SUPPLEMENTARY INSTRUCTIONS

The "Unique" Dualistic, High Speed, Slide Rule (Patented)

(The "Half-Hour" Instructions issued with every "Unique" Slide Rule apply generally to the Dualistic rule, but some modifications are necessary due to the different scale arrangement. All references in the "Half-Hour" instruction to the A and B, Sin, Tan and Reciprocal scales, must be disregarded, as these scales are not included in the Dualistic Slide Rule. This supplementary instruction refers specifically to the 10″ rule, but the "Dualistic" is now being produced in 5″ size, one 5″ model being complete with the three section Log-Log scale and another 5″ model with the C—D and P—Q scales only. In these 5″ models the C—D scales are 5″ long and the P—Q scales are 10″ long. This instruction applies to the 5″ rules without any modification other than the reference to the lengths of the scales.)

The word Dualistic (meaning dual purpose) has been chosen since this new model is a combination of two rules. It has many advantages over the standard rule and should make a strong appeal to those who are familiar with the ordinary type of rule.

Apart from work of a specialised nature, probably 90 per cent. of the computations effected by Slide Rule involve the use of the C and D scales only. In the Dualistic rule these scales occupy their usual positions. They are designated by the symbols C_1 and D_1 and lie along the lower edge of the slide, and on the adjacent edge of the stock, respectively.

The upper margin of the slide, and the edge of the stock adjacent to it, are equipped with modified C and D scales. These are designated by C_2 and D_2 respectively, and are used in conjunction with the C_1 and D_1 scales, as described below.

The extreme margins of the stock, and the centre of the slide are provided with a pair of 20″ scales. These will be recognised as the principal scales of the well-known "Unique" 10/20 Rule. They may be used separately; they are equivalent to a 20″ rule and give the same high degree of accuracy. These scales may be used in conjunction with the C_1 and D_1 scales as will be demonstrated presently.

On the reverse of the slide three scales, LL_1, LL_2 and LL_3 will be found; these are three sections of a continuous log-log scale, extending from 1·01 to 40000, and are used with the slide inverted in conjunction with the D_1 scale.

C_1 AND D_1 SCALES.

As stated above, these are the C and D scales of the standard type slide rule. The directions given in the "Half-Hour" instruction booklet, dealing with the operations of multiplication and division, apply without modification, except that C and D should be read as C_1 and D_1 respectively. In all cases when a calculation involves the use of the C and D scales only, the standard slide rule, or the Dualistic rule, may be used without any discrimination.

C_2 AND D_2 SCALES.

On inspection, it will at once be seen that these two scales are divided in the same manner as the C_1 and D_1 scales, but they are placed differently on the rule. The 1 of the scales C_2 and D_2 is in the middle of the length of the rule. The scales commence at π at the left-hand end of the rule, the readings increase reaching 10 (or 1) at the mid-point, and then increase reaching π at the right-hand extremity of the rule.

Scales C_2 and D_2 should not be used alone for multipliaction and division. A few simple examples will at once show that, although multiplication or division may be effected with their aid, frequently the result is off the scale at the first setting, and cannot be obtained without traversing the slide through its own length. The same dilemma sometimes arises with the C_1 and D_1 scales when multiplying, but the traversing of the slide is rather more easily effected. In any case, there is no advantage gained by using C_2 and D_2 in preference to C_1 and D_1, and since all slide rule users are familiar with the C_1 and D_1 scales, it is advisable to adhere to them. To illustrate this point the reader is asked to compute 6×4 using C_2 and D_2 scales. On setting 1 C_2 to 4 D_2 it will be found that the 6 C_2 lies beyond the D_2 scale at the left-hand end. The result may be obtained by traversing the slide. Set the cursor index (in future referred to as X) to the π near the right-hand end of C_2, and move the slide to bring the π at the left-hand end of C_2 under X. Immediately above 6 C_2 will be found the result, 24 in D_2.

SCALES C_1, D_1, C_2 AND D_2 USED IN CONJUNCTION.

Computations involving multiplication and/or division, are more rapidly effected when using these four scales than when C_1 and D_1 only are employed, and for this reason the term "high speed" is incorporated in the name given to the rule. The following example is designed to illustrate this feature :— Compute the value of $\frac{3{\cdot}1 \times 6{\cdot}4 \times 9{\cdot}2}{1{\cdot}5 \times 11{\cdot}2}$. Using C_1 and D_1 only, and performing division and multiplication alternately since this saves times (see example on pp. 2 and 3 of "Half-Hour Instructions"), the following operations are required :—

Set 15 C_1 to 31 D_1 ; X to 1 C_1 ; 10 C_1 to X ; X to 64 C_1 112 C_1 to X ; X to 1 C_1 ; 10 C_1 to X. Result 109 in D_1 immediately under 92 C_1. Approximation, performed mentally, shows the answer is of the order 10, and the result therefore is 10·9.

