); or, if the angle C′ is known, B = C′ − A, since, as shown in geometry, the exterior angle C′ of a plane triangle is equal to the sum of the two opposite interior angles B and A; whence, C′ = B + A, and B = C′ − A. It is shown in trigonometry that the sines of the angles are proportional to the sides opposite them; in other words, $\frac{a}{b} = \frac{\sin A}{\sin B}$, from which

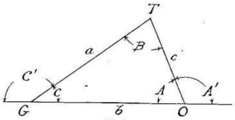


Fig. 1

$$a = \frac{b \sin A}{\sin B}. \quad (1)$$

By means of equation (1), *a* can be readily calculated, but it requires time and a table of trigonometric functions; by the use of the military slide rule, the time is reduced to almost nothing, and the table of trigonometric functions is eliminated. The angles used in computation with the military slide rule are not expressed in degrees, minutes, and seconds, as is customary; instead, they are expressed in mils—a single unit—which greatly facilitates the calculation. Before proceeding further, itis necessary to explain this method of measuring angles.

Measuring Angles in Mils.—The arc of a quadrant is supposed to be divided into 1600 equal parts, each of which is called a mil. Since a quadrant contains 90°, a semicircle, or 180°, contains 2 + 1600 = 3200 mils, and a circle, or 360°, contains 4 × 1600 = 6400 mils. The relations between degrees, minutes, and seconds and mils are easily established. Thus, $1° = \frac{1600}{90} = 17\frac{7}{9} = 17.7778$ mils. Also, 1 mil $= \frac{90 \times 60 \times 60}{1600}$ 202.5 seconds = 3′ 22.5″. Further, 1 minute = 0.2963 mil, and 1 second = 0.00494 mil.

To convert mils into degrees, minutes, and seconds, the easiest way is to proceed as follows: Multiply the number of mils by 200, and add to the product 1/80th of the product; the result will be in seconds, which can then be reduced to degrees, minutes and seconds. Thus, to convert 1444 mils into degrees, minutes and seconds, 1444 × 200 = 288800; 288800 ÷ 80 = 3610; 288800 + 3610 = 292410 seconds = 81° 13′ 30″. The work would be performed as shown in the margin. This conversion is useful in those cases where it is desired to check the results obtained with the military slide rule by making a direct trigonometric calculation. If it is desired to express in mils an angle given in degrees, minutes, and seconds, multiply the degrees by 17-7/9, the minutes by 0.2963, the seconds by 0.00494 and add the several products. Thus, 81° 13′ 30″ = 1444 mils, since 81 × 17-7/9 = 1440 mils; 13 × 0.2963 = 3.8519 mils; 30 × 0.0494 = 0.1482 mil; and 1440 + 3.8519 + 0.1482 = 1440.0001 = 1440 mils.

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       1444
        200
80 | 288800
       3610
60 | 292410"
60 |  4873'  30"
        81°  13'
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Description of Scales on Military Slide Rule.—The military slide consists of two parts—a fixed part, called the **rule**, and a movable (or sliding) part, called the **slide**. The rule (see Fig. 2) contains two scales, the upper one being called scale A and the lower one scale D. These two scales are exactly alike, and the corresponding divisions on each are exactly opposite each other. These scales are divided in practically the same manner as the C and D scales on the ordinary slide rule,

and no special description of them is necessary. To locate a number, such as 387, find the long division mark numbered with the large figure 3; this is the first figure of the number; glancing to the right, note the division mark numbered with a small figure 8; this is the second figure of the number; since there are five spaces between 8 and 9, each interval represents two-tenths of the space between 8 and 9, and each unnumbered division mark within this space represents 2; hence the third mark represents 6 and the fourth represents 8, while half way between represents 7, the third figure of the number. With this explanation no difficulty should be experienced in locating the position of any number on scales A and D.

Scales on the Slide.—The upper scale on the slide is called scale B, and the lower scale is called scale C. These two scales are straight sine scales, but graduated from *right* to *left* instead of from left to right, as in the case of scales A and D. As a consequence of graduating scales B and C in the reverse direction, it follows that when the left-hand division mark on scale B (marked 16, but representing 1600) is placed in line with the left index mark on scale A, the numbers on scales A and D opposite the adjacent divisions on Scales C and B are the reciprocals of the sines of the angles taken on scales C and B, when the angles are expressed in mils; in other words, they are cosecants of those angles. (See paragraph, "To Find the Sine of an Angle".) Thus, the number on D opposite 54 on C is 1885, and the cosecant (abbreviated to csc) of 54 mils is 18.85; the number on A opposite 400 on B is 2613, and csc 400 mils is 2.613. The last reading is obtained as follows: the 400-mark comes about ⅔ ds of the way between 260 and 262 on A; 2 × ⅔ = 4/3 = 1.3; and 260 + 1.3 = 261.3, or 2613.

As before stated, the numbers on scales B and C increase from *right* to *left*, instead of from left to right, as on scales A and D. Therefore, in taking numbers on these scales, after the first figure has been found, the second and subsequent figures for all numbers less than 1600 must be found by going to the *left*. No difficulty should be experienced in locating numbers on these scales. For example, to locate 725 mils, find the 700 mark on B; note that between 700 and 800 there are 10 long marks, and one short mark between two consecutive long ones; hence, the second long mark to the left of 700 represents 2, the second figure of the number, and the next short mark represents 5, the third figure of the number, 725.

It will be noted that there are two sets of numbers printed over the main division numbers; the sum of any two numbers of a set is always 3200. The reason for this is that the cosecant of an angle is equal to the cosecant of its supplement. For instance, csc 27° = csc 180° — 27° = csc 153°; and 153° + 27° = 180°. Since 180° = 3200 mils, csc 700 mils = csc 3200 — 700 = csc 2500 mils, and $700 + 2500 = 3200$. Hence, if an angle is greater than 1600 mils, use the outside row of numbers to determine its position on B or C.

With the slide in the position previously mentioned, that is with 1600 on B opposite the left index on A, note that the right index on D is opposite about 10.2 on C; this is because csc 10.18 (= 10.2 nearly) mils is 100. Note further that the right index on A is opposite 102 on B; this is because csc 102.03 mils = 10. Therefore the cosecant of any number on scale C will have 2 integral places; and the cosecant of any number on scale B will have 1 integral place. Hence, as previously determined, csc 54 mils is 18.85, and csc 400 mils is 2.613. It is well to note that the cosecant of 1600 mils (= 90°) is 1; that it can never be less than 1, and that its value varies between 1 and

For angles between 1600 and 3200 mils, the cosecant increases as the angle increases; consequently, for angles greater than 1600 mils and less than 3200 mils, the readings must be taken from *left* to *right*, as on scales A and D. Thus, csc 2816 mils = 2.715, the mark representing 2816 on scale B coming three-quarters of the way between 270 and 272 on scale A; the space between two short marks here represents 2; hence, $\frac{3}{4} \times 2 = 3/2 = 1.5$; and $270 + 1.5 = 271.5$. Consequently, csc 2816 mils = 2.715.

Scales B and C Form One Scale.—Scales B and C are in reality two halves of a single scale beginning at 10.18 mils on C and extending to 1600 mils on B. In order to keep the length of the rule within reasonable limits, the scale has been divided in the middle, the first half forming scale C and the second half forming scale B. The range of cosecants is, as shown above, from 100 to 1, the lower scale being from 100 to 10, and the upper from 10 to 1.

Hereafter, unless otherwise especially stated, the mil will not be used in connection with angles and cosecants; thus, an angle of 1250 will mean 1250 mils, and csc 358 will mean csc 358 mils.

Index Marks.—The owner of a military slide rule will find it greatly to his advantage to set the slide so that the 1600 mark on B is directly in line with the left index (1) on scale A; then, with a sharp instrument (a scriber or the blade of a penknife) and straightedge, cut a line across the slide from 1 to 1; do the same thing at the other end of the slide. Now, with a sharp lead pencil, go over these lines, so as to blacken them. The left line should pass through and form a continuation of line indicating 1600 mils, and the right line should pass through and be a continuation, the line indicating 102 mils. In the directions that follow, it will be assumed that these two lines have been drawn; the left line will be called the **left index** on the slide or L1S, and the right line will be called the **right index** on the slide or R1S. The lines at the left end of the rule marked 1 will be called the left index on rule or L1R, and those at the right end of the rule will be called the right index on rule or R1R. If it is desired to indicate any particular scale in connection with the indexes, the letter of the scale will be used in place of S and R; thus, L1A means left index on A, R1C means right index on C, etc.

Length of Arc of One Mil.—It is useful to know the relation between the length of an arc of one mil and the radius of the arc; this relation is easily determined. For instance, the length of an arc of a semicircle is $\pi r = 3.1416 \times r$; since a semicircle contains 3200 mils, 1 mil $= \frac{3.1416 \times r}{3200} = 0.000982\,r =$ 0.001 r, very nearly. In other words, an arc of 1 mil is very nearly equal to $\frac{1}{1000}$th of the radius.