Using the four scales :—

Set 15 C_1 to 31 D_1 ; X to 64 C_2 ; 112 C_2 to X
Result—109 in D_2 above 92 C_2.

If the rule is used to carry out these two series of operations, it will be found that by using all four scales the number of movements of slide and cursor is greatly reduced, and the actual distances through which the slide and cursor are moved in these various operations are very much smaller. Greater accuracy will be attained, because in the course of time, all slide rules develop small errors in their scales due to shrinkage or other distortion, and scales which originally were identical, differ slightly in length. Critical inspection will almost invariably show that in slide rules which have been in use for some time, the overall lengths of scales on slide and stock differ slightly. With such a rule, imagine multiplication of 12 $\times$ 4 is being effected using C_1 and D_1 scales. Set 1 C_1 to 12 D_1 and the result—48—appears in D_1 under 4 C_1. If the slide scale has, through shrinkage, become, say, slightly shorter than the stock scale, a small error will be seen, the 4 C_1 falling just below the 48 D_1. Using the Dualistic rule Set 1 C_1 to 12 D_1, and read the result—48—in D_2 over 4 C_2. With this setting the length of slide scale used is only about 1″, and the error will be only about one-sixth of that involved in using the C_1 and D_1 scales, where the length of slide employed is about 6″. The same argument applies to any series of operations.

The principle involved in using the four scales of the Dualistic rule is the same as that employed in slide rules generally. Multiplication and division are effected by adding or subtracting logarithms, but with two sets of scales available, there are alternative scale readings provided, and the manipulation of the rule is easier and speedier than in the case of the standard slide rule.

In using the Dualistic rule the first factor is selected in the D_1 or D_2 scales. The choice of scales is unrestricted, but it is an advantage to start with that scale in which the first factor lies near the middle of the length of the rule. If the first factor lies between 2 and 6 use the D_1 scale, but if it lies between 6 and 2 start in the D_2 scale. For the factors used subsequently there are alternative scale readings, and the one lying nearest should be used. An example will make this selection of factors clear.

Example.—Evaluate $3 \times 1{\cdot}2 \times \frac{9}{4} \times 2{\cdot}5$.
Set 1 C_2 to 3 D_1 (using X) X to 12 C_2 4 C_1 to X X to 9 C_2
1 C_2 to X X to 25 C_1 Result in D_1 under X is 20·2

It will at once be noticed that the movements of slide and cursor are small compared with those necessary if the C_1 and D_1 scales are used alone.

Whether the final result appears in D_1 or D_2 depends upon which scales, C_1 or C_2, were used for the intermediate factors ; the determination presents no difficulty. Very often, especially in short computations, the order of the result is already known, and the slide rule is used to obtain an accurate figure. In such cases it is only necessary to glance at the results lying in D_1 and D_2 under X. These values differ in the ratio of $\sqrt{10}$ to 1, i.e., about 3·16 to 1. In such a case the appropriate reading will be obvious.

In longer computations, when the result is not obvious, a rough approximation should be made to determine the position of decimal point (see "Half-Hour Instruction," p. 3). This approximation will also disclose in which of the two scales D_1 or D_2 the result lies.

The following method for determining in which scale the result lies may be preferred, and the reader is advised to spend a few minutes making himself familiar with it, since it applies also to the 20″ scales which will be dealt with later. The method may, at first reading, sound complicated. It is, in fact, very easy of application.

Every multiplication, or division, or combined multiplication and division, involves using two factors in the C scale. In a multiplication the 1 (or 10) C is set to some value in D, and the result found in D opposite the multiplying factor in C. In division the divisor in C is first set, and the result read opposite the 1 (or 10) C, and in multiplication/division, the divisor in C is set and the quotient obtained opposite another factor in C. In applying the method—which we believe to be original—it is only necessary to observe whether the two factors are both in the same C scale, or whether one is in C_1 and the other in C_2. If the two factors are selected in different sections of the C scale, the result is obtained by crossing from D_1 to D_2, or vice versa ; if both factors lie in the same part of the C scale, the result will be found in that part of the D scale in which the number being multiplied or divided appeared.

A simple example may assist. Suppose multiplication of 8×3 is desired. There are six different ways of obtaining the result, 24, they are :—

(*a*)	Set	1 C_2 to 8 D_2.	Result in	D_2	opposite	3 C_2.
(*b*)	„	1 C_2 to 8 D_2.	„ „	D_1	„	3 C_1.
(*c*)	„	10 C_1 to 8 D_1.	„ „	D_1	„	3 C_1.
(*d*)	„	10 C_1 to 8 D_1.	„ „	D_2	„	3 C_2.
(*e*)	„	1 C_2 to 8 D_1.	„ „	D_2	„	3 C_1.
(*f*)	„	10 C_1 to 8 D_2.	„ „	D_1	„	3 C_2.

the Cursor index X being used in setting where necessary.

In the first and third settings of the slide the two factors 1 and 3 lie in the same section of the C scale, namely, both in C_2, in the first, and both in C_1 in the third method. In both the result lies in the section of the D scale in which the factor 8 was chosen. In the other four methods the factors 1 and 3 lie in opposite sections of the C scale, and the result is always in the opposite section of the D scale from that in which the first factor 8 was selected.

In the simple example just cited it is easy to determine in which part of the D scale the result will be found, but in a longer one it is advisable to record the various operations as now suggested. **When the first slide setting is made, note which section of the D** scale is used and jot down D_1 or D_2 as the case may be. If in the next operation the two factors used are in the same section of the C scale, take no further notice of them, but if they are in different sections of the C scale write a stroke thus, /, following the D_1 or D_2. Proceed in this way, making a stroke each time scale C_1 and C_2 are both used in any one setting of the slide, the second stroke cancelling the first by changing it into a $\times$, so the record starting with, say, D_1, would next become D_1 /, and then $D_1 \times$. At the end of the computation the record will finish either with D_1, or D_1 /, or $D_1 \times$. If the last symbol is a stroke, the final result will lie in the D_2 scale ; in other cases it will lie in D_1.

Example.—Evaluate $\dfrac{2{\cdot}8 \times 93 \times 107 \times 46}{18 \times 52 \times 29}$

Set X to 28 D_1 and jot down D_1
„ 18 C_2 to X
„ X to 93 C_1 } „ „ /
„ 52 C_1 to X
„ X to 107 C_2 } „ „ \
„ 29 C_1 to X
„ X to 46 C_1 } No symbol necessary here.
Under X read 472 in C_1, and 1495 in C_2.

The symbols when written down in line result in $D_1 \times$; the indication is that the result is in D_1. Approximation gives 46 and the result is 47·2.

THE 20″ SCALES.

The scales lying along the top and bottom edges of the face of the stock designated by the symbols P_1 and P_2, respectively, together form a 20″ logarithmic scale, and in combination with a similar pair of scales placed in the middle of the slide and designated by Q_1 and Q_2, form the equivalent of a 20″ slide rule.

When a higher degree of accuracy than can be derived from the 10″ C and D scales is desired, the P and Q scales should be used. Inspection of the rule will show the additional dividing which has been made possible by the use of these long scales.

Slide rules equipped with 20″ continuous scales are sometimes seen in design offices. They are awkward to use on account of their increased length, generally about 23″ overall, and they cannot easily be carried in the pocket or attache case. The manufacturers of

"UNIQUE" Slide Rules have coined the term 10/20 Rule to designate the 10″ rule equipped with 20″ scales; the manipulation of such rules is slightly different from the ordinary type, but the more compact design is a distinct advantage.

Multiplication and division are effected by using the P and Q scales and the cursor index, X. The method given earlier for determining whether the final result should be read in D_1 or D_2 may be adopted when there is any doubt as to whether the result appears in P_1 or P_2. This method has already been dealt with fully and need not be repeated. Two examples are now given to illustrate the use of the 10/20 scales.

Example.—Evaluate $\dfrac{13{\cdot}65 \times 23{\cdot}4}{39{\cdot}6}$

Set X to 1365 P_1 396 Q_2 to X   X to 234 Q_1

Result is 807 in P_2.

The value in P_1 under X is 255 and it is obvious that this result is incorrect. 396 appeared in Q_2 and 234 in Q_1, therefore the result must be in P_2, since the first factor, 1365, is in P_1. It is quite unnecessary to write down the symbols, but if, for illustration only, we do so, they will be P_1 /. The stroke at the end indicates the final result is in the opposite scale to that in which the first factor, 1365, was found: 13 into 39 is 3, and 3 into 24, gives 8, as an approximate result. Now, with two values under X, 807 and 255, there is no difficulty in selecting the correct one and at the same time inserting the decimal point. Result—8·07.