To Find the Sine of an Angle.—The **reciprocal** of a number is 1 divided by the number; thus, the reciprocal of 3.1416 is $\frac{1}{3.1416} = 0.3183$. It is shown in trigonometry that the cosecant is the reciprocal of the sine; consequently, the reciprocal of the sine is the cosecant. It is also shown in works treating on the slide rule that if the slide be inverted, that is, turned upside down, and the index marks on the rule and slide are placed in line, the numbers on the rule will be the reciprocals of those on the slide, and vice versa. Hence, if it is desired to use the military slide rule to find the sine of an angle, remove the slide, turn it upside down, and replace it in the rule. Bring the index marks into line with those on the rule; the index mark passing through 1600 will then be in line with R1D, and the numbers on the slide will be upside down. The number opposite 50 on C is 491 on A, and sin 50 = 0.0491; the number opposite 400 on B is 383, and sin 400 = 0.383; etc. The decimal point is easily determined, since as was previously shown, numbers on scale C have two integral places and those on scale B have one integral place; therefore, the reciprocal of a number on scale C will have one cipher between the decimal point and the first digit (a cipher is not a digit), while the decimal point comes immediately before the first digit in the case of reciprocals of numbers on scale B.

The sine may also be found by setting the angle on scale B or C, as the case may be, to one of the index marks on scale A or D; then, opposite the index mark on scale B or C, according to which scale is used, read the sine on A r D. Thus, to find the sine of 50 mils, set 50-C to L1D (or R1D), and opposite R1C (or L1C) read 491 on A or D; hence, sin 50 mils = 0.0491. Similarly, to find sin 400 mils, set 400-B to L1A (or R1A), and opposite R1B (or L1B), read 383 on A or D; hence, sin 400 mils = 0.383. This method is to be preferred when the sine of only one angle is desired.

Restatement of Formula (1).—It was stated above that the reciprocal of the sine was the cosecant; bearing this in mind, Formula (1) may be written as follows:

$$a = \frac{b \sin A}{\sin B} = \frac{b \times \frac{1}{\sin B}}{\frac{1}{\sin A}} = \frac{b \csc B}{\csc A}.$$

For purposes of calculation on the military slide rule, this last expression may be written

$$a = \frac{b}{\csc A} \times \csc B \qquad (2)$$

The sequence of operations is: divide the distance b by csc A and multiply the quotient by csc B; the result is the value of *a*, which will be obtained with a single setting on the military slide rule. The following examples will illustrate the process. The reader is advised to read very carefully the explanations and to work out each example on his rule, in accordance with the directions.

Example 1.—Referring to Fig. 1, b = 200 yards, A = 1440, and B = 1260; what is the range *a*?

Solution.—Bring 1440-B (this means 1440 on scale B) to line with 2-A (= 200-A); opposite 1260-B, read 209 on A. Hence, *a* = 209 yards.

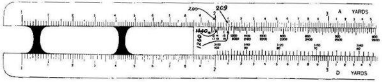

Fig. 2

This setting is shown in Fig. 2. Note that between 1200 and 1300, and between 1300 and 1400, the spaces between the long division lines is divided into 4 parts; whence, each small space represents 25. Therefore, 1260 will be 10 ÷ 25 = 0.4 of the space to the *left* of the second short mark that lies to the *left* of the 1200 mark on B. These two spaces are the only ones divided into 4 parts. The position of 1440 on B lies 40 ÷ 50 = $\frac{4}{5}$ths of the space between the 1400 mark and the next short mark to the left.

Example 2.—When b = 300, A = 1060, and B = 260, what is the range *a*?

Solution.—Set 1060-B to 3-A (= 300-A); note that 260-B is beyond R1A, and does not come opposite any number on A. To provide for this case, take the reading on A (or D) opposite L1S, in this case 159, and shift the slide to the left until R1S comes opposite 159-A (or 159-D); then opposite 260-B read 1025 on A. The range *a* is then 1025 yards, if b is in yards.

Locating the Decimal Point.—In practice, the position of the decimal point will usually be evident, that is, it can be determined by observation whether the value obtained for the range is reasonable or whether it is 10 times too long or too short. It is, however, an easy matter to determine definitely the position of the decimal point without regard to practical considerations, as will now be shown.

The product of any two numbers will contain as many integral places as there are integral places in the two numbers (factors) or that sum less 1. For example, 34.2 × 67.45 = 2306.79, and 2 + 2 = 4, there being 2 integral places in each factor and 4 in the product. Again, 142 × 67.45 = 9577.9, and 3 + 2 − 1 = 4, there being 3 integral places in the first factor and 2 in the second; the sum is 5, and 5 − 1 = 4, the number of integral places in the product. In order to determine whether to subtract 1 or not, note whether the slide projects to the right or left; that is, whether R1S is to the right or left of R1A (or R1D); if it projects to the **right**, subtract 1; otherwise, simply add, as in the first case.