A longer example is now given:—

Evaluate $\dfrac{4{\cdot}4 \times 69{\cdot}2 \times 24{\cdot}6 \times 1{\cdot}246 \times 36}{15{\cdot}1 \times 82{\cdot}2 \times 18{\cdot}6 \times 28{\cdot}1}$

Set X to 44 P_2. Note down P_2

151 Q_1 to X
X to 692 Q_2 } „ „ /

822 Q_2 to X
X to 246 Q_1 } „ „ \

186 Q_1 to X
X to 1246 Q_1 } No symbol required here.

281 Q_1 to X
X to 10 Q_2 } „ „ /

1 Q_1 to X
X to 36 Q_2 } „ „ \

The symbols written in line should appear $P_2 \times \times$, showing the result is in P_2. It is 519.

Approximation.—4·4 into 15 is slightly over 3, which divides into 69 about 20 : 20 into 82, say, 4, 4 into 24 gives 6, 6 into 18 is 3, and 3 into 36 gives 12 : 12 times 1·2 is 14 approximately and we are left with $\frac{14}{28}$ = ·5 as the approximate result. The actual result is, therefore, ·519 ; it lies in P_2, as indicated by the symbols.

The procedure explained in pages 3 and 4 of the "Half-hour" Instruction for determination of position of decimal point, may be used if desired, but we strongly recommend the approximation method as being easier and safer. In a long computation there is a risk that a factor may be inadvertently omitted in the slide rule manipulation. The approximation if carefully made will disclose the error—another sound reason for making it.

The reader is now advised to practise the use of this new rule by working through a few simple examples, the results of which may easily be checked. It is confidently predicted that when familiarity with the scales is attained the rule will make an appeal as being superior to the standard type. The difficulties—if there are any—have now been dealt with and the remaining instruction deals with simple points.

SQUARES AND SQUARE ROOTS.

The relative positions of the 10″ and 20″ scales give a ready means of evaluating squares and square roots. The square of any number is obtained by projecting by means of X, the number from either Q_1 or Q_2 into C_1. For example, 1·6, when projected from Q_1 into C_1, gives 2·56. When projecting from Q_2 to C_1, the squares are 10 times the actual values of the numbers engraved along the C_1 scale. 5 in Q_2 lies immediately above 2·5 in C_1, and this value must be read as 25. Readers familiar with the A, B, C and D scales of a standard rule will notice the similarity in procedure. They will also notice the higher degree of accuracy possible with the longer scales.

Square roots are obtained by the reverse process of projecting from C_1 into Q_1 or Q_2. Square roots of numbers from 1 to 10 are obtained by projection from C_1 into Q_1, and square roots of numbers from 10 to 100 by projection from C_1 into Q_2. When a number whose square root is desired lies outside the range 1 to 100, the rule given on page 4 of the "Half-Hour Instruction" should be used, reading Q_1 for the left-hand half of scale A, and Q_2 for the right-hand half.

Scales P_1, P_2 and D_1 may be used for squares and square roots if preferred ; the procedure will be obvious from the instructions given above.

CUBE AND CUBE ROOTS.

Cubes are easily obtained by setting the 1 (or 10) of C_1 to the number in D_1 ; the cube lies in D_1 immediately below the number in Q_1 or Q_2. To cube 2·2 set 10 C_1 to 2·2 D_1 ; set X to 2·2 Q_1 and read in D_1 under X the result—10·65, the decimal point being inserted by inspection.

Cube roots are evaluated by setting X to the number in D_1 and then moving the slide until the value in D_1 coincident with 1 (or 10) C_1 is the same as the number in Q_1 or Q_2 under X. Suppose the cube root of 2 is required. Set X over 2 in D_1 ; now move the slide about an inch to the right of its mid-position, and then carefully adjust it until the value in D_1 coincident with 1 C_1 is the same as the value in Q_1 under the cursor index X ; this value will be found to be 1·26, which is the cube root of 2 (1·25992).

It may assist to observe that :—

If the number lies within the range 1–10 its cube root will be found in Q_1 and coincident with 1 C_1.

If the number lies within the range 10–31 (π^3) its cube root will be found in Q_1 and coincident with 10 C_1.

If the number lies within the range 31–100 its cube root will be found in Q_2 and coincident with 1 C_1.

If the number lies within the range 100–1000 its cube root will be found in Q_2 and coincident with 10 C_1.

Numbers from 1 to 1000 have cube roots from 1 to 10. If the number whose cube root is required is not within the range 1–1000 it should first be altered by moving the decimal point three, or multiples of three, places to right or left to bring the number within that range. The cube root should then be found as detailed above, and finally, the decimal point of the result should be moved *back* one place for each step of three places made in the original number.

Example.—Find the cube root of 116300.

Moving the decimal point three places to the left alters the figure to 116·3, which lies within the 1 to 1000 range. The cube root of 116·3 is 4·88. The decimal point must now be moved one place to the right giving the actual result as 48·8.

In evaluating cube roots it is a good plan to find the nearest integral result mentally.

Example.—Find the cube root of ·682. First move the decimal point three places to the right, so that the number becomes 682. The cube of 5 is 125, which is well below 682. Try the cube of 7 ; $7 \times 7 = 49$ (say 50) ; $7 \times 50 = 350$, still too small ; try 9 ; $9 \times 9 = 81$; and $9 \times 80 = 720$. The required cube root is less than 9. Set X over 682 D_1 ; and the slide so that 10 C_1 is over 9 D_1. Now move the slide slowly to the left until the reading in D_1 below 10 C is the same as that in Q_2 under X. These identical values are 8·8. The cube root of 682 is 8·8, and of ·682, ·88.

Squares, square roots, cube and cube roots, may be evaluated easily with the aid of the log-log scales, often with a higher degree of accuracy than can be attained with the P and Q scales.

LOG-LOG SCALES.

When the log-log scale is used the slide should be inverted, so that the surface which generally is underneath is brought uppermost, or, if the log-log scale is fitted to a separate slide, the slides should be interchanged. The log-log scale provides a means of effecting unusual computations. *Its most useful property is the ease with which powers and roots may be evaluated, even when the power root is a mixed number.*

Suppose the value of $8{\cdot}4^{1{\cdot}79}$ is required. Set 8·4 LL_3 to 1 D_1, then immediately above 1·79 D_1 will be found the result, 45·1 in LL_3. If the index of the power is negative, e.g., $8{\cdot}4^{-1{\cdot}79}$ the value of $8{\cdot}4^{1{\cdot}79}$ should first be evaluated and the reciprocal of this be found, using the C and D scales ; ie., $8{\cdot}4^{-1{\cdot}79} = \frac{1}{45{\cdot}1} = {\cdot}0222$

To evaluate $\sqrt[4{\cdot}15]{1{\cdot}31}$ set 1·31 LL_2 to 415 D_1, and then use X to project 10 D_1 into LL_1 and read the result, 1·067.

Results outside the range of the log-log scale may be obtained by the methods suggested in page 8 of the " Half-Hour " Instruction.

Logarithms to any base may be obtained by setting the base in LL to 1 of D_1, or 1 of D_2, and projecting the number whose log is required from the log-log scale into D_1 or D_2. The log so obtained will be complete, comprising characteristic and mantissa. Common logarithms are found by setting the 10 LL_3 to 1 D_1. It will be seen that immediately below 100 LL_3 stands 2 D_1, 2 being the log of 100. Below 1000 stands 3 and below 10000 stands 4. To obtain the logs of numbers in LL_1 and LL_2 the cursor index must be used to project into D_1. If the number whose log is required lies towards the left-hand end of the log-log scale, 10 LL_3 should be set to 10 D_1, or 1 D_2.

Natural logarithms are obtained by setting the value 2·7183 near the left-hand end of the LL_3 scale to the 1 of D_1. This setting will enable all the natural logs of numbers within the range of the log-log scale to be read without moving the slide ; the numbers in LL_1 and LL_2 being projected into D_1 by using the cursor index X.

It is useful to remember that the 10th powers of numbers in LL_1 lie immediately below in LL_2, and the 10th power of numbers in LL_2 lie immediately below in LL_3.

The working of a few examples such as those given in pages 6, 7 and 8 of the " Half-Hour " Instruction, will illustrate the uses of the log-log scale and demonstrate its value for certain classes of calculation which would be difficult and sometimes impossible, without its aid. The examples in the " Half-Hour " Instruction are effected by log-log scales on the stock of the rule, whereas in the Dualistic rule the log-log scale is carried by the slide, but the relative positions of the scales in use will be the same for any particular calculations, and the slight change in lay-out should cause no difficulty.

The Dualistic type of Slide Rule is new and represents a distinct advance in the technique of mechanical calculators. It is protected by Patent Rights, and can be obtained only through the agents of the " UNIQUE " Slide Rule Company.

Copies of this supplementary instructions and the "Half-Hour" instructions are supplied free with every Dualistic Slide Rule. Additional copies Sixpence each.