The number of integral places in the quotient is equal to the number of integral places in the dividend minus the number of integral places in the divisor or that difference plus 1. For example, 2306.79 ÷ 34.2 = 67.45, and 4 − 2 = 2, there being 4 integral places in the dividend, 2 in the divisor, and 4 − 2 = 2 in the quotient. Again, 9577.9 ÷ 142 = 67.45, and 4 − 3 + 1 = 2 there being 4 integral places in the dividend, 3 in the divisor, and 4 − 3 + 1 = 2 in the quotient. In order to determine whether or not to add 1, note whether the slide projects to the right or left; if it projects to the **right**, add 1; otherwise, simply subtract, as in the first case.

Referring to example 1, the first operation is to divide 200 by the cosecant of 1440; the slide projects to the right, and as previously pointed out, any number on scale B contains 1 integral place; there will be, therefore, 3 − 1 + 1 = 3 integral places in the quotient. The second operation is to multiply this quotient by csc 1260, which contains 1 integral place, as it comes on scale B. The slide projects to the right and the product contains 3 + 1 − 1 = 3 integral places.

Referring to example 2, the first operation is to divide 300 by csc 1060; the slide projects to the right, and the quotient contains 3 − 1 + 1 = 3 integral places. Multiplying this quotient by csc 260, the slide projects to the left, and the product contains 3 + 1 = 4 integral places.

In practice, write down the number of integral places in b, subtract the number in csc A, add 1 if the slide projects to the right in making the setting, add the number of integral places in csc B, subtract 1 if the slide projects to the left when taking the reading on A or D. Thus, for example 1, 3 — 1 + 1 + 1 — 1 = 3 integral places in *a*; and for example 2, 3 — 1 + 1 + 1 = 4 integral places in *a*.

Example 3.—When b = 300, A = 2456, and B = 544, what is the range *a*?

Solution.—Here 2456 comes on B; setting it opposite 3 (= 300) on A, find 393 on A opposite 544 on B. The slide projects to the right for both operations; hence, there are 3 — 1 + 1 + 1 — 1 = 3 integral places in the result, and the range is 393.

Example 4.—When b = 200, A = 82, B = 724, what is the range?

Solution.—Here A comes on scale C and B on scale B. Set 82-C opposite 2 (= 200) on D; opposite 724-B read 2466 on A. The slide projects to the right for both operations, and since csc 82 contains 2 integral places, the number of integral places in the result is 3 — 2 + 1 + 1 — 1 = 2; therefore the range is 24.66.

It may here be remarked that the range will be in the same units as the distance b, which may be measured in feet, yards, meters, or any other convenient unit.

Example 5.—When b = 400, A = 77, B = 235, what is the range?

Solution.—Set 77-C opposite 4 (= 400) on D; since 235-B lies beyond R1A, it is necessary to shift the slide. The number on A opposite L1S is 321; hence, move slide to left until R1S comes opposite 321 on A or D; the number on A opposite 235-B is 1321. The slide projects to the right in division and to the left in multiplication; therefore, the number of integral places in the product is 3 — 2 + 1 + 1 = 3, and the range is 132.1.

Example 6.—When b = 200, A = 800, B =12, what is the range?

Solution.—Set 800-B opposite 2 (= 200) on A; since 12-C lies beyond R1D, shift the slide and read 12 on D opposite 12-C. Here the slide projects to the right in division and to the left in multiplication; hence, the number of integral places in the result is 3 — 1 + 1 + 2 = 5, and the range is 12000.

Example 7.—When b = 200, A = 60, B = 20, what is the range?

Solution.—Set 60-C opposite 2 (= 200) on D; opposite 20-C, read 6 on D. The number of integral places in the result is 3 — 2 + 1 + 2 — 1 = 3, and the range is 600. Here the slide projects to the right for both operations, and the cosecants of both angles contain 2 integral places.

Example 8—When b = 1500, A = 63, and B = 30, what is the range?

Solution.—Set 63-C opposite 15 (= 1500) on D; opposite 30-C read 315 on D. Here the slide projects to the left for both operations; hence, the number of integral places in the range is 4 — 2 + 2 = 4, and the range is 3150.

Example 9.—What is the range when b = 200, A = 2840, and B = 3125?

Solution.—Setting 2840-B opposite 2 (= 200) on D, it is found that 3125-C falls to the left of L1D; hence, it is necessary to shift the slide to the right. The number on D (or A) opposite R1S is 6925; bringing L1S to 6925-D, the number on D opposite 3125-C is 94. Since the slide projects to the left in division and to the right in multiplication, the number of integral places in the range is 3 — 1 + 2 — 1 = 3, and the range is therefore 940.

Shifting Slide without Using Index Marks.—If the lines have not been drawn across the slide as previously described, the slide may be shifted by making use of the 100 marks on B and C. Placing the slide so that 100-C is in line with L1D, 100-B will be in line with R1A; that is, the distance between 100-C and 100-B is equal to the distance between L1R and R1R, as it should be. Therefore, when it is necessary to shift the slide, note the reading on the rule opposite one of the 100 marks, and then shift the slide until the other 100 mark comes opposite the same reading on the rule. Thus, in example 9, when 2840-B is set to 200-D, 100-B comes opposite 706-A; hence, shift the slide until 100-C comes opposite 706-D, and the number on D opposite 3125-C is 94, as before.

Scales on Back of Slide.—If the slide be turned over, it will be seen that there are two scales on the under side, or back. These scales are exactly the same as the B and C scales on the face, with the exception that they are numbered differently, and are used when the angle is greater than 3210 mils. These scales are not really necessary, since when the angle is greater than 3200, all that is required is to subtract 3200 from the angle. However, by using the scales on the back of the slide, this operation of subtraction is avoided.

To understand how an angle can be greater than 3200 (= 180°), refer to Fig. 3. An angle in surveying and in field operations is always measured in one direction—*counter-clockwise.* Thus, if the instrument is pointing from O towards A and it is desired to find the angle which OB makes with OA, the instrument will be turned counter-clockwise to the position OB, and the angle measured; suppose it equals 850. If, however, the instrument were pointing in the direction OB and it was desired to find the angle BOA, which AO makes with BO, the instrument would still be turned counter-clockwise until it pointed from O towards A, and the angle through it turned would be 6400 — 850 = 5550. In other words, the angle AOB is 850, but the angle BOA is 5550.

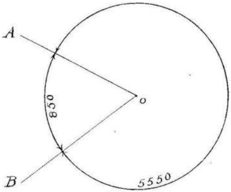

Fig. 3

Index Marks on Back of Slide.—If desired, the index marks may be scribed on the back of the slide in the same manner as on the front. Place 4800-B (note that 4800 = 1600 + 3200) in line with L1A; then with a straight-edge and sharp instrument, draw lines across the slide that will be continuations of the right and left index marks on the rule, and blacken with a sharp lead pencil, as before. If these lines are not drawn, use two 3300 marks when shifting the slide. Note that 3300 = 100 + 3200. As these two scales are used in exactly the same manner as those on the face of the slide, no special instructions regarding their use are necessary.

Multiplication and Division.—Multiplication and division of ordinary numbers may be performed on the military slide rule, though not readily. For this purpose, use scales A and B, the face of the slide being up.

Example 10.—Find the product of 24.3 and 368.

Solution.—Place the index marks on the rule and slide in line. Opposite 243-A, read 2767 on B. (Use the higher numbers on B, as both scales then read the same way—from left to right.) Now move the slide until L1B comes opposite 368 (the other factor) on A, and opposite 2767, the number previously noted, read 894 on A. To locate the decimal point, apply the rule previously given. Here the slide projects to the right; hence, there are $2 + 3 - 1 = 4$ integral places, and $24.3 \times 368 = 8940$. The exact product is 8942.4.

Example 11.—Find the product of 57.3 and 3.14.

Solution.—With the index marks in line, the number on B opposite 314-A is 2870. Bring R1B to 573 A, and the number on A opposite 2870-B is 18. Since the slide projects to the left, the number of integral places in the product is $2 + 1 = 3$, and $57.3 \times 3.14 = 180$. The exact product is 179.922.

Example 12.—Find the value of $180 \div 57.3$.

Solution.—With the index marks in line, the number on B opposite 573-A is 30214. Bring 30214-B to line with 18-A, and the number on A opposite R1B is 314. Since the slide projects to the left, the number of integral places in the quotient is $3 - 2 = 1$, and the quotient is 3.14.

Example 13.—Find the value of $8940 \div 368$.

Solution.—With the index marks in line, the number on B opposite 368-A is 2920. Bring 2920-B to line with 894-A, and the number on A opposite L1B is 243. Since the slide projects to the right, the number of integral places in the quotient is $4 - 3 + 1 = 2$, and the quotient is 24.3.

It will be noted that in example 1, setting 1440-B against 200-A means dividing 200 by csc 1440, and the quotient, 1975, is on A opposite L1B. To multiply this quotient by csc 1260, the slide is already in correct position, and all that is necessary is to read the number on A opposite 1260-B.